THE

DIARIAN MISCELLANY:

CONSISTING OF

All the Useful and Entertaining Parts, both Mathematical and Poetical, extracted from the

LADIES' DIARY,

From the beginning of that work in the year 1704, down to the end of the year 1773.

With many additional

SOLUTIONS and IMPROVEMENTS.

In five Volumes.

VOL. III.

By CHA. HUTTON, F.R.S.
Professor of Mathematics in the Royal Military Academy.

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W. W. Beman
gt.

THE MATHEMATICAL PARTS OF THE LADIES' DIARIES.

1755.

Eclipses in 1755, calculated by Mr. Ra. Hulse.

THERE will, this year, happen four eclipses; two of each luminary, according to the following order.

The first is an invisible eclipse of the sun, on March 12th, at 10 at night.

The second is a visible eclipse of the moon, on March 27th, beg. 11 h. 17 m. middle 12 h. 34 m. end 13 h. 51 m. total duration 2 h. 34 m. digits eclips'd 7° 7′ on the moon's lower limb.

The third is an invisible eclipse of the sun, on September 6th, at 8 in the morning.

The fourth, and last, is of the moon, and likewise invisible, on September 20th, at 11 in the morning.

Mr. Harland Widd (for Whitby) by tables founded on Sir Isaac Newton's theory, computes the time of the lunar eclipse on March 27th to be as follows. Beginning 11 h. 21 m. middle 12 h. 38 m. ecliptic opposition 12 h. 45 m. end 13 h. 55 m. total duration 2 h. 34 m. digits eclips'd 7° 6′.

Mr. John Ramsay of Morpeth, and Mr. Alexander Rowe of Penzance, also favoured us with calculations of this eclipse.

New Questions.

I. QUESTION 391 *by Miss* Maria Atkinson.

(*Addressed to Mr.* E. P. *who took the liberty to ask her age.*)

Five times seven and seven times three
Add to my age, the sum will be
As many above six nines and four
As twice my years exceed a score:
From hence, sweet Sir, my age explore.

II. QUESTION 392 *by* Sylvius.

A ball, descending by the force of gravity from the top of a tower, was observed to fall half the way in the last second of time: Required the tower's height, and the whole time of descent.

III. QUESTION 393 *by Mr.* G. Brownbridge.

(*Addressed to the ingenious Mr.* C. T——te.)

In vain you hope to live in quiet
By stinting of your vixen's diet;
'Tis five to one that scheme won't do;
But I'll exchange a wife with you,
And, if you please, a daughter too,
If you their ages will make known
From what below is fairly shown.

Given $\begin{cases} x^3 z + x z^3 = 546560 \\ x^4 + z^4 = 1086992 \end{cases}$ x and z being the two ages required.

IV. QUESTION 394 *by Mr.* W. Kingston.

The distance of the centers of two circles, whose diameters are 50 each, being given = 30; 'tis required to find the side of the square inscribed in the intersection, or space common to both circles.

V. QUESTION 395 *by Mr.* Walter Trott.

Sailing on a certain course, we observed a head-land to bear due West from us; four hours after which, it was seen at W. S. W. and six hours after this (we still continuing to run at the same rate) its bearing was found to be S. S. W. What was our course at that time?

VI.

VI. Question 396 by Mr. Tho. Moss.

In the three sides of an equiangular field stand three trees, at the distances of 10, 12, and 16 chains from one another; To find the content of the field, it being the greatest the data will admit of?

VII. Question 397 *by Mr.* J. Ash.

Standing at the end of a visto, at the distance of 50 yards from each of the parallel sides, the opening of the said sides, at a sylvan statue before me, appeared to be $\frac{1}{2}$ of the opening of the further end; the part, or length, on this side the statue being to the part on the other side in the ratio of 1000 to 501: From hence I would know the distance of the statue from the point of observation, and the whole length of the visto?

VIII. Question 398 *by Mr.* Richard Gibbons.

Being on a journey, I took a guide at Modbury for Dartmouth; with whom having travelled 66 minutes, I asked him how far we were come? who replied, Just half so far as we are now from Totness. Having jogged on together seven miles farther, I asked him how far we had now to travel? whose answer was still the same, Just half as far as we are from Totness: These indirect answers I did not like, tho' I found when we arrived at Dartmouth, which we reached in 55 minutes more, that all the fellow had said was strictly true, and that the two roads leading from Totness to Dartmouth and Modbury formed a right angle. From hence I would know the true distances of the three towns, and the rate at which we travelled which was uniform?

IX. Question 399 *by Mr.* H. Watson.

The latitudes and longitudes of three places on the earth's surface, suppose London, Moscow, and Constantinople, being given, as below; required the latitude and longitude of that place which is equidistant from the former three?

The latitude of London is 51° 30′ the latitude and longitude of Moscow 55° 45′ and 38° 0′, and those of Constantinople 41° 30′ and 29° 15′, respectively.

X. Question 400 *by Mr.* Hugh Brown.

A lends B 1000 l. for which B repays him as follows, viz. at the end of three months 180l. of five monrhs 150l. of six months 140l. of eight months 100l. of nine months 90l.

of ten months 120l. and at the year's end 250l. The rate of interest is required?

XI. QUESTION 401 *by Mr.* E. Rollinson.

To investigate the value of an annuity, on a life of a given age, according to any table of observations on the degrees of mortality of mankind, by dividing the whole extent of life into different periods, during which the decrements, or numbers dying off yearly, may be esteemed equal; without having any other series to sum than a common geometrical progression.

XII. QUESTION 402 *by Mr.* W. Bevil.

Suppose the ends of a thread, ten feet long, be fastened to two tacks, in the same horizontal line, at the distance of six feet: I would know where two weights, the one three and the other five ounces, must be fixed to the thread, so as to hang at rest in the same horizontal line at the distance of three feet from the level of the tacks?

XIII. QUESTION 403 *by Mr.* Thomas Moss.

Suppose that, from the top of a mountain, in form of a paraboloid, whose perpendicular height is 600 yards and its base-diameter three miles, a cannon ball is to be discharged with a quantity of powder sufficient to carry it to the height of 700 yards in a vertical direction; I would know the elevation of the piece so that the ball may fall at the greatest distance possible from the place of projection?

XIV. QUESTION 404 *by Mr.* H. Watson.

To determine the center of attraction of a semi-spherical body, or that point in the axis where a corpuscle may be placed to remain in equilibrio by the equal and contrary action of the matter of the hemisphere surrounding it.

XV. QUESTION 405 *by Mr.* Patrick O'Cavanah.

Given $\dot{x}\dot{y} - x\ddot{y} - a\ddot{y} - \frac{x\dot{y}^2}{b} = 0$, to find the general relation of the fluents x and y.

PRIZE QUESTION *by Mr.* E. Rollinson.

Three ships, A, B, C, sail at the same time from three different ports: The ship A, from the southermost port, runs due

due eaſt, at the rate of five knots: The ſhip B ſails S. E. by S. at the rate of three knots, her port lying N. N. E. from that of A, and at the diſtance of twelve miles: The other ſhip, C, always bears down upon (or ſteers directly towards) the two former, which ſhe keeps conſtantly in a line. I would know how far ſhe muſt run before ſhe comes up with B, together with the diſtance of her port from the other two, and the path ſhe deſcribes?

A PARADOX *by* Perpendicularius.

Whatever angle any two right lines can poſſibly form that meet with each other, a third line may nevertheleſs be drawn in ſuch manner as to be a perpendicular to them both.

1756.

Queſtions anſwered.

I. QUESTION 391 *anſwered by Miſs* E. S. *to the Propoſer Miſs* Atkinſon.

I happily congratulate
With you in joys of dear EIGHTEEN.
Oh! might we ever theſe partake,
Nor age, nor trouble intervene:
But years, alas! too ſoon will fly,
And all the gaudy ſcene deſtroy.

The ſame anſwered by Mr. W. Litſon.

By Falſe-Poſition it appears,
Your age, fair Miſs, is eighteen years.

Thus it was alſo anſwered by *Emilia*, Mr. *Joſ. Briſcall*, Mr. *J. Wood*, Mr. *M. Hitchins*, and Mrs. *Priſ. Miſterton*, who ſent the proceſs wrote out at length.

Mr. B. Lydal, *addreſſing himſelf to the Propoſer, anſwers it thus:*

Let x denote your age, then will $56 + x - 58 = 2x - 20$; and conſequently $x = 18$. A very good age for matrimony, Miſs.

In this manner it was also answered by Mr. *J. Beresford*, Mr. *Jos. Briscall*, Mr. *J. Carr*, Mr. *Jos. Dawson*, Mr. *John Dodson*, Mr. *G. Dunn*, Mr. *D. Hastings*. Mr. *T. Knight*, Mr. *G. Langley*, Mr. *Jos. Lord*, Mr. *T. Love*, Mr. *W. Richardson*, Mr. *Jos. Scott*, Mr. *T. Sharp*, Mr. *G. Stapley*, Mr. *J. Taylor*, Mr. *W. Tombs*, Mr. *T. Wilkin*, and many others.

II. QUESTION 392 *answered by Mr.* W. Stoker, *of* Fatfield Staiths.

The square roots of the distances being as the times we have (per quest.) as $\sqrt{1}$ to $\sqrt{2}$, so is the time of falling through the first half, to the time of falling through the whole required height: And therefore as $\sqrt{2}-1$ is to $\sqrt{2}$, so is 1 second (the time of descent through the latter half) to $\frac{\sqrt{2}}{\sqrt{2}-1}$ $(=2+\sqrt{2})=3{\cdot}414$ the time of descent through the whole height: Whence the height itself is found $=$ $187{\cdot}48$ feet.

The same answered by Mr. J. Nichols.

Let $t=$ the whole time of descent, so will $t-1=$ the time of descent through the first half of the tower's height; and therefore (the spaces descended being always as the squares of the times) we have $tt:\overline{t-1}|^2::2:1$; whence $tt-2t+1=\frac{1}{2}tt$: From which $t=2+\sqrt{2}=3{\cdot}414$, and the tower's height $=187{\cdot}48$ feet.

The same answered by Mr. J. Boston.

Let $a=16\frac{1}{12}$ feet (the ball's descent in the first second of time) and $x=$ the seconds the ball was falling; then (by the question and the law of the descent of heavy bodies) the tower's height will be $=ax^2$, and the half of that height $=$ $a\times\overline{x-1}|^2$; hence we have $\frac{1}{2}ax^2=a\times\overline{x-1}|^2$: consequently $x=2+\sqrt{2}=3{\cdot}41421$; and $ax^2=187{\cdot}4806$ feet $=$ the tower's height.

In the very same manner it was answered by Mr. *W. Bacon*, Mr. *S. Bamfield*, Mr. *Ja. Beresford*, Mr. *W. Bevil*, *Birchaverensis*, Mr. *G. Brownbridge*, Mr. *R. Butler*, Mr. *Abr. Botham*, Mr. *Lionel Charlton*, Mr. *Tim. Drury*, Mr. *R. Flitton*, Mr. *P. George*, Mr. *R. Gibbons*, Mr. *E. Griffiths*, Mr. *J. Honey*, Mr. *T. Hopkinson*, Mr. *S. King*, Mr. *T. Peart*, Mr. *W. Phipps*, Mr. *Ja. Robinson*, Mr. *Jos. Scott*, Mr. *J. Shipman*, Mr. *W. Spicer*, Mr. *W. Trott*, Mr. *H. Watson*, Mr. *R. Weston*, Mr. *C. Wildbore*, and Mr. *R. Young*.

III. QUESTION 393 *anſwered by Mr.* Rich. Gibbons.

Put $p = xz$, and $s = xx + zz$, $a = 546560$, and $e = 1086992$: Then, by the queſtion, $sp = a$, and $s^2 = e + 2p^2 = \frac{a^2}{p^2}$. Hence, by completing the ſquare and reduction, p is found $= 448$, and $s\ (= \frac{a}{p}) = 1220$. Conſequently $x + z = \sqrt{s + 2p} = 46$, and $x - z = \sqrt{s - 2p} = 18$, and therefore $x = 32$ and $z = 14$.

The ſame anſwered by Mr. W. Phipps.

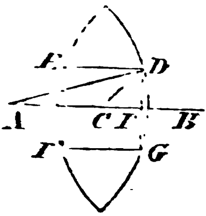
Putting $a = 546560$, and $b = 1086992$, we have, by the firſt equation, $x^2 + z^2 = \frac{a}{xz}$; from the ſquare of which take the ſecond, ſo ſhall $2x^2z^2 = \frac{a^2}{x^2z^2} - b$: whence $x^4z^4 + \frac{1}{2}bx^2z^2 = \frac{aa}{2}$, and conſequently $x^2z^2 = \frac{1}{4}\sqrt{8a^2 + b^2} - \frac{1}{4}b = 20054$; which call m^2, and then $xz = m = 448$; whence, by ſubſtitution, $x^2 + z^2 = \frac{a}{m}$: From which and $xz = m$, we have

$$\left.\begin{matrix} x \\ z \end{matrix}\right\} = \tfrac{1}{2}\sqrt{\frac{a}{m} + 2m} \pm \tfrac{1}{2}\sqrt{\frac{a}{m} - 2m} = \left\{\begin{matrix} 32 \text{ the wife's age.} \\ 14 \text{ the daughter's age} \end{matrix}\right.$$

In this manner it was anſwered by Mr. *Abr. Botham*, Mr. *R. Butler*, Mr. *Lionel Charlton*, Mr. *R. Flitcon*, Mr. *J. Hemingway*, Mr. *J. Honey*, Mr. *Step. King*, Mr. *S. Koit*, Mr. *J. Nichols*, Mr. *C. Tate*, Mr. *T. Wilkin*, Mr. *R. Young*, and ſome others.——Mr. *W. Bacon*, Mr. *J. Eaden*, Mr. *P. George*, Mr. *J. Goodhead*, Mr. *E. Griffiths*, Mr. *W. Stoker*, and ſome others, ſolve it by ſubſtituting for the ſum and difference of the two unknown quantities. Thus, making $v + y = x$, $v - y = z$, $546560 = 2a$, and $1086992 = 2b$, the given equations are reduced to $v^4 - y^4 = a$, and $v^4 + y^4 + 6v^2y^2 = b$; and, by ſubtracting the former from the latter, and putting $b - a = 2c$, there comes out $3y^2v^2 = c - y^4$ or $9y^4 \times v^4\ (= 9y^4 \times \overline{y^4 + a}) = \overline{c - y^4}^2$, or $8y^8 + \overline{9a + 2c} \times y^4 = c^2$: Which equation, ſolved, gives $y = 9$; whence $v = 23$, $x = 32$, and $z = 14$.

IV.

IV. QUESTION 394 answered by Mr. Cha. Tate.



CONSTRUCTION. Let the given distance AB of the two centers be bisected in C; and make the $\angle BCD = \frac{1}{2}$ a right angle; then a perpendicular DP, let fall from the intersection D, will evidently be half the side of the inscribed square $DEFG$.

CALCULATION. The radius AD being drawn, in the $\triangle ACD$ will be given $AD = 25$, $AC = 15$, and the $\angle ACD = 135°$; whence DAC is found $= 19° 54'$, and from thence $DP = 8·5078$, the double of which 17·0156, is the side of the square.

In this manner it was constructed and solved by Mr. *P. George*, Mr. *R.* [illegible], Mr. *J.* [illegible], Mr. *T. Moss*, Mr. *J.* [illegible], Mr. *J. Stephens*, and Mr. *Abr. Stone*.

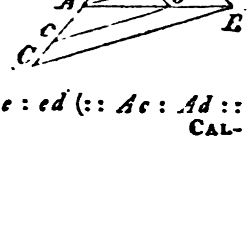
The same answered algebraically by Mr. W. Harrison, of Wigan.

Putting the radius $AD = a = 25$, $AC (= BC) = b = 15$, and $CP (= PD) = x$, we have (per Euc. 47. 1.) $\overline{b+x}|^2 + x^2 = a^2$; whence $x^2 + bx = \frac{1}{2}a^2 - \frac{1}{2}b^2$, and consequently $x = \sqrt{\frac{1}{2}a^2 - \frac{1}{4}b^2} - \frac{1}{2}b = 8·5078$: Therefore ED $(= DG) = 17·0156$. *W. W. R.*

In the same manner it was answered by Mr. *J. Boston*, Mr. *G. Brownbridge*, Mr. *D. Hastings*, Mr. *Step. King*, Mr. *S. Koit*, Mr. *B. Lydal*, Mr. *W. Pepps*, and many others.

V. QUESTION 395 *answered by Mr.* Philip George.

CONSTRUCTION. Draw the meridian AN; and, supposing A to be the place of the headland, draw AB, AE, and AD to represent the given bearings of the ship therefrom; in DAC take Ac to Ad in the given ratio of the times, or as 4 to 6, and draw cb parallel to AE, meeting AB in b; then draw bed, to which the ship's course BED (let AB be what it will) must be parallel: Because (by Euc. 6. 4.) $be : ed$ (:: $Ac : Ad$ :: $4 : 6$) :: $BE : ED$.



CAL-

CALCULATION. In the triangle Abc are given all the angles and the side Ac (= 4), to find $AB = 7\cdot391$; and then from Ab, Ad (= 6) and the contained angle dAb, we get Abd (= ABD) = 47° 25′, which is the complement of the course. *W. W. R.*

The same answered by Mr. J. Shipman, *of* Hull.

Let A represent the headland, and AN the meridian; and through A draw the given bearings AB, AE, and AD; make $AC = 4$, $AD = 6$, and draw CB parallel to AE; then from B draw BD, so will ABD be the ship's course from the west. For, because of the parallel lines, $AC : AD :: EB : ED$. From hence the course is found, by calculation, to be N. 42° 35′ westerly.

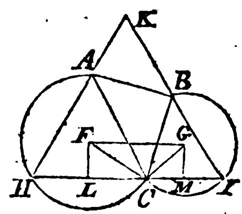
The same answered by Mr. J. Milbourn.

Let A represent the headland, and B, E, and D the places of the ship at the three observations. Put the sine of BAE (22° 30′) = a, and that of DAE (45°) = b; then will $AB = \frac{BE \times \text{sin.}\,E}{a}$, and $AD = \frac{DE \times \text{sin.}\,E}{b}$ (by trig.): whence we have $AB : AD \left(:: \frac{BE}{a} : \frac{DE}{b} :: \frac{4}{a} : \frac{6}{b}\right) :: 4b : 6a$. From which known ratio of the sides, and the included angle BAD, the course ABD, from the west, is found to be 47° 25′.

Mr. *G. Dunn*, Mr. *R. Gibbons*, Mr. *D. Hastings*, Mr. *T. Moss*, Mr. *T. Peart*, Mr. *Abr. Stone* (Land Surveyor), Mr. *W. Trott* (the proposer), and two or three others, likewise sent very neat constructions of this problem.

VI. QUESTION 396 *answered by Mr.* W. Bevil.

CONSTRUCTION. The given points A, B, C being joined, upon AC and BC let two segments of circles be described, each to contain an angle of 60°; join their centers by the line FG, and parallel thereto draw HCI cutting the two circles in H and I; then through A and B draw HK and IK, so shall HIK be the triangle required.



For, supposing FL and GM to be perpendicular to HI, and HI (always terminated by the circles)

to revolve about the point C, it is plain that, when HI is a maximum, LM, being the half thereof, must also be a maximum; and this will evidently be when LM is parallel to FG.

CALCULATION. In the triangle ABC are given all the sides, to find the angle $C = 38^\circ\ 38'$; then in the triangle FCG are given the angle $C = 98^\circ\ 38'$, and the ratio of the containing sides, as AC to BC (or 16 to 12); whence is found the angle F $(= HCF) = 33^\circ\ 41'$, and the angle G $(= ICG) = 47^\circ\ 4'$; therefore $HCA = 63^\circ\ 41'$, $ICB = 77^\circ\ 4'$, $CAH = 56^\circ\ 19'$, $CBI = 42^\circ\ 56'$, $HI\ (= HC + CI) = 24{\cdot}7$, and the area $= 26.42 = 26$ acres, 1 rood, 27 perches.

Algebraic Solution to the same by Mr. W. Spicer.

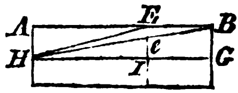
Put $a = BC = 12$, $b = BA = 10$, $s =$ sine of 60°, and p and $-q$ equal to the sine and cosine of $\angle BCI + \angle BAK = 152^\circ\ 51'\ 58''$, and x and y those of $\angle BAK$: Then will $py + qx =$ sine $\angle BCI$; and (per trig.) $s : b :: x : \frac{bx}{s} = BK$; and $s : a :: py + qx : \frac{apy + aqx}{s} = BI$: Therefore $\frac{bx + aqx + apy}{s} = IK$, a max. In fluxions, $b\dot{x} + aq\dot{x} + ap\dot{y} = 0$: But $\dot{y} = -\frac{x\dot{x}}{y}$; whence $b\dot{x} + aq\dot{x} - \frac{apx\dot{x}}{y} = 0$: Solved, $\frac{x}{y}$ $(=$ tang. $\angle BAK) = \frac{b + aq}{ap}$. Hence $\frac{1}{s}\sqrt{aa + 2abq + bb} = IK = 24{\cdot}7002$; and $\sqrt{\frac{1}{3}} \times \overline{aa + 2abq + bb} = 26{\cdot}42$, the area required.

According to the former of the two preceding methods it was also answered by Mess. *Moss* (the proposer), *Peart*, *Rollinson*, *Trott*, and *Watson*; and according to the latter, by Mess. *Botham*, *Charlton*, *George*, *Nichols*, and some others. ——Many contributors have solved this question upon a supposition that one side of the required triangle is parallel to the longest side of the given one; which, though very near, is not strictly true; the said sides, when the area is the greatest possible, being inclined to each other in an angle of $3^\circ\ 41'$.

VII. QUES-

VII. QUESTION 397 *anſwered by Mr.* Abr. Botham.

Suppoſing HG to be the length of the viſto, and I the place of the ſtatue; let t expreſs the tang. of $\frac{1}{3}$ of $\angle EHI$, which (by the queſt.) is $\frac{1}{2}$ of $\angle BHG$; alſo let $a = 1501$, and $b = 1000$: Then the tangents of EHI and BHG, being expreſſed by $\frac{3t - t^3}{1 - 3tt}$

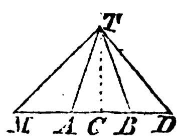


and $\frac{2t}{1 - tt}$, (vide theor. p. 41, Diary 1755) and theſe tangents being to each other in the given proportion of a to b, we therefore have $\frac{3bt - bt^3}{1 - 3tt} = \frac{2at}{1 - tt}$; which, properly reduced, becomes $t^4 + \overline{\frac{6a}{b} - 4} \times t^2 = \frac{2a}{b} - 3$; and this, ſolved, gives $t = \cdot 019987$: From whence the length of the viſto is found $= 1250$ yards.

According to the ſame method it was alſo anſwered by Meſſis. *Aſh* (the propoſer), *Butler*, *Charlton*, *Griffiths*, *Moſs*, *Peart*, *Rollinſon*, *Spicer*, and *Watſon*.——Some very ingenious Gentlemen have taken a good deal of pains, in order to convince us, that this queſtion was not properly limited; and others have hinted the ſame thing; not conſidering, that, from the ratio of any two angles (EHI, eHI) and the ratio of their tangents (which ratios are here given) both the angles and tangents may, always, be determined. ——The ſmallneſs of the angles in the caſe propoſed (where the tangents are nearly in the ſame ratio with them) is, we preſume, what embarraſſed theſe gentlemen: The method of trial-and-error (as it is called) whereby moſt of them attempted the ſolution, being of little or no uſe here.

VIII. QUEST. 398 *anſwered by Mr.* G. Brownbridge.

The places of the three towns being repreſented by M, D, and T, let the three parts of MD (as ſpecified in the problem) be denoted by $6x$, a, and $5x$, reſpectively; then a : $22x$ :: $2x$: $\frac{44xx}{a} = AC - BC$; $\therefore$ $AC = \frac{a}{2} + \frac{22xx}{a}$, $BC = \frac{a}{2} - \frac{22xx}{a}$,



MC

$MC = \frac{a}{2} + 6x + \frac{22xx}{a}$, and $DC = \frac{a}{2} + 5x - \frac{22xx}{a}$. Now $MC \times DC (= CT^2) = AT^2 - AC^2$; whence we have $44x^3 + 184ax^2 - 11a^2x = a^3$, or $44x^3 + 1288x^2 - 539x = 543$; which, solved, gives $x = \cdot 75284$. Therefore $MD = 15\cdot 281$, $MT = 12\cdot 236$, and $DT = 9\cdot 155$.

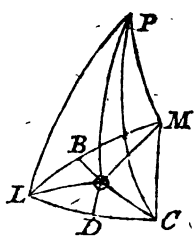
The same answered by Mr. R. Butler.

Supposing M, D, and T to represent the three towns, let $MA = 6x$, $TA = 12x$, $BD = 5x$, $BT = 10x$, and $AB (= 7) = a$: Then (by trig.) $a : 22x :: 2x : \frac{44x^2}{a} =$ diff. segments AC and BC; whence $CA = \frac{a}{2} + \frac{22x^2}{a}$, and $CB = \frac{a}{2} - \frac{22a^2}{a}$. But (by Euc. 12. 2.) $MA^2 + AT^2 + 2MA \times AC + DB^2 + BT^2 + 2BD \times BC (= MT^2 + DT^2) = MD^2$ (by Euc. 47. 1.); that is, in species, $305x^2 + 11ax + \frac{44x^3}{a} = \overline{a + 11x}|^2$, or $44x^3 + 184ax^2 - 11a^2x = a^3$. Whence x is found $= .752846$: Therefore $MD = 15,2813$, $MT = 12,2365$, $TD = 9,1534$, and the rate at which he travelled was 4,1064 miles per hour.

To this question upwards of 30 true answers have been received.

IX. QUESTION 399 *answered by Mr.* H. Watson, *the Proposer.*

Let P represent the pole of the earth, and L, C, and M the three places proposed: From the latitudes and longitudes of which the distances LM, LC, and the contained angle CLM, will (by common proportions) be found 22° 38′, 22° 17′, and 40° 55′, respectively. Then supposing OB and OD perpendicular to LM and LC (O being the required place) we have given $LB (= \frac{1}{2} LM) = 11° 19′$, $LD (= \frac{1}{2} LC) = 11° 8\frac{1}{2}′$, with the sum of the angles BLO, DLO, at the bases (the hypothenuse LO being common); whence it will be (*per Spherics*) sin.



$\overline{LD' + LD}$

$\overline{LB+LD}$: Sin. $\overline{LB-LD}$: : co-tang. $\frac{DLO+BLO}{2}$ ($20^\circ\ 27\frac{1}{2}'$): tang. of $\frac{DLO-BLO}{2}=1^\circ\ 13\frac{1}{2}'$; whence $BLO = 19^\circ\ 14'$; from which, and LB, we have LO ($=MO=CO$) $= 11^\circ\ 58'$, and then, in the triangle LOP, will be given LO, LP, and the angle OLP ($=83^\circ\ 27' = PLM + BLO$); whence $OP = 38^\circ\ 43'$, and $\angle LPO = 19^\circ\ 13'$, being the complement of the latitude, and the longitude from London, respectively.

In the very same manner it was answered by Mr *T. Moss*

Algebraic Solution to the same by Mr. R. Young.

Having found (by common spherics) LB, LD, and the angle DLB, put the tang. of $LB=a$, that of $LD=b$, the sine and co-sine of DLB equal to m and n, and those of BLO equal to x and y, respectively: Then, the co-sine of DLO being expressed by $mx+ny$, it will be, *per spherics*, $\left\{\begin{matrix} y : a :: 1 \text{ (rad.)} : \text{tang. } LO \\ mx+ny : b :: 1 : \text{tang. } LO \end{matrix}\right\}$; whence $\frac{a}{y} = \frac{b}{mx+ny}$; and consequently $\frac{x}{y} = \frac{b-na}{ma} = $ tang. $BLO = 19^\circ\ 14'$: From which the latitude and longitude of the place are found to be $51^\circ\ 17'$, and $19^\circ\ 13'$, respectively.

Mess. *Bamfield*, *Birchoverensis*, *Botham*, *Charlton*, *Hopkinson*, *Spicer*, and some others, have also given very neat algebraic solutions to this problem. Mr. *P. George*, by producing the perpendicular DO to meet the side LM, solves it by common spherics: And Mess. *Peart* and *Rollinson*, after finding LM, LC, and the angle CLM (as above) compute the difference of the other two angles LMC, LCM, which (because $OMC=OCM$) will also be the difference of OML and OCL, or of their equals OLB and OLD; whence both these angles are known, and from thence LO.

X. QUESTION 400 *answered by Mr.* Hugh Brown, *the Proposer.*

Putting $z = \frac{1}{12}$ of the interest of 1l. for one year, we shall (by discounting at simple interest) have $\frac{250}{1+12z} + \frac{120}{1+10z} + \frac{90}{1+9z} + \frac{100}{1+8z} + \frac{140}{1+6z} + \frac{150}{1+5z}$

$+ \frac{180}{1+3z} = 1000$. The fractions reduced into series, and properly ordered, give $z - \frac{3605z^2}{397} + \frac{36133z^3}{397}$, &c. $= \frac{3}{794}$; whence $z = \cdot0037783 + \cdot0001296 + \cdot0000040 = \cdot0039122$: Therefore the required rate is 4·694, or 4l. 13s. 10½d. per cent.

The same answered by Mr. John Honey.

Let x^{12} = rate per annum; then x (according to compound interest, will be the rate for 1 month, x^2 for 2 months, &c. whence the present value of 180l. to be received at the end of 3 months will be $\frac{180}{x^3}$ &c. Hence we have $\frac{180}{x^3} + \frac{150}{x^5} + \frac{140}{x^6} + \frac{100}{x^8} + \frac{90}{x^9} + \frac{120}{x^{10}} + \frac{250}{x^{12}} = 1000$ (per quest.); and therefore $x^{12} - \cdot18x^9 - \cdot15x^7 - \cdot14x^6 - \cdot1x^4 - \cdot09x^3 - \cdot12x^2 = \cdot25$: Solved, $x = 1\cdot003852$; and $x^{12} = 1\cdot047216$ and consequently 4l. 14s. 5d. the rate per cent. required.

In the very same manner it was answered by Mess. *W. Bevil, E. Rollinson*, and *H. Watson*.

Mr. R. Flitcon *answers it thus:*

Let x, $2x$, and $3x$ be the interest of one pound for 1, 2, and 3 months, respectively; then the interest of 1000l. for 3 months will be $3000x$, and its amount $1000 + 3000x$, from which subtracting 180l. then paid, the remainder $820 + 3000x$ will be a new principal; which, in 2 months more, amounts to $820 + 4640x + 6000x^2$; from whence subtracting 150l. then paid, the remainder $670 + 4640x + 6000x^2$ will be the principal (or debt) at the end of 5 months: And by proceeding in the same manner with all the rest of the payments, the equation resulting (according to the question) will, in its least numbers, be $-3 + 758x + 4871x^2 + 14503x^3 + 24016x^4 + 22692x^5 + 11456x^6 + 2400x^7 = 0$. Hence $x = \cdot0038609$, and the rate, per cent. per annum, 4l. 12s. 7¾d.

According to this last method it was likewise solved by Mess. *Bamfield, Butler, Goodhead*, and *Trott*, whose equations exactly agree.——A great number of other solutions have been received, upon principles somewhat different, which however come very near the truth.

XI. QUEST.

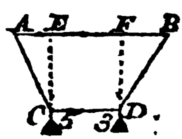
XI. QUEST. 401 *answered by the Proposer, Mr.* Rollinson.

Let a denote the number of the living corresponding, in the table, to the given age; and let the succeeding decrements, or the numbers that die off yearly (for as long as they continue equal) be represented, each, by b; also let the next succeeding decrements (for as long as they continue equal) be denoted, each, by c; and the next after those by d; and so on: Putting r to represent the rate, and P the value of the perpetuity corresponding: Moreover, suppose that, after the decease of the proposed life A, the estate, or annuity, is to go to another person B, and his heirs, for ever. Then the probability that the life A fails the first year being $\frac{b}{a}$, the value of B's expectation on that contingency will therefore be $P \times \frac{b}{a}$: And the probability of A's dropping the second year being also $\frac{b}{a}$, the expectation of B thereon will be $\frac{P}{r} \times \frac{b}{a}$, ($\frac{P}{r}$ being the value of the perpetuity discounted for one year). In the same manner the expectation of B on the third year will appear to be $= \frac{P}{r^2} \times \frac{b}{a}$, and on the fourth year $= \frac{P}{r^3} \times \frac{b}{a}$, &c. Whence it is evident, that B's whole expectancy, on the contingency of the life's failing in the first interval, (m) during which the decrements are equal, each, to b, is truly defined by $rP \times \frac{b}{a} \times : \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} \ldots\ldots + \frac{1}{r^m}$, or its equal $rP \times \frac{b}{a} \times \frac{1 - r^{-m}}{r - 1}$, which is also $= rP^2 \times \frac{b - br^{-m}}{a}$, because $P = \frac{1}{r - 1}$. Again, the decrements during the second interval (n) being each $= c$, the probability of the life's failing in any assigned year of this interval will therefore be denoted by $\frac{c}{a}$; and the value of B's expectation, on the whole interval, by $\frac{P}{r^m} \times \frac{c}{a} + \frac{P}{r^{m+1}} \times \frac{c}{a} + \frac{P}{r^{m+2}} \times \frac{c}{a}$, &c. $= \frac{rP}{r^m} \times \frac{c}{a} \times : \frac{1}{r} + \frac{1}{r^2}$

$+\frac{1}{r^3}\ldots+\frac{1}{r^n}=rP^2\times\frac{c-cr^{-n}}{ar^m}$. In the very same manner the expectancy on the third interval (p) appears to be $=rP^2\times\frac{d-dr^{-p}}{ar^{m+n}}$; and on the fourth ($q$) $=rP^2\times\frac{e-er^{-q}}{ar^{m+n+p}}$, &c. &c. Therefore, by collecting all these values together, the whole expectation of B and his heirs comes out $=\frac{rP^2}{a}\times : b+\frac{c-b}{r^m}+\frac{d-c}{r^{m+n}}+\frac{e-d}{r^{m+n+p}}+$, &c. which, subducted from P, the value of the perpetuity, leaves $P-\frac{rP^2}{a}\times : b+\frac{c-b}{r^m}+\frac{d-c}{r^{m+n}}+\frac{e-d}{r^{m+n+p}}$, &c. for the true value of the life A: But rP is $=P+1$, and $\frac{1}{r^m}$, $\frac{1}{r^{m+n}}$, &c. are the present values of 1 l. to be received at the end of m, $m+n$, &c. years; which being found (from the tables) and represented by M, N, Q, &c. respectively, the value of the annuity will be $P-\frac{PP+1}{a}\times : b+\overline{c-b}\times M+\overline{d-c}\times N+\overline{e-d}\times Q+\overline{f-e}\times R$, &c. where the series is to be continued till it terminates, and where the co-efficient of the last term will be the last of the quantities b, c, d, &c. with a negative sign, the next letter in order being equal to nothing.

XII. QUESTION 402 answered by Mr. S. Bamfield.

Supposing CE and DF to be perpendicular to the horizontal line AB, it is evident, from mechanics, that $AE : BF :: 3 : 5$. If therefore $EC\,(=FD)=b=3$, $AB=a=6$, $AC+CD+DB=c=10$, and $AE=3x$, then will $BF=5x$; $EF\,(=CD)=a-8x$; and $\sqrt{bb+9xx}\,(AC)+\sqrt{bb+25xx}\,(BD)+a-8x=c$. From the resolution of which equation x is found $=\cdot31786$; and from thence $AC=3\cdot1479$, $BD=3\cdot3950$, and $CD=3\cdot3951$.



The

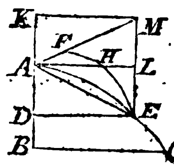
The same answered by Birchoverensis.

Put AB $(=6)=l$, CE $(=3)=b$, $AC+CD+DB$ $(=10)=s$, and $AC=x$; then $AE=\sqrt{xx-bb}$, and, by the property of the center of gravity, $BF=\frac{5}{3}\sqrt{xx-bb}$: Consequently CD $(=EF)=l-\frac{8}{3}\sqrt{xx-bb}$, and $BD=\frac{1}{3}\sqrt{25xx-16bb}$. Whence we have $x+l-\frac{8}{3}\sqrt{xx-bb}+\frac{1}{3}\sqrt{25xx-16bb}=s$. This equation, properly reduced, will give $x^3-3{\cdot}35x^2-10{\cdot}8x+36=0$; where x is found $=3{\cdot}1479$.

Mess. *Bevil* (the proposer), *Botham*, *Moss*, *Peart*, *Rollinson*, *Watson*, and *Young*, sent elegant solutions to this problem.

XIII. QUESTION 403 *answered by Mr.* Tho. Peart.

From the given height of the mountain AB, and its semi-diameter BC, the parameter of the parabola will be found $=11616$ yards; from whence the curve AEC may be constructed. Take $AK=$ twice the height of the perpendicular shot, and draw KM perpendicular thereto; in KM take the point M, so that a perpendicular let fall from thence, meeting the curve in E, shall be equal to the straight line AE; then join A, M, which will be the line of direction.



CALCULATION. Let $d=$ the height of the perpendicular shot, and $a=11616=$ the parameter, $AD=x=LE$; then ME $(=AD+AK)=2d+x$, $DE=\sqrt{ax}$, and $AE=\sqrt{ax+xx}=2d+x$ $(=ME)$; therefore $x=\frac{4dd}{a-4d}=222{\cdot}323$ yards, and AE $(2d+x)=1622{\cdot}323$; from whence the angle AED is found $=7^\circ\ 52'\ 36''$; the half of which being taken from 45°, there remains $41^\circ\ 3'\ 42''$ for the angle of elevation LAM. *W.W.R.*

The same answered by Mr. Walter Trott.

Put the height AB of the mountain $(=600)=a$, the semi-diameter BC of the base $(=2640)=b$, and the dou-

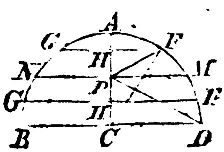
 ble

ble impetus ($= 1400$) $= c$: Then, supposing E to be the place where the ball (whose path is AHE, and first direction AF) impinges on the surface, let AE and ED be drawn, the latter parallel to CB, and let $AD = x$; and then, by the property of the parabola, $DE^2 = \frac{b^2 x}{a}$, and therefore $AE = \sqrt{\frac{bbx}{a} + x^2}$. Hence because the dist. is to be a max. we have (by Simpson's Exercises, p. 199) $\sqrt{\frac{bbx}{a} + xx} - x$ ($= AE - AD = c$; from which $x = \frac{acc}{bb - 2ac} = 222{\cdot}323 = AD$. Whence the angle $EAD = 82^\circ\ 8'$; the half of which ($= 41^\circ\ 4'$) is the true angle of elevation (HAF). *W. W. R.*

Mess. *Botham*, *Moss* (the proposer), *Rollinson* and *Watson* answer it in the same manner. Mess. *Bamfield* and *Charlton* give the investigation by a neat fluxional process, which room will not admit.

XIV. QUESTION 404 *answered by Mr.* Lionel Charlton, *of* Whitby.

Let BAD be the given hemisphere, P the required point, NPM and GHF sections of the solid perpendicular to the axis AC; let PF, CF, and PD be drawn; putting $AC = r$, $PC = a$, $AP = e$, $PM = f$, $PD = g$, $PH = x$, and $PF = z$. Then the effect of the attraction of all the particles in the plane of the circle GHF, on a corpuscle at P, will, it is well known,



be as $1 - \frac{x}{z}$; and consequently the fluxion of the attraction of the segment $NGFM$ as $\dot{x} - \frac{x\dot{x}}{z}$: Which (because $a^2 + z^2 + 2ax = r^2$, and therefore $x = \frac{ff - zz}{2a}$, and $\dot{x} = -\frac{z\dot{z}}{a}$) will be reduced to $\dot{x} + \frac{f^2 - z^2}{2a} \times \frac{\dot{z}}{a}$; whereof the corrected fluent is $x + \frac{3f^2 z - z^3 - 2f^3}{6a^2}$: Which, when

$x =$

$x = e = z$, becomes $e + \frac{3ef^2 - e^3 - 2f^3}{6aa}$ for the force of the upper segment NAM in the direction PA.

In the same manner, taking GF below NM, the fluxion of the attraction of the part $NGFM$ will be, *still*, expressed by $\dot{x} + \frac{f^2 - z^2}{2a} \times \frac{\dot{z}}{a}$, and the corrected fluent by $x + \frac{3f^2 z - z^3 - 2f^3}{6aa}$, which, by taking $x = a$ $(= PC)$ and $z = g$ $(= PD)$ gives $a + \frac{3f^2 g - g^3 - 2f^3}{6aa}$ for the whole force of the lower segment $NBDM$ in the opposite direction PC. Hence we have $e + \frac{3ef^2 - e^3 - 2f^3}{6aa} = a + \frac{3f^2 g - g^3 - 2f^3}{6aa}$, or $6a^2 \times \overline{e - a} + 3ef^2 - e^3 = g \times \overline{3ff - gg}$. But $e = r - a$, $ff = rr - aa$, and $g = \sqrt{rr + aa}$, so that, by substitution, $2r^3 - 8a^3 = \sqrt{rr + aa} \times \overline{2r^2 - 4a^2}$; which, order'd, gives $12a^4 - 8r^3 a + 3r^4 = 0$; where, taking $r = 1$, we have $a = 0.4230428$. Therefore the ratio of AP to CP will be that of 0.5769572 to 0.4230428, or as 4 to 3, nearly.

The author of a very neat solution to this proplem (whom we should be sorry to disoblige by an improper step) will, we hope, candidly excuse our giving preference to the above; as room would not possibly admit of both, and as the fluents, here, may be comprehended by common readers, for whose improvement we are solicitous.

XV. QUESTION 405 *answered by* Mr. T. Moss.

Divide the given equation by $\frac{\dot{y}y}{\dot{x}}$, and you will have $\frac{x^2\dot{y} - x\dot{x}\dot{y}}{\dot{y}y} - \frac{a\dot{x}y}{\dot{y}y} - \frac{x\dot{x}}{b} = 0$; which, by taking the fluent, becomes $\frac{xx}{y} + \frac{ax}{y} - \frac{xx}{2b} = d$ (a constant quantity); whence $y = 2b \times \frac{ax + xx}{2bd + x^2}$, and consequently $y = a\sqrt{\frac{2b}{d}}$.

$\sqrt{\frac{2b}{d}} \times$ arc whose tangent is $\frac{x}{\sqrt{2bd}}$ (to radius 1) $+ b \times$ hyp. log. $\frac{2bd + x^2}{e}$; where d and e may denote any constant quantities at pleasure.

The same answered by Mr. Patrick O'Cavanah, *the Proposer.*

In the given equation $(\dot{x}\dot{y} - x\ddot{y} - a\ddot{y} - \frac{x\dot{y}^2}{b} = 0)$ let $-\frac{\dot{y}\ddot{x}}{\dot{x}}$* be wrote in the place of $\ddot{y}$ (so that $\dot{y}$ may become the quantity flowing uniformly) then will $\dot{x}\dot{y} + x \times \frac{\dot{y}\ddot{x}}{\dot{x}} + a \times \frac{\dot{y}\ddot{x}}{\dot{x}} - \frac{x\dot{y}^2}{b} = 0$, or $\dot{x}\dot{x} + x\ddot{x} + a\ddot{x} - \frac{x\dot{x}\dot{y}}{b} = 0$; whose fluent, it is evident, will be $x\dot{x} + a\dot{x} - \frac{x^2\dot{y}}{2b}$ = some constant quantity, let it be $c\dot{y}$, so shall $\dot{y} = 2b \times \frac{a\dot{x} + x\dot{x}}{2bc + xx}$; and consequently $y = a\sqrt{\frac{2b}{c}} \times A + b \times L$; A being = the arch whose radius is 1, and tangent $\frac{x}{\sqrt{2bc}}$; L = the hyp. log. of $\frac{2bc + xx}{dd}$; and c and d any constant quantities.

Mr. *R. W.* (whom for particular reasons, we should be glad to oblige) will not, we hope, take it amiss that his solution to this problem was not inserted, as his reasoning upon the correction of the fluent (tho' very ingenious) is, nevertheless, defective.

* That the substitution of $-\frac{\dot{y}\ddot{x}}{\dot{x}}$ for $\ddot{y}$, in any fluxionary equation of the second (or any higher) degree, causes no difference in the equation of the fluents, may be thus demonstrated. Suppose the relation of the fluents to be expressed by the general equation $y = Ax^m + Bx^n + Cx^p$, &c.

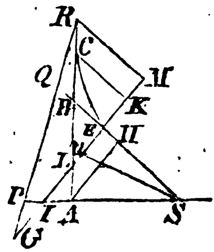
&c. then 'tis evident that $\frac{\dot{y}}{\dot{x}} = mAx^{m-1} + nBx^{n-1} + pCx^{p-1}$, &c. And, if $\dot{x}$ be made constant, we shall, by taking the fluxion a second time, have $\frac{\ddot{y}}{\dot{x}} = m.\overline{m-1}.Ax^{m-2}\dot{x} + n.\overline{n-1}Bx^{n-2}\dot{x}$, &c. But if $\dot{y}$ be made constant, we shall then have $-\frac{\dot{y}\ddot{x}}{\dot{x}\dot{x}} = m.\overline{m-1}.Ax^{m-2}\dot{x} + n.\overline{n-1}.Bx^{n-2}\dot{x}$, &c. Therefore, seeing $\frac{\ddot{y}}{\dot{x}}$ and $-\frac{\dot{y}\ddot{x}}{\dot{x}^2}$ are both equal to one and the same quantity ($m.\overline{m-1}.Ax^{m-2}$, &c.) they must necessarily be equal to each other, and consequently $\ddot{y} = -\frac{\dot{y}\ddot{x}}{\dot{x}}$.

In like manner it will appear that $\dddot{y} = \frac{3\dot{y}\ddot{x}^2}{\dot{x}\dot{x}} - \frac{\dot{y}\dddot{x}}{\dot{x}}$. For, by taking again, the fluxions of the preceding equations, $\frac{\ddot{y}}{\dot{x}\dot{x}} = m.\overline{m-1}.Ax^{m-2} + n.\overline{n-1}.Bx^{n-2}$, &c. and $-\frac{\dot{y}\ddot{x}}{\dot{x}^3} = m.\overline{m-1}.Ax^{m-2} + n.\overline{n-1}.Bx^{n-2}$, &c. (making $\dot{x}$ constant in the former and $\dot{y}$ in the latter) there arises $\frac{\dddot{y}}{\dot{x}^2}$ ($= m.\overline{m-1}.\overline{m-2}.Ax^{m-3}\dot{x}$, &c.) $= -\frac{\dot{y}\dddot{x}}{\dot{x}^3} + \frac{3\dot{y}\ddot{x}^2}{\dot{x}^4}$; and from thence $\dddot{y} = \frac{3\dot{y}\ddot{x}^2}{\dot{x}\dot{x}} - \frac{\dot{y}\dddot{x}}{\dot{x}}$.

M. H.

PRIZE QUESTION answered by Κυβερνήτης.

Let P, Q, and R represent the three ports, and A, B, and C the ships sailing from thence, the former two in the right lines PS, QS, and the latter C in a curve line RCE, whose nature must be such that a right line ABC passing through A and B, may always touch the curve in that very point where the ship C then is.——Let E be the place of the ship B when A arrives at S, the intersection of the courses, and C is come up with B: Let also SI be to SE in the ratio of the given celerities, or as PS to QE: Draw $GIEM$, and make AH parallel thereto, and CK and RM parallel to QES.



It is evident, that SH and EB will be always equal to each other, being each to AS in the constant ratio of SE to SI, or of the celerity in BS to that in AS; whence BH and ES must likewise be equal: But, by similar triangles, $AH : BH\ (ES) :: EL : BE$, and $EI : AH :: ES : SH\ (BE)$: From the composition of which propositions we have $BE^2 = \frac{ES^2}{EI} \times EL = En \times EL$ (Sn being drawn to make the $\angle ESn = EIS$) which is a known property of the parabola: For in the parabola (supposing p to denote the parameter of any diameter EM) it is well known that $CK^2 = p \times EK$, and that $EL = EK$, and consequently $EB = \frac{1}{2}CK$; so that $BE^2\ (= \frac{1}{4}CK^2) = \frac{1}{4}p \times EL$; whence, it not only appears that the curve is a parabola, but that En is $\frac{1}{4}$ of the parameter thereof, corresponding to the diameter EKM.

To determine now the length of the arch ECR, &c. we have, in the $\triangle ESI$, the ratio of SI to SE (as 5 to 3), and the contained $\angle S\ (= 56^\circ\ 15')$: Whence the $\angle SEI$ (or MKC) which the ordinate makes with the diameter EM, is found $= 86^\circ\ 56'$. Again, since $\angle S = \angle Q$, we have $PS = PQ = 12$, and $QE\ (= \frac{3}{5}PS) = 7{\cdot}2$; therefore $RM\ (= 2QE) = 14{\cdot}4$, and $QR\ (= GQ = QE \times \frac{\text{sin. } E}{\text{sin. } G}) = 14{\cdot}086 =$ the required distance of the port R from the port Q. Again, (by trigonometry) $QS = 13{\cdot}3337$, $SE\ (= QS - QE) = 6{\cdot}1337$, and $En = 4{\cdot}42$; which last put $= \frac{1}{2}b$,

$= \frac{1}{2}b$, and let the cosine of the angle MKC ($86° 56\frac{1}{2}'$) $= m$: Then (supposing $EK = x$, and $CK = y$) we have (by the property of the parabola) $2bx = y^2$; whence $\dot{x} = \frac{y\dot{y}}{b}$, and consequently $\sqrt{\dot{y}^2 + 2m\dot{y}\dot{x} + \dot{x}^2}$) the flux. of the arch EC $= \frac{\dot{y}}{b}\sqrt{b^2 + 2bmy + y^2}$; which, by putting $v = mb + y$ and $h = b\sqrt{1 - mm}$, will be transformed to $\frac{\dot{v}}{b}\sqrt{h^2 + v^2}$; whereof the (corrected) fluent will be $\frac{v}{2b}\sqrt{hh + vv} - \frac{1}{2}mb + \frac{h^2}{2b} \times$ hyp. log. $\frac{v + \sqrt{hh + vv}}{mb + b}$: From whence (by taking $y = RM = 14{\cdot}4$) the distance RE run by the ship C is found $= 19{\cdot}77$ miles.

Mr. L. *Charlton*, *Micromegas*, Mr. E. *Rollinson*, (the proposer), and Miss *Frances Harris* (whose performances are an honour to her sex) have given elegant solutions to this problum, by a fluxionary calculus founded on the properties of tangents; some of which solutions we should have inserted, could we possibly have found room for them.—— Mr. *Bamfield* truly determines the distance QR, and, by an ingenious approximation, makes the distance sailed by the ship C to be 19·89 miles.

The Prize of 12 Diaries was won by Miss Frances Harris.

The PARADOX *answered by Mr.* C. Wildbore.

A right line perpendicular to the plane of the two given lines, at the point of their concourse, will be perpendicular to them both.

Eclipses in 1756, calculated by Mr. Ra. Hulse.

There will be only two eclipses this year, both of the sun, and both invisible to the inhabitants of Great Britain.

1. March the 1st, at 2 in the morning, the sun will be eclipsed in ♓ 11 degrees. This will be a great eclipse, total and central, in the Indian sea.

2. August the 25th, at 7 at night, the sun will be totally eclipsed in ♍ 3 degrees; visible at New Spain, Jamaica, Terra-Firma, and places adjacent.

New

New Questions.

I. QUESTION 406, *by* Rusticus.

Two quondam friends at market meet,
(John sheep had bought, Hodge pigs and geese)
Who, whilst a chearful pot they took,
A change propos'd, and bargain struck:
For ev'ry sheep, they did agree,
A pig and goose the price should be.
Hodge had ten pigs less than he'd geese,
Which last were charg'd, six groats a-piece:
By computation shrewd 'twas found,
That pigs and geese were worth five pound;
And that they were in number more
Than all John's sheep, by four times four.
——Now sirs, from hence, please to declare
What number of each sort there were?
Supposing that, for John's whole flock,
Hodge, in exchange, gave all his stock.

II. QUESTION 407, *by* Philo-Pesos.

In a given triangle, to inscribe a rhombus, having one of its angular points coincident with a given point in the base.

III. QUESTION 408, *by Mr.* R. Young, *Writing-master, in* Chester.

The sides of a quadrangular field are known to be 9, 10, 11, and 12 chains, respectively: And, if a diagonal be therein drawn, the part included by it and the two shortest sides will be to the remaining part in the given proportion of 3 to 5: From hence the content of the field is required.

IV. QUESTION 409, *by Mr.* H. Watson.

To determine a point in a given triangle (whose sides are 16, 20, and 24 inches) from whence perpendiculars being let fall on all the sides, the solid (or continual product) contained under them, shall be the greatest possible.

V. QUESTION 410, *by Mr.* Walter Trott.

The distances of the three corners of a right-angled triangular field, from a watering-place within the field, are known to be 38, 50, and 62 perches, respectively: To find, from thence, the content of the field, it being the greatest the data will admit of.

VI. QUES-

VI. Question 411, *by Mr.* J. Ash.

Being on a journey, in lat. 52° 30′ N. the third of April, 1755, in the afternoon, I observed the two intersections of the inner part of the interior rain-bow with the horizon, to form an angle, at the place where I stood, of 60°: From which the time of observation is required.

VII. Question 412, *by Mr.* Chr. Mason.

The human stage is three-score years and ten;
So short the span! yet seldom reach'd by men.
As man's quietas ought to be his care,
'Tis now high time for Mason to prepare.
The rolling years that o'er my head have flown,
If you would know, they're in the margin * shown.

* Viz. $v+x+y+z=1734$, $v^2+x^2+y^2+z^2=2850372$, $vxyz=3240960$, and $x=z$; v being the year of my birth, x the month, y the day, and z the hour *P. M.* all which are required.

VIII. Quest. 413, *by Mr.* Pat. O'Cavanah, *of* Dublin.

(*Addressed to the ingenious Authors of the 378th and 393d Questions.*)

Ah! much, my friends, I mourn your lot;
Who have such woeful help-mates got:
Both sluts and scolds! oh, dreadful curse!
Bad daughters too! What can be worse?
Yet hear me, and to ease your grief,
I'll teach you what will yield relief;*
But like rare nostrums, little known,
In mystic terms it must be shown—
For brother Philomaths alone.

This powerful specific is denoted by a word consisting of five letters, and these have their places in the alphabet expressed by the values of u, w, x, y, and z, in the subjoined equation: † By means whereof the important mystery may be disclosed, and that, without having the root of any equation to extract, higher than a quadratic.

† Viz. $w^2+z^2=89$; $wz+w+z=53$; $\frac{xx+yy}{x+y}=18\frac{1}{10}$; $\frac{x^3+y^3}{x^2+y^2}=18\frac{172}{181}$; $\frac{u^5+x^5}{ux\times\overline{u+x}|^3}=2\frac{1}{10}$.

IX. QUESTION 414, *by Mr.* E. Rollinson.

A beam BC (*see fig. to the solution*) is to be supported in a given position, by means of a prop DE, of a given length, insisting on the horizontal beam AB: 'Tis proposed to determine the position of the prop, so that the beam AB whereon it stands may be the least subject to break, or so that the force, whereby it actually tends to break, may be to the whole force it can sustain in E at the least ratio possible, the thickness of the beam being every-where the same.

X. QUESTION 415, *by Mr.* Hugh Brown.

An usurer lends 2000 l. at 5 per cent. per annum compound interest, which the borrower is to clear off by quarterly payments, viz. 1 l. at the end of the first quarter, 2 l. at the end of the second, 3 l. at the end of the third, and so on. Quere, At what time will the debt be the greatest possible?

XI. QUESTION 416, *by Mr.* Tho. Moss.

Suppose 12 half-pence to be thrown up, and those that come up heads to be taken away, and the remaining ones to be thrown up again, and so on, in the same manner, till all the half-pence have been thrown up heads: 'Tis proposed to find in what number of throws, according to an equality of chances, this may be effected.

XII. QUESTION 417, *by Mr.* W. Bevil.

Suppose that a chain is to be suspended, at its extremes, by two tacks, in the same horizontal line, at the distance of 10 feet: To find the length of the chain, such that the stress or force thereof, upon the tacks, may be a minimum.

XIII. QUESTION 418, *by Mr.* Tho. Peart.

Suppose a mountain, formed by the rotation of the catenaria, whose superficial content is equal to the square of its base diameter; and suppose two equal pendulums, one at the foot, and the other at the vertex of the mountain, to be put in motion the same instant; and that at the end of 24 hours, measured by the former, a cannon ball is there discharged in such a direction, as to fall the farthest possible upon the mountain, and arrives at the vertex at the same instant the pendulum there has measured 24 hours: It is required to find the mountain's height, and the velocity with which the ball is discharged.

XIV.

XIV. QUEST. 419, *by Mr.* R. Weston, Discip. Landenii.

From the equation $a^3\ddot{y} = a^2\dot{y}^2 - y^2\dot{x}^2$ it is proposed to find x in terms of y, without first finding y in terms of x, and then reverting the series.

XV. QUESTION 420, *by Mr.* Pat. O'Cavanan, *of* Dublin.

To find the sum of the infinite series $z - \frac{z^3}{2.3} + \frac{z^6}{2.3.4.5.6} - \frac{z^9}{2.3.4.5.6.7.8.9}$, &c. by means of circular arcs and logarithms.

PRIZE QUESTION, *by Mr.* H. Watson.

A person being to pass from one place A to another B, at the distance of three miles, between which places there lies a morass, is desirous to know the path he must describe, and also how far he must travel, to perform his journey in the shortest time possible; the way, by reason of the morass, being rendered so bad, that he can only move with a celerity every-where proportional to his distance from the center thereof, lying one mile distant from A, and two from B.

1757.

Questions answered.

I. QUESTION 406 *answered by Master* Jonath. Kimbell, *Of* Leicester.

John chang'd ten sheep, at ten* a-piece, * *Shillings*,
For eight small pigs and eighteen geese.

The same answered by Mr. T. Barker, *of* Westhall, *in* Sussex.

Let x denote the number of pigs, and y the price of a pig; then will $x + 10$ be the number of geese, and $2x - 6$ the number of sheep; and therefore, by the question, $xy + \overline{x + 10} \times 2 = 100$, and $\overline{2x - 6} \times \overline{y + 2} = 100$. From the first equation $y = \frac{80}{x} - 2$; and from the second $y =$

 100

$\frac{100}{2x-6} - 2$; $\because 80 \times \overline{2x-6} = 100x$: whence $x = 8$, and $y = 8$. Therefore there were 8 pigs, 10 sheep, and 18 geese.

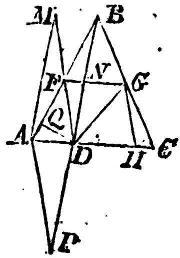
The same answered by Mr. Tho. Wilkin.

Let x = number of pigs; then $x + 10$ = number of geese, and $2x - 6$ = number of sheep; whence $2x + 20$ = price of all the geese; $100 - 2x - 20$ ($= 80 - 2x$) = price of all the pigs; and $\frac{80 - 2x}{x}$ ($\frac{80}{x} - 2$) the price of one pig; and consequently $\frac{80}{x}$ (= the price of one sheep) which multiplied by the number of sheep gives $\frac{80}{x} \times \overline{2x - 6} = 100$: Hence $160x - 480 = 100x$, and $x = 8$. Therefore there were 8 pigs at 8 shillings each, 18 geese at 2 shillings each, and 10 sheep at 10 shillings each.

In the same manner it was also answered by Mr. *G. Armstrong*, Mr. *W. Baxter*, Mr. *J. Beresford*, Mr. *Turner Boston*, Mr. *G. Crabbe*, *Darcy Donought*, Mr. *Jos. Farrington*, Mr. *T. Fletcher*, Mr. *Ja. Giles*, Mr. *E. Griffiths*, Mr. *J. Hampson*, Mr. *W. Harvey*, Mr. *G. Hicks*, Mr. *J. Hudson*, Mr. *T. Hudson*, *Juvenis*, Mr. *W. Lee*, Mr. *W. Litson*, Mr. *Jos. Lord*, Mr. *B. Lydal*, Mr. *R. Marsh*, Mr. *R. Morris*, Mr. *W. Patrick*, Mr. *W. Rippon*, Mr. *Alex. Rowe*, Mr. *T. Sandling*, Mr. *W. Stoker*, Miss *S. T.* Mr. *W. Terrell*, Mr. *R. Terry*, Mr. *W. Thompson*, Mr. *J. Vicary*, Mr. *R. Walton*, Mr. *J. Woolcott*, and several others.

II. QUESTION 407 *answered by Mr.* Abr. Botham.

CONSTRUCTION. From the vertex B, of the triangle, through the given point D, draw BDP; to which, from A, apply AP equal to the base AC; draw DF, FG, GH parallel to AP, AC, and FD, respectively; and the thing is done.



For, by sim. Δs, $AP : FD$ (:: $BA : BF$) :: $AC : FG$; but $AP = AC$ (by construction); whence $FD = FG$.

The same answered by Mr. T. Peart.

Draw BD from the vertex to the given point D; and make AM parallel thereto

thereto, and equal to the base AC of the triangle; draw MD cutting AB in F; then draw FG and GH parallel to AC and DF, for the other sides of the rhombus.

DEMONSTRATION. Let BD cut FG in N. The triangles ADM and DFN (because of the parallel lines) will be similar; whence AM $(AC) : DF :: AD : FN :: AC : FG$; and consequently $DF = FG$.

METHOD OF CALCULATION. From AB, AD, and the $\angle DAB$, the $\angle BDC$ $(= MAD)$ will be known; from which and the given sides AD and AM, the $\angle ADF$ will also be known, and consequently the side DF.

An algebraic Solution to the same by Mr. W. Smith, *of* Irthlingborough.

Put $AB = b$, $AC = c$, DQ (perp. to AB) $= d$, $BQ = e$ and DF $(= FG) = x$: Then $c : b :: x$ $(FG) : BF = \frac{bx}{c}$; whence $FQ = e - \frac{bx}{c}$, and consequently $\overline{e - \frac{bx}{c}}|^2 + d^2 = x^2$.

Which solved gives $x = \frac{bce}{cc-bb} + \sqrt{c^2 \times \frac{dd+ee}{cc-bb} + \overline{\frac{bce}{cc-bb}}|^2}$.

According to the former of these methods the problem was constructed by Mr. *O'Cavanah*, *Philo-Pesos*, Mr. *E. Rollinson*, Mr. *Walter Trott*, and Mr. *H. Watson*.——Algebraic solutions to the same have also been received from Mr. *S. Bamfield*, Mr. *W. Baxter*, *Birchoverensis*, Mr. *L. Charlton*, Mr. *J. Vicary*, and many others.

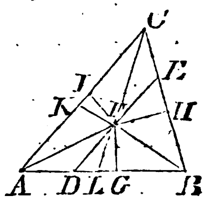
III. QUESTION 408 *answered by Mr.* Wm. Kingston, *of Bath*.

Let $DFGH$ be the quadrangular field (see last figure); then, since by a well-known theorem, the area of the triangle DFG is $\frac{1}{4}\sqrt{\overline{DF+FG}|^2 - DG^2 \times DG^2 - \overline{DF-FG}|^2}$; if DG be put $= x$, we shall have (because FG is given $= 9$, and $DF = 10$) $DFG = \frac{1}{4}\sqrt{361 - x^2 \times x^2 - 1}$. And in the very same manner, the area of DHG will be $= \frac{1}{4}\sqrt{529 - x^2 \times x^2 - 1}$: which being to that of DFG as 5 to 3 (by the quest.) it is evident that $25 : 9 :: 529 - x^2 : 361 - x^2$; whence $xx = 266.5$; and the area $DFG = \frac{1}{4}\sqrt{94.5 \times 265.5} = 39.5994$: and consequently, $\frac{8}{3} \times 39.5994 = 105.5984$ (= 10 ac. 2 r. $9\frac{1}{2}$ p.) the content of the field.

In

In the very same manner it is answered by Mr. *Ja. Beresford*, Mr. *J. Boston*, Mr. *Abr. Botham*, and Mr. *W. Trott*. —It was also truly and concisely answered by Mr. *W. Allen*, Mr. *S. Bamfield*, Mr. *T. Barker*, Mr. *W. Baxter*, Mr. *Jos. Farrington*, Mr. *J. Fletcher*, Mr. *R. Flitcon*, Mr. *Ja. Giles*, Mr. *J. Hampson*, Mr. *G. Hicks*, Mr. *J. Hudson*, Mr. *W. Smith*, Mr. *W. Stoker*, Mr. *W. Terril*, Mr. *R. Terry*, Mr. *W. Thompson*, Mr. *J. Vicary*, Mr. *T. Wilkin*, Mr. *J. Woolcott*, and several others.

IV. QUESTION 409 *answered by* Birchoverensis.



When $FG \times FH \times FI$ is a maximum, the product thereof by the constant quantity $\frac{1}{2}AB \times \frac{1}{2}BC \times \frac{1}{2}AC$, will also be a maximum; that is, the product of the three parts ABF, BCF, and ACF of the given triangle, will be a maximum, and consequently those parts equal among themselves; since it is evident (from Euc. 5. 2.) that the continual product of the parts of any given quantity (whatever their number is) will be the greatest when the parts are all equal: Therefore, ABF being $= \frac{1}{3} ABC$, it is evident that FG will be $= \frac{1}{3}$ of the perpendicular falling from C upon AB, &c. and consequently, that F will be the center of gravity of the given triangle ABC. Hence the three required perpendiculars are found to be 5·2915, 4·4095, and 6·6143, and their continual product $=$ 154·33. *W. W. R.*

The same answered by Mr. Walter Trott.

From any point F, in $DE \parallel$ to one side AC of the triangle, conceive $\perp$s FG, FH, and FI to be let fall; then FG being in a constant ratio to DF, and FH to FE, it is evident, that, when $DF \times FE$ is a maximum, $FG \times FH$ or $FG \times FH \times FI$ (because FI is supposed to continue the same) will likewise be a maximum; which therefore is known to be when $FD = FE$, or when $AK = CK$, supposing the right line BFK drawn meeting AC in K. Hence it appears that (let the distance between DE and AC be what it will) the quantity $FG \times FH \times FI$ cannot be a maximum, unless $AK = CK$: Neither can it be a maximum (by the very same argument) unless AL (supposing CFL drawn) is $= BL$; therefore it must be so when $AK = CK$, and $AL = BL$: Whence the construction is manifest; the point F required being the center of gravity of the triangle, and the three

three required perpendiculars equal to $\frac{5}{3}\sqrt{7}$, $2\sqrt{7}$, and $\frac{5}{2}\sqrt{7}$, or 4·4096, 5·2915, and 6·6144, and the content of the solid contained under them $= \frac{175}{3}\sqrt{7} = 154{\cdot}33$ cubic inches.

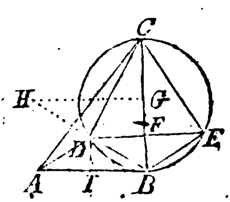
A Fluxionary Solution to the same, by Mr. W. Allen.

Put $AB = a$, $BC = b$, $AC = c$, $FG = x$, $FH = y$, $FI = z$, and the area ABC ($= 158{\cdot}745$) $= d$; then $ax + by + cz = 2d$; and xyz a max. which last, when z is exterminated by means of the former, will give $2dxy - ax^2y - bxy^2$, a max. From which, supposing y invariable, we have $2dy\dot{x} - 2ayx\dot{x} - by^2\dot{x} = 0$; whence $y = \frac{2d - 2ax}{b}$. Again, by making x invariable, we get (after reduction) $y = \frac{2d - ax}{2b}$. Therefore $\frac{2d - ax}{b} = \frac{2d - ax}{2b}$; whence $x = \frac{2d}{3a} = 4{\cdot}40958$; also $y = \frac{2d}{3b} = 6{\cdot}61437$; $z = \frac{2d}{3c} = 5{\cdot}2915$; and $xyz = 154{\cdot}33$.

According to the first of the three foregoing methods the problem is resolved by Mr. *E. Rollinson*, and Mr. *H. Watson*; and according to the last of them, by Mr. *W. Bevil*, Mr. *R. Flitcon*, Mr. *T. Peart*, *Penovius*, Mr. *W. Smith*, Mr. *W. Spicer*, and some others.

V. QUESTION 410 *answered by Mr.* Pat. O'Cavanah.

Let ABC represent the field, and D the watering-place. Let DE be $=$ and $\parallel$ to AB, intersecting CB in F, and let BE and CE be drawn: Then will the trapezium $BDCE$ ($= DE \times \frac{1}{2} BC$) and the $\triangle ABC$ ($= AB \times \frac{1}{2} BC$) be mutually equal to each other. Moreover, since $CE^2 - CD^2$ ($= EF^2 - DF^2$) $= BE^2 - BD^2$, it is evident that $CE^2 = BE^2$ (AD^2) $- BD^2 + CD^2$; from which CE is given ($= 70$). Therefore, all the four sides of



the trapezium $DCEB$ being given, the area will be a maximum when the figure is inscribed in a circle. But the rectangle of the two diagonals of any trapezium inscribed in a circle is equal to the sum of the rectangles of the opposite sides: Therefore $DE \times BC = DC \times BE + DB \times CE$; the half of which is ($= 2880$ perches, or 18 acres) the true area sought. The geometrical construction from hence is very

very easy; for (by sim. Δ's) DF is to BF in the given ratio of CD to BE (or AD). Therefore, having first drawn two $\perp$ lines at pleasure, in the one of them take $BG = AD$, and $\parallel$ to the other draw $GH = CD$; in BH (when drawn) take BD of the given length; and from D, to BA and BC, apply DA and DC also of the given lengths; from whence the points A and C, and consequently the $\Delta\ ABC$ itself, will be determined.

A Fluxionary Solution to the same, by Mr. J. Honey.

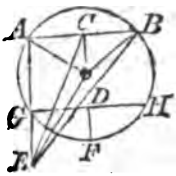
Let $AD = 62 = a$, $BD = 38 = b$, $CD = 50 = c$, and $AB = x$; then $x : a + b :: a - b : \frac{aa - bb}{x} =$ diff. segments of the base; whence, putting $aa - bb = n$, we get $\frac{xx - n}{2x} = IB = DF$; and (p. 47. 1 Euclid) $BF = \frac{1}{2x}\sqrt{4b^2x^2 - x^4 + 2nx^2 - n^2}$; also $CF = \frac{1}{2x}\sqrt{4c^2x^2 - x^4 + 2nx^2 - n^2}$; and consequently $BC = \frac{1}{2x}\sqrt{rx^2 - x^4 - n^2} + \frac{1}{2x}\sqrt{mx^2 - x^4 - n^2}$, by putting $r = 4bb + 2n$, and $m = 4cc + 2n$. Hence $\sqrt{rx^2 - x^4 - n^2} + \sqrt{mx^2 - x^4 - n^2} = 4$ times the area ABC; whose fluxion being taken and made $= 0$, we thence get $x^4 - \frac{mr + 4nn}{m + r} \times x^2 = - n^2$; solved, $x = 78\text{·}3435$ perches: Whence $AC = 73\text{·}5223$, and the area $ABC = 18$ acres.

By this last method, and the same substitution, the problem is also answered by Mr. *W. Allen*, Mr. *Abr. Botham*, Mr. *L. Charlton*, *Penovius*, and Mr. *W. Spicer*; whose solutions are equally neat with that above.——Mr. *Peart*, (who gives the solution without fluxions) says the angles BAD and BCD must be equal to each other; the truth of which is evident from the foregoing construction: For BED, which is $= BAD$, must necessarily be $= BCD$, standing on the same arch BD.

VII.

VI. QUESTION 411 *answered by Mr.* L. Charlton.

Let O be the center of the Iris, E the eye of the ſpectator, and $EACB$ the plane of the horizon: Then, if AE be taken as the radius, it is evident (from the writers on optics) that OA will be the ſine of an $\angle (AEO)$ of $40° 17' = \cdot 6465$; from whence and AC, which is given $=$ ſin. $30° = \cdot 5$, OC is found $= \cdot 4099$. Then in the $\triangle\ EOC$ (right-angled at O) it will be, as EO ($= \cdot 7628 =$ coſ. $40° 17'$) is to OC, ſo is rad. (1) to the tang. of $OEC = 28° 15' =$ the depreſſion of the center of the bow, or the ſun's altitude: From which the time is found 3 h. 15 m. afternoon.



The ſame anſwered by Mr. Samuel Bamfield.

The arch (AO or BO) of a great circle of the ſphere, drawn from the interior ſide of the bow ABF to the center thereof, is (according to the writers on optics) $= 40° 17'$ Therefore in the iſoceles ſpherical $\triangle\ AOB$ (where AB repreſents the horizon) we have given $AO = OB = 40° 17'$, and $AB = 60°$; whence the perpendicular OC ($=$ the ſun's altitude) is found $= 28° 15'$; and from thence the time of obſervation 3 h. 15 m. afternoon.

VII. QUESTION 412 *anſwered.*

Making $a = 1734$, $b = 2850372$, and $c = 3240960$, we have $v + y = a - 2x$, $vv + yy = b - 2xx$, and $vy = \frac{c}{xx}$. From the ſquare of the firſt of which equations ſubtract the double of the laſt, ſo ſhall $v^2 + y^2 = a^2 - 4ax + 4x^2 - \frac{2c}{xx} = b - 2x^2$; and conſequently $x^4 - \frac{2ax^3}{3} + \frac{aa - b}{6} \times x^2 = \frac{c}{3}$; that is, $x^4 - 1156x^3 + 26064x^2 = 1080320$: Whence $x = 8$, $v = 1688$, $y = 30$, and the time of the propoſer's birth Oct. 30, 1688, 8 hours P.M.

Thus the problem was anſwered by Mr. *W. Allen*, Mr. *G. Armſtrong*, Mr. *S. Bamfield*, Mr. *T. Barker*, Mr. *W. Baxter*, Mr. *Tho. Baxtonden*, Mr. *J. Beresford*, *Birchoverenſis*, Mr. *Abr. Botham*, Mr. *J. Boſton*, Mr. *L. Charlton*, Mr. *G. Crabbe*, Mr.

Mr. *R. Flitcon*, Mr. *T. Fletcher*, Mr. *J. Goodhend*, Mr. *E. Griffiths*, Mr. *W. Harrison*, Mr. *G. Hicks*, Mr. *J. Honey*, Mr. *J. Hudson*, *Juvinis*, Mr. *W. Kingston*, Mr. *B. Lydal*, Mr. *T. Peart*, *Penovius*, Mr. *Alex. Rowe*, Mr. *W. Smith*, Mr. *W. Spicer*, Mr. *W. Stoker*, Mr. *W. Terrill*, Mr. *R. Terry*, Mr. *W. Thompson*, Mr. *W. Trott*, Mr. *M. Ward*, Mr. *T. Wilkin*, and Mr. *J. Woolcott*; to none of whom, in particular, we have a right to aſcribe the ſolution here put down.

VIII. QUESTION 413 *anſwered.*

Meſſ. *Barker*, *Bevil*, *Birchoverenſis*, *Botham*, *Juvenis*, *Honey*, *Peart*, *Penovius*, *Smith*, and *Trott*, ſolve this problem by ſubſtituting for the half-ſum, and the half-difference of the quantities ſought in the ſeveral given equations: Thus, let $w = s + d$ and $z = s - d$; then the two firſt equations ($w^2 + z^2 = 89$ and $wz + w + z = 53$) will become $2ss + 2dd = 89$, and $ss - dd + 2s = 53$; whence $ss + s = 48\frac{3}{4}$, and $s = 6\frac{1}{2}$: Therefore $d\ (= \sqrt{44\frac{1}{2} - ss}) = 1\frac{1}{2}$, $w = 8$, and $z = 5$.

Again, by making $x = s + d$, and $y = s - d$, the 3d and 4th equations ($xx + yy = a \times \overline{x + y}$, and $x^3 + y^3 = b \times \overline{xx + yy}$) will be $2ss + 2dd = 2as$, and $2s^3 + 6sdd = b \times \overline{2ss + 2dd} = b \times 2as$ (a being $= 18\frac{1}{10}$, and $b = 18\frac{172}{181}$): Whence $dd = as - ss = \frac{1}{3}ab - \frac{1}{3}ss$, and $ss - \frac{3}{2}as = -\frac{1}{2}ab$. From which $s = \frac{3}{4}a \pm \frac{1}{4}\sqrt{9a^2 - 8ab} = 10$, and $d\ (= \pm\sqrt{as - ss}) = \pm 9$; therefore $x = 10 \pm 9$, and $y = 10 \mp 9$; that is, the greateſt number will be 19, and the leſſer 1; but which of theſe x muſt be, depends on the other given equation $\frac{x^5 + x^5}{ux \times \overline{u + x}^3} = 2\frac{1}{20}$; which equation, by ſubſtituting in like manner ($u = s + d$, and $x = s - d$) becomes $= \frac{s^4 + 10ssdd + 5d^4}{\overline{ss - dd} \times 4ss} = 2\frac{1}{20}$; whence $d^4 + 3{\cdot}64ssdd = 1{\cdot}44s^4$. Now, by compleating the ſquare, and taking the root, $dd = 0{\cdot}36ss$, and $d = \frac{6s}{10}$: Therefore $x\ (= s - d) = s - \frac{6s}{10}$; whence $s = \frac{10x}{4}$; alſo $d\ (\frac{6s}{10}) = \frac{6x}{4}$; and conſequently $u\ (s + d) = \frac{16x}{4} = 4x = 4$ or 78. But, if u be taken as the leſſer of the two numbers, it is evident that, then,

then, $x = 4u$, or $u = \frac{x}{4}$; that is, $u = \frac{1}{4}$, or $\frac{19}{4}$; but of these four different values of u, the first only can fulfil the conditions of the problem: So that the five numbers required will be 4, 5, 1, 19, and 8; and the letters corresponding D, E, A, T, H.

Another Answer to the same, by Mr. H. Watson.

1. By adding the double of the second equation to the first, we have $\overline{w+z}|^2 + 2 \times \overline{w+z} = 195$; whence $w+z = \sqrt{196} - 1 = 13$: From which, and $w^2 + z^2 = 89$, the greater number is found $= 8$, and the lesser $= 5$.

2. By multiplying together the third and fourth equations, we have $\frac{x^3+y^3}{x+y} = ab$; that is, $xx - xy + yy = ab$ (a being $= 18\frac{1}{10}$, and $b = 18\frac{1}{2}\frac{7}{8}\frac{1}{1}$): The double of which, taken from the triple of the third equation, $xx + yy = a \times \overline{x+y}$, gives $xx + 2xy + yy = 3a \times \overline{x+y} - 2ab$, or $\overline{x+y}|^2 - 3a \times \overline{x+y} = -2ab$; whence $x+y = \frac{3a}{2} \pm \frac{1}{2}\sqrt{9aa - 8ab} = 20$: From which, and $xx + yy = 18\frac{1}{10} \times 20$ $(= 362)$, the greater quantity is found $= 19$, and the lesser $= 1$.

3. It is plain that $\overline{u+x} \times \overline{u^4 - u^3x + uuxx - ux^3 + x^4} = u^5 + x^5$; whence $\frac{u^5+x^5}{ux \times \overline{u+x}|^3} = \frac{u^4 - u^3x + uuxx - ux^3 + x^4}{ux \times \overline{u+x}|^2} = 2\frac{1}{20}$; and consequently $u^4 - u^3x + uuxx - ux^3 + x^4 = 2.05ux \times \overline{uu + 2ux + xx}$; that is, $\overline{uu+xx}|^2 - ux \times \overline{uu+xx} - uuxx = 2.05ux \times \overline{uu+xx} + 4.1uuxx$, or $\overline{uu+xx}|^2 - 3.05ux \times \overline{uu+xx} = 5.1uuxx$; whence, by compleating the square, and extracting the root, $uu + xx - 1.525ux = 2.725ux$; and therefore $uu - 4.25xu = -xx$; and, by compleating the square again, $u = 2.125u \pm 1.875x = 4x$ or $\frac{x}{4}$; that is, $u = 4$, 76, $\frac{1}{4}$, or $\frac{19}{4}$; but the first of these values must be the required one; and the letters, answering the conditions of the problem, D, E, A, T, H.

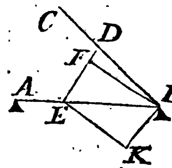
Oh! cruel case! how fix'd the grief!
When DEATH alone can yield relief.

Mr. *W. Allen* and Mr. *Lionel Charlton*, by substituting for the sum and product in each case, bring out the same conclusion from the resolution of quadratics only.——Many contributors have answered all the equations, except the last, by means of quadratics.

X. QUES-

IX. QUEST. 414 *answered by the Proposer, Mr.* Rollinson.

Since the stress or pressure upon the prop ED is (by mechanics) as $\frac{1}{BF}$ (supposing BF perpendicular to ED), the force in the perpendicular direction, whereby the beam tends to break at E, will be as $\frac{1}{BE}$ (it being to the absolute force, in the direction DE, as BF to BE). But the strength of (or the whole force necessary to break) the beam at E, is known be as $\frac{1}{AE \times BE}$; which is to $\frac{1}{BE}$ the force above-mentioned in the proportion of $\frac{1}{AE \times BE}$ to $\frac{AE}{AE \times BE}$, or of 1 to AE; whence (by the question) AE is to be a minimum, and consequently BE a maximum: But BE is to the sine of the $\angle D$ in the given ratio of ED to the sine of B; whence it is evident that BE will be a maximum when the sine of D is so, or when D itself is a right angle.——Therefore having made $BK \perp BC$ and $=$ the given length of DE, draw $KE \parallel BC$, meeting AB in E, so shall E be the place where the end of the prop must stand.



Much after the same manner the problem was solved by Mess. *Botham*, *Charlton*, *Holliday*, *Trott*, and *Watson*; all of whom determined the angle BDE to be a right one.

X. QUESTION 415 *answered by Mr.* W. Bevil.

Let $S = 2000 =$ the sum proposed, $r\ (1{\cdot}05^{\frac{1}{4}}) =$ the amount of 1 l. in one quarter, and $n =$ the number of quarters required; then the amount of the sum S will be Sr^n, and the amount of all the quarterly payments (exclusive of that due at the end of n quarters) $= r^{n-1} + 2r^{n-2} + 3r^{n-3} + 4r^{n-4}$, &c. continued to $n-1$ terms; the sum of all which will be found $= \frac{r}{\overline{r-1}|^2} \times \overline{r^n - rn + n - 1}$ *; and consequently the money then owing $= Sr^n - \frac{r}{\overline{r-1}|^2} \times \overline{r^n - rn + n - 1}$; whereof the interest for one quarter is

$r-1$

$\overline{r-1} \times Sr^n - \frac{r}{\overline{r-1}|^2} \times r^n - rn + n - 1$; and this, by the nature of the question, must be equal to rn (the sum r paid at the end of n quarters, and its interest for the same quarter): Hence we have $\overline{r-1}|^2 \times Sr^{n-1} - r^n + 1 = 0$, and $n = \frac{\log. a}{\log. r} = 28{\cdot}99$ (a being put equal to $\frac{r}{r - S.\overline{r-1}|^2} = 1{\cdot}42361$): Therefore at the end of 29 quarters the debt will be the greatest possible (the moment before the 29th quarter is paid).

* *See* Turner's *Mathematical Exercises*, *No.* 2.

The same answered by Mr. Hugh Brown.

Put $a = 2000$, $r = 1{\cdot}05^{\frac{1}{4}} = 1{\cdot}0122722$, &c. and z the number of quarters at the end of which the debt will be the greatest; then (by the question, and known principles) $ar^z - r^{z-1} - 2r^{z-2} - 3r^{z-3} \ldots\ldots\ldots - \overline{z-1} \times r$ $= ar^z + \frac{rz}{r-1} - \frac{r^{z+1} - r}{\overline{r-1}|^2}$ a max. Therefore $a\dot{z}r^z n$ $+ \frac{r\dot{z}}{r-1} - \dot{z}r^z n \times \frac{r}{\overline{r-1}|^2} = 0$; n being $=$ ($\cdot 0121975$) the hyperbolic log. of r: Whence $rz = \frac{r}{n} \times \frac{r-1}{r - a \times \overline{r-1}|^2}$ $= 1{\cdot}43233$, &c. and $z = \frac{\log. 1.43233, \&c.}{\frac{1}{4} \log. 1{\cdot}05} = \frac{{\cdot}156042}{{\cdot}005297} = 29{\cdot}45$, &c. Hence, as the answer by the nature of the question is restrained to a whole number, it is manifest, that at the end of 29 quarters (the moment before the payment then due is made) the debt will be the greatest possible.

Mess. *Bamfield*, *Birchoverensis*, *Botham*, *Rollinson*, *Smith*, *Trott*, *Watson*, and some others, also answered this question, in a concise and elegant manner.

XI. QUESTION 416 *answered by Mr.* E. Rollinson.

The probability of missing a head x times together with a single halfpenny being $\frac{1}{2}^x$, the probability of throwing any halfpenny a head, in x trials, will therefore be expressed by $1 - \frac{1}{2}^x$; and consequently that of throwing all 12 assigned heads, in x trials, $= \overline{1 - \frac{1}{2}^x}|^{12}$ (for the throwing all the 12 heads may be considered as 12 independent events;

since the happening of some of these, sooner or later, no ways influences the happening of the others). Hence, by the conditions of the problem, we have $\overline{1-\overline{\tfrac{1}{2}^x}\,\big|^{12}} = \frac{1}{2}$; and consequently $x = \frac{\log. \overline{1-\tfrac{1}{2}^{\frac{1}{12}}}}{\log. \frac{1}{2}} = 4{\cdot}155$, the number of throws required; which, not being an integer, shews there can be no exact equality of chance in the case proposed; 4 being a small matter too little, and 5 considerably too great.

COROLLARY. It appears from hence that the exact odds of bringing up all the heads of any number (n) of halfpence, in x throws, will be as $\overline{1-\tfrac{1}{2}^x}\,\big|^n$ to $1-\overline{1-\tfrac{1}{2}^x}\,\big|^n$, universally.

In this manner it was also answered by Mr. *O'Cavanah*, and Mr. *Moss* the proposer.

XII. QUESTION 417 *answered by Mr.* Lionel Charlton.

If half the length of the chain or curve (GF or HF) be denoted by z [*see last fig. but one*], then will the abscissa $DF = \sqrt{aa+zz} - a$, and the given semi-ordinate GD $(= c) = a \times$ hyp. log. $\frac{z+\sqrt{aa+zz}}{a}$ (see Landen's Mathematical Lucubrations, p. 34). Moreover, by mechanics, the stress or force upon both the pins, at G and H, will be to the weight of the chain, as the radius is to the sine of the $\angle G$ or H, that is, as $\dot{z}$ to $\dot{x}$ $\left(\frac{zz}{\sqrt{aa+zz}}\right)$ or as $2\sqrt{aa+zz}$ to $2z$: Therefore the weight of the chain being as the length $2z$, the stress upon the pins will consequently be as $2\sqrt{aa+zz}$; which quantity is to be a minimum, by the question: From whence, and the above equation, $c = a \times$ hyp. log. $\frac{z+\sqrt{aa+zz}}{a}$, both the values of a and z will be found for the hyp. log. $\frac{z+\sqrt{aa+zz}}{a}$ being $= \frac{c}{a}$, supposing N to denote the number whose hyp. log. $= 1$, it is evident that $N^{\frac{c}{a}} = \frac{z+\sqrt{aa+zz}}{a}$; whence, by reduction, z is found $= \frac{1}{2}aN^{\frac{c}{a}} - \frac{1}{2}aN^{-\frac{c}{a}}$, and from thence

$2\sqrt{aa+zz}$

$2\sqrt{aa+zz}=aN^{\frac{c}{a}}+aN^{-\frac{c}{a}}$: This, in fluxions, &c. gives $\dot{a}N^{\frac{c}{a}}-\frac{c\dot{a}}{a}N^{\frac{c}{a}}+\dot{a}N^{-\frac{c}{a}}+\frac{c\dot{a}}{a}N^{-\frac{c}{a}}=0$; whence $N^{\frac{2c}{a}}=\frac{c+a}{c-a}$, or $N^{2v}=\frac{v+1}{v-1}$ (by putting $a=\frac{c}{v}$); which (in logarithms) becomes $2v=$ hyp. log. $\frac{v+1}{v-1}$; whence $v=1{\cdot}1996$; and from thence $a=c\times{\cdot}8336=4{\cdot}168$, and $2z$ $(GFH)=2c\times1{\cdot}2578=12{\cdot}578$.

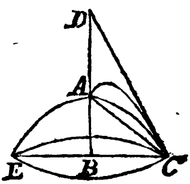
The same answered by Penovius.

Let (GF) half the length of the chain $=z$; GD $(HD)=b$, and $DF=x$; then will $b=a\times$ hyp. log. $\frac{z+\sqrt{aa+zz}}{a}$, by the property of the curve. Also as $\dot{x}$ $\left(\frac{z\dot{z}}{\sqrt{aa+zz}}\right):\dot{z}$ :: z (half the chain) : $\sqrt{aa+zz}=$ the stress on each pin, which put $=n$; then, by exterminating z, we have $\frac{b}{a}=$ hyp. log. $\frac{n+\sqrt{nn-aa}}{a}=$ hyp. log. $\overline{n+\sqrt{nn-aa}}-$ hyp. log. a; whereof the fluxion, when n is a minimum, will be $-\frac{b\dot{a}}{aa}=\frac{-a\dot{a}}{\sqrt{nn-aa}\times\overline{n+\sqrt{nn-aa}}}-\frac{\dot{a}}{a}$; whence by reduction, $n=\frac{ba}{\sqrt{bb-aa}}$: Therefore $\sqrt{nn-aa}=\frac{aa}{\sqrt{bb-aa}}$, and $n+\sqrt{nn-aa}=\frac{ba+aa}{\sqrt{bb-aa}}=a\sqrt{\frac{b+a}{b-a}}$; which value substituted above gives $\frac{b}{a}=$ hyp. log. $\sqrt{\frac{b+a}{b-a}}$; or $a\times$ hyp. log. $\overline{b+a}-$ hyp. log. $\overline{b-a}=2b$: Whence $a=4{\cdot}1677$; and from thence $2z$ $(GFH)=b\times1{\cdot}2578=12{\cdot}578$.

COROLLARY. Hence the length of the chain GFH (when the stress is a minimum) is to the given distance of the tacks G, H, as $1{\cdot}2578$ to 1; and the stress on the tacks is to the weight of the chain, as $1{\cdot}1996$ to 1, or as 6 to 5, nearly.

This question was also answered, in an elegant manner, by Mr. *O'Cavannah*, Mr. *W. Bevil* (the proposer), Mr. *E. Rollinson*, and Mr. *H. Watson*.

XIII. QUEST. 418 *answered by Mr.* Peart, *the Proposer.*



From the equation $zz = 2ax + xx$ of the generating curve, the surface of the mountain $AEBC$ is easily found to be $4c \times \overline{yz - ax}$ (where $c = \frac{3.1416}{2}$), which being given $= 4yy$, we thence have (by compleating the square) $y = \frac{1}{2}cz + \sqrt{\frac{1}{4}c^2z^2 - cax} = \frac{1}{2}c\sqrt{2ax + xx} + \sqrt{\frac{1}{4}c^2 \times \overline{2ax + xx} - cax}$: but y is also $= a \times$ hyp. log. $\frac{a + x + \sqrt{2ax + xx}}{a}$ (by property of the curve): From which equal values, by substituting $ux = a$, we have $\frac{1}{2}c\sqrt{2u + 1} + \sqrt{\frac{1}{4}c^2 \times \overline{2u + 1} - cu}$ $=$ hyp. log. $\frac{u + 1 + \sqrt{2u + 1}}{u}$; whence u is found $= 1.779 +$, and from thence $y = 1.8078\,x$. Having, therefore, made BC to AB in the given proportion of 1.8078 to 1, and taken $AD =$ the right-line $AC = 2.066\,x$, let CD be drawn, which (because the range is a *maximum*) will be the line of direction.

Now let $r =$ the earth's semi-diameter, in feet; $b = 16\frac{1}{12}$ feet; and $s =$ the number of seconds in 24 hours: Then it will be as $r : r + x :: s : s + \frac{sx}{r} =$ the seconds taken up in performing an equal number of vibrations by the pendulum on the top of the mountain: Therefore $\frac{sx}{r}$ is the time of the ball's flight; and consequently $\frac{s^2x^2}{rr} \times b = 2.066\,x$ ($= AD$); whence x is given $= \frac{2.066\,rr}{bss} = 7598$ feet, the mountain's height; and the time of flight ($\frac{sx}{r}$) $= 31.24$ seconds: By which number dividing CD ($= 27046$), the quotient $865\frac{3}{4}$ feet will be the velocity, *per second*, with which the ball is discharged.

In this solution I have supposed the gravitation to be proportional to the square of the distance from the earth's center,

ter, inverſely, without having regard to the attraction of the mountain: But if *this laſt* (which is to the former as the height of the mountain to the ſemi-diameter of the earth, nearly) be alſo taken into the conſideration, the time loſt by the pendulum on the top of the mountain will then be only the half of $(\frac{sx}{r})$ what it is above found to be: Therefore the height of the mountain will come out four times as great here, as when the mountain's attraction is neglected.*

XIV. QUESTION 419 *anſwered by Mr.* R. Weſton, *the Propoſer.*

Since it appears, by what is done in the laſt diary, that $-\frac{\dot{y}\dot{x}}{\dot{x}}$ may be ſubſtituted for $\ddot{y}$, if ſuch ſubſtitution be made in the given equation, the required value of x may then be readily obtained by the common method of finding fluents by infinite ſeries: Or that value may be found in the following manner:

Aſſume $x = b + fy + py^2 + qy^3 + ry^4$, &c. ſuppoſing that when y is $= 0$, x is $= b$, and $\frac{\dot{x}}{\dot{y}} = f$: Then the given equation ($a^3\ddot{y} = a^2\dot{y}^2 - y^2\dot{x}^2$) being multiplied by the invariable quantity $\dot{x}$, we have $a^3\dot{x}\ddot{y} = a^2\dot{x}\dot{y}^2 - y^2\dot{x}^3$; whence ſubſtituting for $\dot{x}^3$ its value found by the aſſumed equation, we have, after dividing by $\dot{y}^2$, $\frac{a^3\dot{x}\ddot{y}}{\dot{y}\dot{y}} = a^2\dot{x} - f^3y^2\dot{y} - 6f^2py^3\dot{y}$, &c. Hence, by taking the fluents, we find $-\frac{a^3\dot{x}}{\dot{y}} = a^2x - \frac{f^3y^3}{3} - \frac{6f^2py^4}{4} - 9f^2q +$

* This may be explained thus. Suppoſe the earth's attraction to be 1, and that it is to that of the mountain as r to x; then $\frac{x}{r}$ will be the attraction of the mountain, and $1 + \frac{x}{r}$ the ſum of both; and, becauſe the times of *vibration* are as the roots of the forces reciprocally, $\sqrt{1} : \sqrt{1 + \frac{x}{r}} = 1 + \frac{x}{2r}$ nearly $:: s : s + \frac{sx}{2r}$ nearly, and $\frac{sx}{2r}$ = time loſt nearly.

$\frac{9f^4q+12fp^2}{5}y^5$, &c. But, x being $= b$, and $\frac{\dot x}{\dot y} = f$, when y is $= 0$, the correct equation of the fluents is — $\frac{a^3\dot x}{\dot y} = -a^2 b - a^3 f + a^2 x - \frac{f^3y^3}{3} - \frac{6f^2py^4}{4}$ &c. which, multiplying by $-\frac{\dot y}{a^3}$ and substituting for x its assumed value, becomes $\dot x = f\dot y - \frac{fy\dot y}{a} - \frac{py^2\dot y}{a} + \overline{\frac{f^3}{3a^3} - \frac{q}{a}} \times y^3\dot y$, &c. whence, taking the correct fluents, we get $x = b + fy - \frac{fy^2}{2a} - \frac{py^3}{3a} + \frac{f^3-3a^2q}{3\cdot4a^3}y^4 + \frac{6f^2p-4a^2r}{4\cdot5a^3}y^5$, &c. Consequently, by comparing the two values of x; p, q, r, &c. will be known.

The same answered by Mr. Henry Watson.

Put $\dot y = z\dot x$, then $\ddot y = \dot z\dot x$; which values being substituted in the given equation, $a^3\ddot y = a^2\dot y^2 - y^2\dot x^2$, and the whole divided by $\dot x$, we have $a^3\dot z = a^2z^2\dot x - y^2\dot x = a^2z^2\times\frac{\dot y}{z} - y^2\times\frac{\dot y}{z}$ (because $\dot x = \frac{\dot y}{z}$) and consequently $a^3z\dot z = a^2z^2\dot y - y^2\dot y = \overline{a^2z^2-y^2}\times\dot y$: Put $v = a^2z^2 - y^2$; then $zz = \frac{v+yy}{aa}$ and $z\dot z = \frac{\frac{1}{2}\dot v + y\dot y}{aa}$; and so, by substitution, $a\times\overline{\frac{1}{2}\dot v + y\dot y} = v\dot y$, or $\frac{1}{2}a\dot v = \overline{v - ay}\times\dot y$: Put now $w = v - ay$ ($= a^2z^2 - y^2 - ay$) then $v = w + ay$, and $\dot v = \dot w + a\dot y$; whence again, by substitution, $\frac{1}{2}a\times\overline{\dot w + a\dot y} = w\dot y$, or $\frac{1}{2}a\dot w = \overline{w - \frac{1}{2}aa}\times\dot y$, and therefore $\dot y = \frac{\frac{1}{2}a\dot w}{w-\frac{1}{2}aa}$; whereof the fluent is $y = \frac{1}{2}a\times$ hyp. log. $\frac{w-\frac{1}{2}aa}{d}$ (d being any constant quantity at pleasure) $= \frac{1}{2}a\times$ hyp. log. $\frac{a^2z^2-y^2-ay-\frac{1}{2}aa}{d}$. Hence, putting $M =$ the number whose hyp. log. $= 1$, we have $M^{\frac{2y}{a}} =$

$= \frac{a^2z^2 - y^2 - ay - \frac{1}{4}aa}{d}$, and consequently $z^2 = \frac{y^2 + ay + \frac{1}{4}aa + dM^{\frac{2y}{a}}}{aa}$; from whence $\dot{x}\ (= \frac{\dot{y}}{z}) = \frac{a\dot{y}}{\sqrt{y^2 + ay + \frac{1}{4}aa + dM^{\frac{2y}{a}}}}$: From which, when $d = 0$, x will be found $= a \times \text{hyp. log.} \frac{y + \frac{1}{2}a + \sqrt{y^2 + ay + \frac{1}{4}aa}}{c}$, wherein c may be any constant quantity at pleasure.

In a manner very little different from this last *it* was answered by Mr. *Rollinson*, who brings out the very same conclusion.

XV. QUESTION 420 *answered by* Κυβερνήτης.

Conceive the given series $1 - \frac{z^3}{2.3} + \frac{z^6}{2.3.4.5.6}$, &c. to be composed of three others,

$$A \times \overline{1 + pz + \frac{p^2z^2}{2} + \frac{p^3z^3}{2.3} + \frac{p^4z^4}{2.3.4}}, \text{ \&c.}$$

$$B \times \overline{1 + qz + \frac{q^2z^2}{2} + \frac{q^3z^3}{2.3} + \frac{q^4z^4}{2.3.4}}, \text{ \&c.}$$

$$C \times \overline{1 + rz + \frac{r^2z^2}{2} + \frac{r^3z^3}{2.3} + \frac{r^4z^4}{2.3.4}}, \text{ \&c.}$$

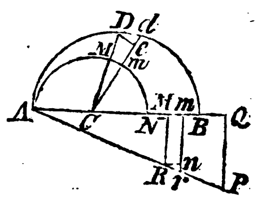
Then, by taking A, B, C each $= \frac{1}{3}$, and equating the homologous terms, we shall have $p + q + r = 0$, $pp + qq + rr = 0$, $p^3 + q^3 + r^3 = -3$, $p^4 + q^4 + r^4 = 0$, &c. &c. Make now $p^3 = -1$, $q^3 = -1$, and $r^3 = -1$; that is, let p, q, and r be the three roots, $(-1, \frac{1}{2} + \sqrt{-\frac{3}{4}}, \frac{1}{2} - \sqrt{-\frac{3}{4}})$ of the cubic equation $x^3 = -1$, or $x^3 + 1 = 0$; then, as both the second and third terms of this equation are wanting, not only the sum of all the roots $(p + q + r)$ but the sum of all their squares $(pp + qq + rr)$ will vanish, or be equal to nothing, as they ought, to fulfil the conditions of the two first of the preceding equations. Moreover, because $p^3 = -1$, $q^3 = -1$, and $r^3 = -1$, it is likewise evident that $p^4 + q^4 + r^4 = -p - q - r = 0$, $p^5 + q^5 + r^5 = -p^2 - q^2 - r^2 = 0$, and $p^6 + q^6 + r^6 = -p^3 - q^3 - r^3 = 3$; which equations being nothing more than the three first repeated, the values of p, q, r, above determined, will

will equally fulfil the conditions of these also: So that the series arising from the addition of the three assumed ones will agree, in every term, with that propounded. But $1 + pz + \frac{p^2z^2}{2} + \frac{p^3z^3}{2.3}$, &c. the first of those series is known to express the number whose hyperbolic logarithm is pz: Therefore, if N be taken to denote the number (2·71828, &c.) whose hyp. log. is unity, then will $1 + pz + \frac{ppzz}{2}$, &c. $= N^{pz}$: And in the same manner $1 + qz + \frac{qqzz}{2}$, &c. $= N^{qz}$, &c. and consequently $1 - \frac{z^3}{2.3} + \frac{z^6}{2.3.4.5.6}$, &c. $= \frac{1}{3}$ into $N^{pz} + N^{qz} + N^{rz} = \frac{1}{3}N^{-z} + \frac{1}{3}N^{\frac{1}{2}z} \times \overline{N^{nz\sqrt{-1}} + N^{-nz\sqrt{-1}}}$ (making $n = \sqrt{\frac{3}{4}}$). But $N^{nz\sqrt{-1}} + N^{-nz\sqrt{-1}}$ is known to express the double of the cosine of the arch nz (the radius being 1); which cosine let be denoted by S, and let the number whose hyp. log. is z be represented by T; then we shall have $1 - \frac{z^3}{2.3} + \frac{z^6}{2.3.4.5.6}$, &c. $= \frac{1}{3T} + \frac{2ST^{\frac{1}{2}}}{3}$. *Q. E. I.*

From the same methods, and the known roots of the equation $x^n \pm 1 = 0$, the series $1 \pm \frac{z^n}{1.2.3\,(n)} + \frac{z^{2n}}{1.2.3\,(2n)}$, &c. may be summed, n being any whole positive number.

PRIZE QUESTION *answered by Mr.* O'Cavanah.

From C, the center of the morass, let a semi-circle AMN be described; and from any two points D, d in the required curve ADB, indefinitely near to each other, conceive DC and dC to be drawn, intersecting the semi-circle in M and m: Calling CA, 1; CB, b; AM, z; CD, y; Mm, $\dot{z}$; and de, $\dot{y}$. Then CM (1) : Mm ($\dot{z}$) :: CD (y) : $De = y\dot{z}$. Whence Dd ($= \sqrt{De^2 + de^2} = \sqrt{y^2\dot{z}^2 + \dot{y}^2}$. This, divided by y, gives $\sqrt{\dot{z}^2 + \frac{\dot{y}\dot{y}}{yy}}$; which



(by

(by the queſtion) is the time of deſcribing Dd, and which by making $\dot{x} = \frac{\dot{y}}{y}$ (or $x =$ hyp. log. y) will be reduced to $\sqrt{\dot{z}\dot{z} + \dot{x}\dot{x}}$.

Make now the right line $AQ =$ the ſemi-circumference AMN, and QP (perpendicular thereto) = hyp. log. b (= the laſt value of x, when $y = b = CB$): And let the line ARP be of ſuch a nature, that, taking the abſciſſa $AM =$ the arch AM, the correſponding ordinate MR ſhall be every-where equal to x: Then Rr (the fluxion of AR) being alſo expreſſed by $\sqrt{\dot{z}\dot{z} + \dot{x}\dot{x}}$, it is evident that AR will truly expreſs the time of deſcribing the arch AD; and, conſequently, AP the whole time of deſcribing the arch ADB; which will evidently be the ſhorteſt poſſible, when AP is a ſtraight line (as being the ſhorteſt that can poſſibly be drawn between the two given points A and P).

Hence, putting AQ ($= 3{\cdot}1416$) $= p$, and QP (= hyp. log. b, or of $\frac{CB}{CA} = {\cdot}693147$) $= q$, we have, by ſimilar triangles, $p : q :: \dot{z} : \dot{x} = \frac{q\dot{z}}{p} = \frac{\dot{y}}{y}$ (per above); and conſequently $De = y\dot{z} = \frac{p\dot{y}}{q}$; which being to de ($\dot{y}$) in the conſtant ratio of p to q, the ange Dde muſt be every-where the ſame, and therefore the curve ADB, the proportional, or logarithmical ſpiral; wherein Dd being to ed in the conſtant ratio of $\sqrt{pp + qq}$ to q, we ſhall therefore have $q : \sqrt{pp + qq} :: NB$ (the whole increaſe of the diſtance CD) : $\sqrt{\frac{pp}{qq} + 1} \times NB = 4{\cdot}641444$ the true length of the ſpiral arch ABD. *W. W. R.*

COROLLARY. It appears from hence that the time of deſcribing the ſpiral ADB will be to the time of uniformly deſcribing the arch of the ſemi-circle AMN, with the firſt velocity at A, in the given ratio of AP to AQ, or of $\sqrt{pp + qq}$ to p.

The prize queſtion was alſo truly and conciſely anſwered by *Birchoverenſis*, Mr. *Abr. Botham*, Mr. *L. Charlton*, Mr. *J. Goodhead*, Mr. *E. Rollinſon*, Mr. *Walter Trott*, Mr. *H. Watſon*, and Mr. *R. Weſton*, from the prob. in p. 496 of Simpſon's Fluxions.——The anſwer by *Plus-Minus* (though a ſmall

a ſmall miſtake is therein committed) ſufficiently diſcovers the author to be a man of genius.

The prize of 12 *Diaries was won by Mr.* Pat. O'Cavanah, *of* Dublin.

Eclipſes in 1757, *calculated by Mr.* Ra. Hulſe.

There will happen four eclipſes this year, viz. two of each luminary; whereof thoſe of the moon will only be viſible to the inhabitants of Great Britain.

1. February 4th, the moon will be eclipſed at 7 h. 6 m. morning, near 7 digits on the upper ſide; the beginning of the eclipſe 5 h. 27 m. middle 6 h. 48 m. end 8 h. 9 m. total duration 2 h. 42 m.

2. February 18th, at 1 h. afternoon, the ſun will be 8 dig. eclipſed, on the ſouth ſide, in ♓ 0° 9′, vertical to the Ethiopian ocean, lat. 21° ſouth, long. 15° weſt.

3. July 30th, the moon will be eclipſed at 11 h. 54 m. at night; the beginning 10 h. 20 m. Middle 11 h. 54 m. End 1 h. 18 m. total duration 2 h. 58 m. digits eclipſed 11° 30′.

4. Auguſt 14th, at 11 at night, the ſun will be eclipſed 5 digits on the upper ſide, in ♌, 22 deg. vertical to the great ocean, lat. 14° north, long. 165° weſt.

The times and quantities of the two viſible eclipſes, for the meridian of London are calculated by Mr. *Edward Greenſted*, as below.

1. The moon will be eclipſed the 4th of February, in the morning; beginning 5 h. 20 m. middle 6 h. 45 m. end 8 h. 10 m. duration 2 h. 50 m. digits eclipſed 6° 41′.
2. The moon will be eclipſed the 30th of July, in the evening; beginning 10 h. 16 m. middle 11 h. 48 m. end 1 h. 19 m. duration 3 h. 3 m. digits eclipſed 11° 32′.

New Queſtions.

I. QUESTION 421, *by* Juvenis.

Three men to ſhare a ſtock agree, of fifteen hundred pound:
The part of *A* to that of *B*, as four to three was found;
But *C*'s exceeding that of *A* by pounds juſt ten times ſeven.
What each man ſhar'd, pray, ladies, ſay, from what above is given.

II. QUES-

II. Question 422, *by Mr.* William Spry, *Engineer.*

Suppose that a cannon ball is discharged to hit a target (or other obstacle) at the distance of 500 yards: How far must I stand from the target, in a perpendicular to the line drawn between it and the cannon, so as to hear the report of the shot and the explosion of the piece at the same instant of time, allowing the velocity of the ball to be to that of sound in the proportion of 3 to 2?

III. Question 423, *by* Birchoverensis.

To determine the position of a point with respect to four given points, so that lines being drawn from thence to the given points, the sum of the four squares formed upon them shall be the least possible.

IV. Question 424, *by Mr.* Tho. Baxtonden, *of* Liverpool.

At a station due south of a tower, I observed the altitude of the top of the tower to be 30°, and that of its base 12°: Proceeding from thence 100 yards, north-east, down a path making an angle of 5° 30′ with the plane of the horizon, I again took the altitude of the tower's summit, which I then found to be 38° 30′. From whence I desire to know the height of the tower, and the distance thereof from my first station.

V. Question 425, *by Mr.* Ja. Beresford.

The four sides of a field, whose diagonals are equal, are known to be 25, 35, 31, and 19 perches, in a successive order; from whence the content of the field is required.

VI. Question 426, *by Mr.* W. Spicer.

A man laid out 60 pounds in sheep, of three different sorts; for the first sort he paid 9 shillings a-piece, for the second 12, and for the third 15: And the number he bought of each sort was such, that the sum of their three squares was less than it could possibly have been, had he bought more of any one sort and less of another. What number of sheep did he buy?

VII. Question 427, *by Mr.* T. Moss.

The rectangle under the two diagonals of any trapezium drawn into twice the cosine of the angle contained by them (the radius being 1) is equal to the difference of the aggregates of the squares of the opposite sides of the trapezium; and

and the area of the trapezium is equal to one-fourth of the ſame difference drawn into the tangent of the ſaid angle. A demonſtration of this is required.

VIII. QUESTION 428, *by Mr.* Lionel Charlton.

The ſum (200) of the two extremes, and the ſum (300) of the four means of ſix numbers in continued geometrical proportion being given; to find the numbers themſelves, by an equation not exceeding a quadratic.

"This queſtion, Mr Charlton obſerves, was propoſed to "him by a Gentleman at the mathematical college at Edin-"burgh."

IX. QUESTION 429, *by Mr.* Walter Trott.

Two ſhips, *A* and *B*, ſail from a certain port, in north latitude, to two other ports lying under the equinoctial line, at the diſtance of 666$\frac{2}{3}$ leagues from each other. The ſhip *A*, ſteering full ſouth (which was her direct courſe), made her port in 15 days; but *B*, though ſhe ſteered the ſhorteſt courſe poſſible, and run at the ſame rate as *A*, did not arrive at her port till the end of 25 days. Now I demand the latitude of the port ſailed from, and the true diſtance run by each ſhip.

X. QUESTION 430, *by Mr.* Henry Watſon.

In a given triangle (whoſe three ſides are 40, 50, and 60) to deſcribe the greateſt ellipſis poſible; and to determine the area, and the principal diameters thereof.

"This problem, or one like it, was printed in the Ladies "Diary for 1739, but never was anſwered in any ſucceed-"ing Diary, or elſewhere, that I have been able to diſcover. "Your propoſing of it, at this time, will oblige many of "your readers, and particularly your humble ſervant,

"HENRY WATSON."

XI. QUESTION 431, *by Mr.* W. Bevil.

Suppoſe a round poſt one foot in diameter, and fixed perpendicular to the horizon, on which are wound 100 rounds of Mancheſter binding (one upon another) whoſe thickneſs is one twenty-eighth of an inch. Now, if a perſon takes hold of the extremity of the outward end, and moves round the ſaid poſt until he has unwrapt it all, how many yards will he have travelled when he arrives at the end of his journey, always keeping as far from the poſt as the binding will admit him?

XII. QUES-

XII. QUESTION 432, *by Mr.* Edw. Rollinſon.

From the equation $m x^3 \dot{z}^2 - n x^2 \dot{z}^2 = \overline{p+zz}|^2 \times q \dot{x}^2$ to determine the general relation of x and z; and alſo to find in what circumſtances of the coefficients m, n, p, q, that relation can be expreſſed in finite terms.

'Dr. BROOK TAYLOR, in his INCREMENTS, after having given a ſolution of that caſe where $m = 4$, $n = 4$, $p = 1$, and $q = 1$, (in which there ſeems to be a miſtake) ſays, that if the coefficients be changed, it does not appear to him that x can be determined, in terms of z, by a finite equation.'

XIII. QUEST. 433, *by Mr.* R. Weſton, *Diſcip. Landenii.*

If a ſtraight, uniform, ſlender rod, or bar, of heavy metal, of a given length, be left to deſcend after being ſet leaning, in a given poſition, with its lower end (n) on the immoveable horizontal plane AB, [*See the fig. to the ſolut.*] and its upper end (m) full againſt the immoveable vertical plane AC (the lower end being at liberty to ſlide freely along the firſt-mentioned plane, while the upper end is deſcending), what will be the poſition of the rod when it ſhall ceaſe to touch the ſaid vertical plane? how long will it then have been in motion? and how far from the point A will the end (m) ſtrike the horizontal plane?

PRIZE QUESTION, *by Mr.* O'Cavanah, *of* Dublin.

A pert young exciſeman, who boaſted his knowledge
In gauging of veſſels, and taking an ullage,
A wager would lay, his ſkill to make good;
And the caſe we propos'd for his trial thus ſtood:
'Eighteen inches, five tenths, a caſk's length is given:
'The heads, which are equal, are each thirty-ſeven*.
" It likewiſe is known, that ſuch is the make,
'The caſk it is formed the leaſt † wood to take.
To find the content quite baffled his art;
But he hopes you the method next year will impart.

* Each head diam. is 37 inches.

† The ſuperficies is the leaſt poſſible.

1758.

Questions answered.

I. QUEST. 421 *answered by Mr.* W. Bacon, *of* Ipswich.

Let the share of A be $= x$; then that of B will be $= \frac{3x}{4}$, and that of $C = x + 70$ (by the quest.); whence $2x + \frac{3x}{4} + 70 = 1500$ (by the quest.) and consequently $x = \frac{1430 \times 4}{11} = 520$. Therefore the share of A was 520l. that of B 390l. and that of C 590l.

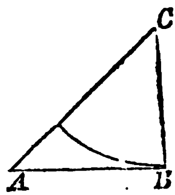
The same answered by Mr. R. Flitcon.

Let $4x$, $3x$, and $4x + 70$ denote the shares of A, B, and C, respectively; then will $11x + 70 = 1500$ (per question): Whence $x = 130$; and the three shares 520, 390, and 590 pounds.

Thus the problem is also answered by Mr. *W. Bamfield*, Mr. *Edw. Barrass*, Mr. *Fra. Bell*, Mr. *Ja. Beresford*, Mr. *J. Chapman*, Mr. *G. Clarke*, Mr. *E. Cock*, Mr. *T. Corbett*, Mr. *Ja. Fellowes*, Mr. *Ja. Giles*, Mr. *J. Hampson*, Mr. *E. Hare*, Mr. *W. Harrison*, Mr. *G. Hickes*, Mr. *W. Honnor*, Mr. *T. Howe*, Mr. *R. Hudson*, Mr. *T. Jeffery*, Mr. *J. Johnson*, Mr. *T. Knight*, Mr. *E. Langworth*, Mr. *J. Lewin*, Mr. *W. Litson*, Mr. *R. Marsh*, Mr. *W. Mathewson*, Mr. *Herbert Nokes*, Mr. *J. Rennard*, Mr. *R. Richardson*, Mr. *Jos. Rose*, Mr. *Alex. Rowe*, Mr. *T. Sandling*, Miss *T. S—l*, Mr. *Jos. Scott*, Mr. *F. Sims*, *Sinbad*, Mr. *G. Stapley*, Mr. *H. Stephens*, Mr. *J. Stothart*, Mr. *R. Walton*, and many others.

II. QUEST. 422 *answered by Mr.* J. Hampson, *of* Leigh.

Let A be the place of the cannon, B that of the target, and C that of the spectator; and in CA take $CD = CB$: Then the sound of the cannon will be at D, on the ball's striking the target at B; and so AD will be to AB in the given proportion of 2 to 3 (by the question). Putting, therefore, $AB = 3a$, $AD = 2a$, and $BC (= CD) = x$, we have $AC = x + 2a$; and from thence (by Euc. 47. 1.) $xx + 4ax + 4aa = xx + 9aa$:

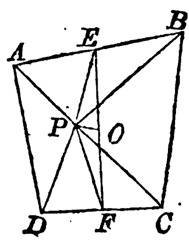


From which $x = \frac{5a}{4} = 208\frac{1}{3}$ yards.

In the very same manner it is answered by Mr. *W. Allen*, Mr. *J. Bank*, Mr. *Ja. Beresford*, Mr. *W. Bevil*, *Birchoverensis*, Mr. *J. Chapman*, Mr. *T. Corbett*, Mr. *J. Herrington*, Mr. *G. Hicks*, Mr. *J. Honey*, Mr. *W. Honnor*, Mr. *T. Howe*, Mr. *R. Hudson*, Mr. *J. Milbourn*, Mr. *J. Rennard*, Mr. *T. Sims*, Mr. *H. Stephens*, Mr. *W. Stoker*, Mr. *W. Terrill*, Mr. *J. Wilson*, and several others.

III. QUESTION 423 *answered by Mr.* Will. Kingston, *of* Bath.

Let A, B, C, and D be the four given points: Bisect AB in E, and CD in F; then assuming P for the required point, and putting $AE = BE = a$, $DF = CF = b$, $EP = x$, and $FP = z$, we shall (by the 12th of the 2d of Simpson's Geometry) have $AP^2 + BP^2 = 2aa + 2xx$, and $DP^2 + CP^2 = 2bb + 2zz$: Hence $AP^2 + BP^2 + DP^2 + CP^2 = 2aa + 2bb + 2xx + 2zz$, a minimum: And consequently $xx + zz$ ($EP^2 + FP^2$) a minimum also; which will evidently be when $EP + FP$ is a min. that is, when EP and FP make a right line; and EP will then be $= FP$; because (by Euc. 9. 2.) the sum of the squares of the two parts of a line equally divided is always less than the sum of the squares of the parts, when the line is divided unequally.



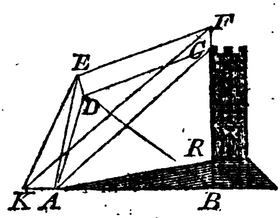
The same answered by Mr. W. Lucas.

Let A, B, C, and D represent the four given points, and P any point taken at pleasure; let PE and PF be drawn to bisect AB and DC in E and F; then (by a theorem in geometry) $AP^2+BP^2=2AE^2+2EP^2$, and $DP^2+CP^2=2DF^2+2FP^2$; therefore $AP^2+BP^2+DP^2+CP^2=2\times\overline{AE^2+DF^2+EP^2+FP^2}$, a minimum (per quest.); and consequently EP^2+FP^2 a minimum also: But, drawing PO to bisect EF, we get $EP^2+FP^2=2EO^2+2PO^2$: Therefore PO^2 a minimum, and so PO being $=0$, O must be the point required.

In this manner, and from the very same property, the problem is elegantly solved by Mr. *Rob. Butler*, Mr. *Patrick O'Cavanah*, and Mr. *W. Davies*.

IV. QUESTION 424 *answered by Mr.* Tho. Baxtondan, *the Proposer.*

Let the $\angle$s BAC and BAR be the given elevations of the top and base of the tower above the plane of the horizon AB, at the first station A; let D be the second station, and DE its perpendicular distance below the plane ABE of the first. In BC produced, suppose CF to be taken $=DE$; let $FK \parallel CA$, and let EF, EB, FK, and AD be drawn. Then, because DE is $=$ and $\parallel FC$, EF will also be $=$ and $\parallel DC$, and consequently the $\angle EFB$ $(=DCB)=$ the complement of the given elevation at the second station D. Now, from AD (100) and the $\angle DAE$ (5° 30'), we have given $AE=99{\cdot}5396$, and DE $(FC)=9{\cdot}5846$: and it will be as rad. : tang. BCA (60°) (: : $BC : BA$) : : $FC : KA=16{\cdot}601$: from which, together with AE and the contained $\angle KAE$ (135°), the $\angle AKE$ will be found $=38°\ 59'$. Moreover the $\angle$s FBE and FBK being both right ones, it will be tang. BFE (51° 30') : tang. BFK (60°) : : $(BE : BK)$: : sin. BKE (38° 59') : sin. $BEK=60°\ 5'$: whence ABE is also given $=80°\ 56'$, $AEB=54°\ 4'$: from which, and AE $(=99{\cdot}5396)$ will be found $AB=81{\cdot}61$; and from thence $BC=47{\cdot}12$, $BR=17{\cdot}34$, and $RC=29{\cdot}77=$ the tower's true height.

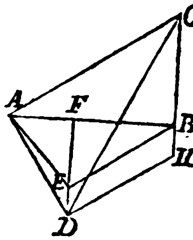


An

An Algebraic Solution to the ſame by Mr. Rich. Mallock.

'I have ſome reaſon to believe that this queſtion is wrong 'printed, viz. north for ſouth, or north-eaſt for ſouth-eaſt; 'which being corrected, I ſolve it thus.'

Let DE be the perpendicular diſtance of the ſecond ſtation D, below the horizontal plane (ABE) of the firſt ſtation A; in CB (the tower's height) produced, take $BH = ED$; join D, H, and upon AB let fall the perpendicular EF.

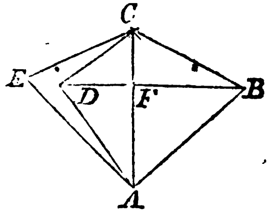


By means of AD ($= 100$) and the given $\angle DAE$ ($= 5^\circ\ 30'$) I find ED ($= BH$) $= 9{\cdot}58458$, and $AE = 99{\cdot}5395$; from which laſt AF (EF) $= 70{\cdot}3853$. Now put $BH = b$, AF ($= EF$) $= c$, and $BC = x$; and let m and n denote the tangents of the complements (BCA and HCD) of the tower's elevation at the two ſtations A and D: Then (by trigonometry) $BA = mx$, and $HD = n \times \overline{x+b} = BE$ (becauſe BH being ∥ and $= ED$, BE will alſo be ∥ and $= HD$). Hence $BF = mx - c$; and conſequently $\overline{mx-c}^2 + cc = \overline{nx+nb}^2$ (Eu. 47. 1.): which ordered, gives $xx - \frac{mc+nnb}{mn-nn} \times 2x = -\frac{2cc-nnbb}{mm-nn}$; in numbers, $xx - 193{\cdot}11x = -6877{\cdot}8$; whence $x = 96{\cdot}555 \pm 49{\cdot}44$; that is, the tower's height above AB is either $47{\cdot}11$ yards, or 146 yards; from the former whereof the required diſtance AB is found $= 81{\cdot}5$ yards.

In the very ſame manner the problem is anſwered by Mr. *W. Allen*, Mr. *R. Butler*, *Birchoverenſis*, Mr. *W. Marſhall*, Mr. *J. Milbourn*, Mr. *S. Towndrow*, and Mr. *W. Stoker*.

V. QUESTION 425 *anſwered by Mr.* O'Cavanah.

In the caſe propoſed, where the ſums of the ſquares of the oppoſite ſides of the trapezium are equal (and where the diagonals do, therefore, cut each other at right angles) the problem may be thus conſtructed.



Having made AB and AE perpendicular to each other, and equal each to any one of the given ſides (as 35) from the centers B and E with radii equal to the two oppoſite ſides (25 and 31) let two arcs be deſcribed,

intersecting in C; draw AC, and also BD $\perp$ and $=$ thereto; so shall $ABCD$ (when AD and CD are drawn) be the trapezium sought.

For the $\angle$s ABD and EAC are equal, being both complements of CAB, to a right angle (by construction), and the sides AB, BD, AE, AC, containing them, are also respectively equal (by construction): Whence the remaining sides AD and EC must necessarily be equal. Also $AB^2 - AD^2$ $(= BF^2 - DF^2) = BC^2 - DC^2$, and therefore $AB^2 + DC^2 = AD^2 + BC^2$, that is, in the present case, $35^2 + DC^2 = 31^2 + 25^2$, and consequently $DC = 19$, as it ought to be. The numerical solution is from hence very easy, whereby the area comes out $=$ 4 A. 1 R. $38\frac{1}{4}$ P.*

Mr. *W. Harrison*, Mr. *J. Honey*, Mr. *R. Mallock*, Mr. *J. Fearce*, Mr. *W. Stoker*, and Mr. *W. Terrill* (whose solutions agree in almost every step) give the following analytical investigation of this problem.

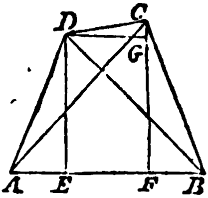
'Since the sums of the squares of the opposite sides of the 'trapezium $ABCD$ are equal (by the question), the dia'gonals will therefore cut each other at right angles in F: 'so that, putting $BC (= 25) = a$, $BA (= 35) = b$, $DA (= 31)$ '$= c$, $CD (= 19) = d$, and $AC (= BD) = x$, we shall have '$x : b + a :: b - a : AF - CF = \frac{bb - aa}{x}$; and $x : b + c$ '$:: b - c : BF - DF = \frac{bb - cc}{x}$; whence $AF = \frac{x}{2}$ '$\frac{bb - aa}{2x}$, and $BF = \frac{x}{2} + \frac{bb - cc}{2x}$; and consequently '$\overline{\frac{x}{2} + \frac{bb - aa}{2x}}\Big|^2 + \overline{\frac{x}{2} + \frac{bb - cc}{2x}}\Big|^2 = bb$; from which equa'tion $2x^4 - \overline{aa + cc} \times 2xx + \overline{bb - aa}|^2 + \overline{bb - cc}|^2 = 0$. 'This solved, gives $x = \sqrt{\frac{aa + cc}{2} \pm \sqrt{bbdd - \frac{1}{4} \times \overline{cc - aa}|^2}}$ '$= 37{'}9$; and the area sought $(\frac{xx}{2}) =$ 4 A. 1 R. 38 P.'

A general Solution of the same Problem by Birchoverensis.

Let the given sides DA, AB, BC, CD of the trapezium be denoted by a, b, c, and d respectively; and let each of the equal diagonals AC, BD, be called x: Then (supposing

* The numerical calculation at large may be seen at p. 129 Hutton's Mensuration.

(supposing DE and CF to be $\perp AB$, and $DG \parallel AB$) we shall have $b : x + a :: x - a : BE - AE = \frac{xx - aa}{b}$; whence $AE = \frac{bb + aa - xx}{2b}$, and $DE = \sqrt{\frac{-x^4 + \overline{aa + bb} \times 2xx - \overline{bb - aa}^2}{4bb}}$.



In the same manner $BF = \frac{bb + cc - xx}{2b}$, and $CF = \sqrt{\frac{-x^4 + \overline{cc + bb} \times 2xx - \overline{bb - cc}^2}{4bb}}$; and consequently DG $(EF = AB - AE - BF) = \frac{2xx - aa - cc}{2b}$; From which (because $\sqrt{DC^2 - DG^2} = GC = CF - DE$) we get $\sqrt{4bbdd - 4x^4 + \overline{aa + cc} \times 4xx - \overline{aa + cc}^2} = \sqrt{-x^4 + \overline{cc + bb} \times 2xx - \overline{bb - cc}^2} - \sqrt{-x^4 + \overline{aa + bb} \times 2xx - \overline{bb - aa}^2}$; whence, by involution and proper reduction, there results $2x^6 - \overline{aa + bb + cc + dd} \times x^4 - \overline{2aacc + 2bbdd - \overline{aa + cc} \times \overline{bb + dd}} \times xx + \overline{aacc - bbdd} \times \overline{aa + cc - bb - dd} = 0$; from which x may be found, let the values of a, b, c, and d be what they will: But, in the case proposed, $aa + cc$ being $= bb + dd$, the last term will vanish, and we shall have $2x^4 - \overline{2aa + 2cc} \times xx - \overline{2aacc - bbdd + \overline{aa + cc}^2} = 0$; which solved, gives $x = \sqrt{\frac{aa + cc}{2} \pm \sqrt{bbdd - \frac{1}{4} \times \overline{cc - aa}^2}}$ $= 37{\cdot}90025$. Whence the area $ABCD$ $(= \frac{1}{2}xx) = 718{\cdot}2145$ $= 4$ A. 1 R. $38{\cdot}2145$ P.

Mess. *W. Allen*, *Ja. Bank*, *T. Baxtonden*, *T. Barker*, *Ja. Beresford*, *R. Butler*, *R. Flitcon*, *Jos. Herrington*, *J. Hampson*, *R. Hudson*, *W. Kingston*, *W. Litson*, *W. Lucas*, *W. Marshall*, *W. Mathewson*, *Penovius*, *R. Richardson*, *Alex. Rowe*, *W. Spicer*, *J. Thompson*, and *J. Wilson*, also favoured us with solutions to this problem.

VI. QUESTION 426 *answered by Mr.* L. Charlton.

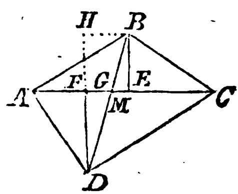
Let x, y, and z represent the sheep bought of each sort, and a, b, and c the given prices paid for each sort per head respectively; then will $ax + by + cz = 1200 (= d)$, and $xx + yy + zz$ a minimum (by the question). Now, seeing any two

two of the quantities x, y, z, may be here varied independently of the other, we shall (by making x and y to flow, while z remains constant) have $a\dot{x} + b\dot{y} = 0$, and $2x\dot{x} + 2y\dot{y} = 0$; from whence $\dot{x} = -\frac{b\dot{y}}{a} = -\frac{y\dot{y}}{x}$; and therefore $y = \frac{bx}{a}$. In the very same manner $z = \frac{cx}{a}$: Whence, by substitution, $ax + \frac{bbx}{a} + \frac{ccx}{a} = d$; and consequently $x = \frac{ad}{aa+bb+cc} = 24$; whence y is also given $= 32$, and $z = 40$.

In the very same manner the problem is resolved by Mr. *T. Baxtonden*, Mr. *R. Butler*, and Mr. *W. Lucas*; and in a manner very little different, by Mr. *W. Allen*, Mr. *W. Bevil*, Mr. *J. Fellows*, Mr. *R. Flitcon*, Mr. *J. Honey*, Mr. *T. Howe*, Mr. *R. Hudson*, Mr. *W. Kingston*, *Penovius*, Mr. *Jos. Scott*, Mr. *W. Spicer*, Mr. *W. Terrill*, and Mr. *J. Thompson*.

VII. QUESTION 427 *answered by Mr.* J. Thompson, *of* Witherly Bridge.

In the trapezium $ABCD$, put $AG = m$, $BG = n$, $CG = p$, $DG = q$, area $ABCD = A$, x and y for the sine and cosine of the $\angle AGD$; then, by a known



theorem, $\begin{cases} AB^2 = mm + nn + 2mny \\ DC^2 = pp + qq + 2pqy \end{cases}$

And $\begin{cases} AD^2 = mm + qq - 2mqy \\ BC^2 = nn + pp - 2npy \end{cases}$

Whence, by equal subtraction, $AB^2 + DC^2 - AD^2 - BC^2 = \overline{mn + pq + mq + np} \times 2y = \overline{m+p} \times \overline{n+q} \times 2y = AC \times BD \times 2y$. Again, by a known theorem, $mnx + npx + pqx + qnx = 2A$, or $\overline{m+p} \times \overline{n+q} \times x^* = 2A$; hence $\frac{2A}{x} = \frac{AB^2 + DC^2 - AD^2 - BC^2}{2y}$, and $\overline{AB^2 + DC^2 - AD^2 - BC^2} \times \frac{x}{4y} = A$. Q.E.D.

COROLLBRY. From the equation * the area of any trapezium is equal to half the rectangle under the two diagonals drawn into the sine of the angle contained by them.

The

The same answered by Mr. Moss, *the Proposer.*

Suppose the diagonal AC to be bisected in M; and having let fall the $\perp$s BE and DF, draw $BH \parallel AC$, meeting DF, produced, in H. It is well known, that $AB^2 - BC^2 = 2AC \times ME$, and $CD^2 - AD^2 = 2AC \times MF$: Whence, by equal addition, $AB^2 + CD^2 - BC^2 - AD^2 = 2AC \times \overline{ME + MF} = 2AC \times BH$: But $BD = \frac{BH \times \text{rad.}}{\text{cos. } AGD}$; whence $AC \times BD = AC \times BH \times \frac{\text{rad.}}{\text{cos. } G} = \overline{AB^2 + CD^2 - BC^2 - AD^2} \times \frac{\frac{1}{2}\text{rad.}}{\text{cos. } G}$. Again, $DH = BH \times \frac{\text{tang. } G}{\text{rad.}}$; whence $\frac{1}{2}AC \times DH$ (= area $ABCD$) $= \frac{1}{2}AC \times BH \times \frac{\text{tang. } G}{\text{rad.}} = \overline{AB^2 + CD^2 - BC^2 - AD^2} \times \frac{\frac{1}{4}\text{tang. } G}{\text{rad.}}$. *Q.E.D.*

Mess. *Ja. Bank, W. Bevil, Birchoverensis, R. Butler, W. Davies, W. Kingston, W. Lucas, Penovius, W. Spicer,* and *J. Wilson,* have also obliged us with answers to this problem; all whereof have a near resemblance to the one, or the other, of those above exhibited.

VIII. QUESTION 428 *answered by Mr.* Robert Butler.

Let a = the first term, r = common ratio; then $a + ar + ar^2 + ar^3 + ar^4 + ar^5$ $(= a \times \frac{r^6 - 1}{r - 1}) = 500$, and $a \times \overline{r^5 + 1} = 200$ (per question). Multiply these two equations cross-wise; whence $2 \times \overline{r^6 - 1} = 5 \times \overline{r^5 + 1} \times \overline{r - 1}$. Now put $2x = r + 1$, and $2z = r - 1$; then will $x + z = r$, and $x - z = 1$: therefore, by substitution, $\overline{x + z}^6 - \overline{x - z}^6 = 5z \times \overline{\overline{x + z}^5 + \overline{x - z}^5}$; which, being expanded, gives $19z^4 = x^4 - 30xxzz$; whence $xx = 15 + \sqrt{244} \times zz$; therefore $z = \frac{1}{\sqrt{15 + \sqrt{244}} - 1} = .22057$. Hence $r = 1.4412$, and $a = 27.527$: Therefore the six numbers answering the question are 27.527, 39.94, 57.56, 82.95, 119.55, and 172.473.

In this manner the problem is resolved by Mr. *W. Lucas.*

The same answered by Mess. Ja. Bank *and* J. Wilson.

Let x and z represent the two middle terms, and put $a = 200$, and $c = 300$: Then, by the question, $x + z + \frac{xx}{z} + \frac{zz}{x} = c$, and $\frac{x^3}{z^2} + \frac{z^3}{x^2} = a$, or (by reduction) $\overline{x+z} \times xz + x^3 + z^3 = cxz$, and $x^5 + z^5 = ax^2 z^2$; which, by putting $s = x + z$ and $p = xz$, will become $s^3 - 2ps = cp$, and $s^5 - 5ps^3 + 5p^2 s = ap^2$; from the former whereof we have $p = \frac{s^3}{2s+c}$; which value, substituted in the latter, gives $s^5 - \frac{5s^6}{2s+c} + \frac{5s^7}{\overline{2s+c}|^2} = \frac{as^6}{\overline{2s+c}|^2}$: Whence $s^2 + \overline{a+c} \times s = c^2$, and consequently $s = \frac{\sqrt{4c^2 + \overline{a+c}|^2} - a - c}{2} = 50\sqrt{61} - 250 = 140{,}51248$.

Hence $p\ (= \frac{s^3}{2s+c}) = \frac{2590000 - 320000\sqrt{61}}{19}$, and $x - z\ (= \sqrt{ss - 4p}) = \frac{\sqrt{805000\sqrt{61} - 6275000}}{19} = 25{\cdot}39269$; from whence the numbers are found to be 27·714, 39·939, 57·559, 82·952, 119·55, and 172·286.

The same Question answered by Mr. William Davies.

Put $2a = 200$ the sum of the two extremes, $3a = 300$ the sum of the four means, $a - ax =$ the first term, $a + ax =$ the last term, and $y =$ the common ratio: Then $\overline{a - ax} \times y^5 = a + ax$, and therefore $y^5 = \frac{1+x}{1-x}$; also $\overline{a - ax} \times \overline{y + y^2 + y^3 + y^4}$, or $\overline{a - ax} \times \frac{y - y^5}{1-y} = 3a$; whence $y^5 = \frac{4y - 3 - xy}{1-x} = \frac{1+x}{1-x}$; and consequently $y = \frac{4+x}{4-x} = \frac{c+x}{c-x}$ (putting $c = 4$.) Hence we have $\frac{\overline{c+x}|^5}{\overline{c-x}|^5} = \frac{1+x}{1-x}$; from which, by reduction, $\overline{5c - 1} \times x^4 + \overline{10c^3 - 10c^2} \times x^2 = 5c^4 - c^5$, or $19x^4 + 480x^2 = 256$; solved $x = \sqrt{\frac{32\sqrt{61} - 240}{19}}$; whence the rest may be found.

In

In this manner the solution is also given by Mr. *Richard Mallock*.

The same answered by Mr. T. Barker, *of* Westhall, *in* Suffolk.

Let $x - y$ and $x + y$ denote the third and fourth numbers (putting $a = 200$ and $b = 300$). Then (by the question) we shall have $\frac{\overline{x-y}|^3}{\overline{x+y}|^2} + \frac{\overline{x+y}|^3}{\overline{x-y}|^2} = a$, and $2x + \frac{\overline{x-y}|^2}{x+y} + \frac{\overline{x+y}|^2}{x-y} = b$; which, cleared of fractions, give $2x^5 + 20x^3y^2 + 10xy^4 = a \times \overline{x^4 - 2x^2y^2 + y^4}$, and $4x^3 + 4xy^2 = b \times \overline{xx - yy}$; from the last whereof we obtain $y = \frac{\frac{1}{4}bx^2 - x^3}{\frac{1}{4}b + x}$. Now substitute the value of yy in the other equation; from whence, after proper reduction, there will come out $xx + \frac{a+b}{2} \times x = \frac{bb}{4}$; which, solved, gives $x = \frac{\sqrt{aa + 2ab + 5bb} - a - b}{4} = 70{,}256$; therefore $y = 12{\cdot}696$; and the six numbers sought are 27·72, 39·964, 57·564, 82·952, 119·44, and 172·28.

In this last way, and by the very same substitution, the problem is also answered by Mr. *W. Spicer*; and in a manner very little different, by Mr. *W. Bevil*, *Birchoverensis*, and Mr. *W. Kingston*.

The same answered by Mr. Hugh Brown.

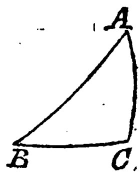
Let the two extremes be denoted by $a - x$ and $a + x$; then, putting the given sum of the four means $= b$, the sum of the 1st, 2d, 3d, 4th, and 5th terms will be $= b + a - x$, and that of the 2d, 3d, 4th, 5th, and 6th terms $= b + a + x$: Whence (by the nature of proportionals) it is manifest, that the first term is to the second as $b + a - x$ to $b + a + x$, or as 1 to $\frac{b+a+x}{b+a-x}$: But the ratio of the first term to the sixth is the quintuplicate of that of the first to the second, that is, $\frac{a+x}{a-x} = \overline{\frac{b+a+x}{b+a-x}}\Big|^5$. From whence, putting $c = b + a = 400$, we get $x \times \overline{c+x}|^5 + \overline{c-x}|^5 = a \times \overline{c+x}|^5 - \overline{c-x}|^5$, or $x \times 2c^5 + 20c^3x^2 + 10cx^4 = a \times$

$10c^4x$

$\overline{10c^4x+20c^2x^3+2x^5}$; and consequently $x^4+\frac{c-a}{5c-a}\times 10c^2x^2=\frac{5a-c}{5c-a}\times c^4$: Whence x will be found, and, from thence, all the other quantities fought.

IX. QUESTION 429 *anfwered by Mr.* W. Spicer.

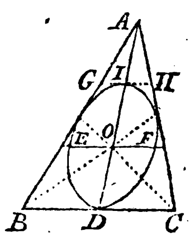
Let A be the port failed from, and B and C the ports to which the two fhips are bound. Then, in the right-angled fpherical triangle ABC, we have given the bafe BC, and the ratio of AB to AC, as 5 to 3: Therefore, putting $a=$ cofine BC ($33^\circ 20'$) and $x=$ cofine $\frac{1}{3}$ AC (or $\frac{1}{5}$ AB) we fhall (by a well known theorem) have $4x^3-3x=$ cofine of AC, and $16x^5-20x^3+5x=$ cofine of AB; and therefore (per fpherics) 1 (rad.) : a : : $4x^3-3x:16x^5-20x^3+5x$: and confequently $16x^4-20x^2+5=4ax^2-3a$; which folved, gives $x=\sqrt{\frac{5+a+\sqrt{24+\overline{1-a}|^2}}{8}}=0{\cdot}990091$; anfwering to an arch of $8^\circ 4' 21''$. Therefore $AC=24^\circ 13' 3''$, or 1453·05 miles; and $AB=40^\circ 21' 45''$, or 2421·75 miles.



Mr. *O'Cavanah*, *Penovius*, and Mr. *W. Stoker*, anfwer it exactly in the fame manner.

X. QUESTION 430 *anfwered by* Penovius.

Draw the tangent GH, and the conjugate diameter EF, parallel to BC. Put $BD=DC=20=b$, $AD=51{\cdot}478=a$, the fine of the angle at D ($74^\circ 30' 30''$) $=s$, and $DI=x$; then will $AI=a-x$, $GI=IH=\frac{b}{a}\times\overline{a-x}$, by fimilar triangles; and, by the property of the ellipfis, $EO=OF=b\sqrt{\frac{a-x}{a}}$; whence $bx\sqrt{\frac{a-x}{a}}$ is a maximum; in fluxions $2ax\dot{x}-3x^2\dot{x}=0$, and $x=\frac{2a}{3}$; whence $EF=\frac{2b}{\sqrt{3}}$, and the area of the ellipfis $=\frac{3{\cdot}1416sab}{3\sqrt{3}}=599{\cdot}8582$.



Now

Now put the transverse axis $=y$, and the conjugate $=z$; then, by conics, $yz=\frac{4ab}{3\sqrt{3}}=2n$, and $yy+zz=\frac{4aa}{9}+\frac{4bb}{3}=4m$; from which $y+z=2\sqrt{m+n}$, and $y-z=2\sqrt{m-n}$; whence $\sqrt{m+n}\pm\sqrt{m-n}=\begin{cases}y=35\cdot229236\\ z=21\cdot67976\end{cases}$ the principal diameters required.

COROLLARY 1. All the sidas of the triangle are bisected by the points of contact; and the center of the ellipsis coincides with the center of gravity of the triangle.

COROLLARY 2. The area Δ : the area of the ellipse :: $3\sqrt{3}:3\cdot1416$. Hence the areas of all triangles circumscribing the same ellipsis, having their sides bisected by the points of contact, are equal: When $s=1$, the triangle is isosceles; and equilateral, when the ellipsis becomes a circle.

Mr. *W. Kingston* solves it in a manner very little different.

Mr. *O'Cavanah*, after demonstrating geometrically, that the least triangle that can be described about any oval figure, will have all its sides bisected by the points of contact, derives, by a different method, the very same conclusions above exhibited; and farther adds, by way of note, 'That the greatest parabola that can be described in a given 'triangle, may be determined by the same method; the area 'thereof being to that of the triangle, as $\sqrt{3}$ is to 2; which 'proportion is general, whatever the species of the triangle 'is, or which ever of its sides the base of the parabola is 'supposed to stand upon.' 'Those persons (continues he) 'are therefore mistaken, who make the area to be greater 'or less, according as the base is made to coincide with this, 'or that side of the triangle.——The axis of the parabola 'will not be perpendicular to the base of the triangle, but 'parallel to a line drawn from the vertex to bisect the base.'

XI. QUESTION 431 *answered by the Proposer, Mr.* Bevil.

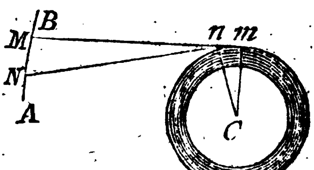
Let $a=$ diameter of the post, $b=$ twice the thickness of the binding, $n=$ number of rounds, $f=a+nb$, and $c=3\cdot14159$ &c. then will f, $f-b$, $f-2b$, $f-3b$, &c. be the diameter of the post and binding, after 0, 1, 2, 3, &c. rounds are disengaged; and cf, $c\times\overline{2f-b}$, $c\times\overline{3f-3b}$, &c. will be the length disengaged in 1, 2, 3, &c. rounds respectively. But, if d be taken to denote the diameter, and s the length of the part disengaged, at the end of any number (r) of rounds; then (the length unwrapped at the end of the next

round after, being $= s + cd$) it appears (by p. 163 of Simpson's Fluxions) that the distance moved over in unwrapping that round, will be $= \frac{\overline{s+cd}^2 - ss}{d} = 2cs + ccd$. Now, when $r = 0$, then $d = f$, and $s = 0$; therefore, in the first round, $2cs + ccd = ccf$. When $r = 1$, then $d = f - b$, $s = cf$, and consequently $2cs + ccd = 2ccf + ccf - ccb = cc \times \overline{3f - b}$. When $r = 2$, then $d = f - 2b$, $s = c \times \overline{2f - b}$; and therefore $2cs + ccd = cc \times \overline{5f - 4b}$: In the same manner, when $r = 3$, then $2cs + ccd = cc \times \overline{7f - 9b}$, &c. Now, by collecting all these values of $2cs + ccd$ together, we get the two following series, viz. $ccf \times : 1 + 3 + 5 + 7 +$ &c. and $-ccb \times : 0 + 1 + 4 + 9$, &c. where each series is to be continued to n terms: These seriefes being summed, we have $ccf \times nn - ccb \times \frac{1}{6}n . \overline{n-1} . \overline{2n-1} = ncc \times \overline{nf - \frac{1}{6}b . \overline{n-1} . \overline{2n-1}} = 26$ M. 1 F. $=$ the distance sought.

In this manner the solution is also given by Mr. *R. Butler.*

The same answered by Mr. H. Watson.

Let $a =$ the semi-diameter of the post, $b =$ the thickness of the binding wrapped thereon, $n =$ the whole number of rounds at first, $x =$ the number of rounds unwrapped at the end of any time, and $u =$ the length of the part disengaged at that time: And, supposing N and M to be two positions of the end of the line indefinitely near to each other, to the points of contact n and m, from the center C, let Cn and Cm be drawn.



It is evident, that $a + nb$ will be the semi-diameter of the post and binding together, and consequently that the length of the first round (computed at the middle of the binding's thickness) will be $= p \times \overline{a + nb - \frac{1}{2}b}$ (p being put $= 2 \times 3{\cdot}1416$). In the same manner the length of the last of x rounds will be $= p \times \overline{a + \overline{n - x - \frac{1}{2}} . b}$. Therefore we have here the first and last terms of an arithmetical progression, whereof the number of terms is x; whereby the sum of the progression is found $= p \times \overline{2a + \overline{2n - x} . b} \times \frac{x}{2} = u$.

Now,

Now, since the angles NnC and Mmc are both right ones, the angle NmM will be $= nCm$; and therefore the figures NmM and nCm being similar, we have as Cn

$$(a+\overline{n-x}.b) : nm(\dot{u}) :: Nm(u) : NM = \frac{u\dot{n}}{a+n-x.b}:$$

But $\dot{u} = p\dot{x} \times \overline{a+\overline{n-x}.b}$; and consequently $NM = p\dot{x} \times u = pp \times \overline{a+n}.x\dot{x} - \frac{1}{2}bx^2\dot{x}$; whose fluent ($pp \times \overline{a+nb}.\frac{1}{2}xx - \frac{1}{6}bx^3$) when $x = n$, will be $= nnpp \times \frac{1}{2}a + \frac{1}{3}nb = 1654320$ inches ($= 26$ M. 1 F. nearly) $=$ the distance sought.

N.B. If the curve AMB be considered as the involute of *Archimedes*'s (or any other kind of) spiral, the length thereof will be always found, by multiplying Nn and its fluxion together, dividing the product by the radius of the curvature of the evolute (at n), and then taking the fluent of the quotient.

XII. QUESTION 432 *answered by Mr.* O'Cavanah.

By division, and extracting the square root on both sides of the proposed equation, we have $\sqrt{\frac{q}{n}} \times \frac{\dot{x}}{x\sqrt{\frac{mx}{n}-1}}$ $= \frac{1}{p} \times \frac{\dot{z}}{1+\frac{zz}{p}}$; which, by substituting $u = \frac{z}{\sqrt{p}}$, and $y = \sqrt{\frac{mx}{n}-1}$, is further transformed to $\frac{\dot{y}}{1+yy} = \frac{1}{2}\sqrt{\frac{n}{pq}} \times \frac{\dot{u}}{1+uu}$: whence, supposing A and B to denote the two circular arcs whose tangents are y ($\sqrt{\frac{mx}{n}-1}$) and u ($\frac{z}{\sqrt{p}}$), we shall, by taking the fluent, have $A + C = \frac{1}{2}B\sqrt{\frac{n}{pq}}$; C being a constant arch, serving to correct the fluent. From this equation, when either of the quantities z or x is given, the other may be determined (from a table of tangents) in all cases, except when imaginary quantities enter into the consideration: Thus, z being supposed given, the tangent u ($\frac{z}{\sqrt{p}}$) will also be given; and from thence (by the

table) the arch B corresponding; whereby the arch A $(= \frac{1}{2}B\sqrt{\frac{n}{pq}} - C)$, and its tangent y, will be known, and consequently x $(= \frac{n}{m} \times \overline{1+yy})$.

When n, p, and q are such, that $\frac{1}{2}\sqrt{\frac{n}{pq}}$ is a rational quantity, then may the relation of x and z be algebraically expressed by a finite equation: For, here the fraction $\frac{r}{s}$, where r and s are integers, may be assumed $= \frac{1}{2}\sqrt{\frac{n}{pq}}$; and we shall have $s \times \overline{A+C} = rB$: But, if c be taken to denote the tangent of the arch C, that of $A+C$ will be $= \frac{c+y}{1-cy}$. Whence (by the theorem in the solution of Q. 388) the tangent of $s \times \overline{A+C}$ will be had $=$ $\overline{s \cdot \frac{c+y}{1-cy} - \frac{s}{1} \cdot \frac{s-1}{2} \cdot \frac{s-2}{3} \cdot \overline{\frac{c+y}{1-cy}}\Big|^3 + \&c.} \div \overline{1 - \frac{s}{1} \cdot \frac{s-1}{2} \cdot \overline{\frac{c+y}{1-cy}}\Big|^2 + \&c.}$ which must necessarily be equal to the tangent of $rB = \overline{ru - \frac{r}{1} \cdot \frac{r-1}{2} \cdot \frac{r-2}{3} \cdot u^3 \ \&c.} \div \overline{1 - \frac{r}{1} \cdot \frac{r-1}{2} \cdot u^2 \ \&c.}$ where r and s being integers, the serieses will terminate, and the relation of y $(= \sqrt{\frac{mx}{n}} - 1)$ and u $(\frac{z}{\sqrt{p}})$ will therefore be expressed in finite terms.

Mr. *Rollinson* (the proposer) answers this question, in almost the very same manner. *Peter Walton* has brought out an elegant solution to it, from p. 66 of Landen's Mathematical Lucubrations; and, in a way not greatly different, the solution is also given by Mr. *R. Butler*, and Mr. *L. Charlton*.

XIII. QUESTION 435 *answered by* Peter Walton.

Let A denote the force which, acting at the end n, at right angles to the rod, would accelerate the velocity of that end about (G) the center of gravity of the rod, at the same rate as the said velocity is accelerated by the action of the rod against AB; and let A' denote the force which, acting at the end m, at right angles to the rod, would retard the

the velocity of that end about G, at the same rate as the said velocity is retarded by the action of the rod against AC: Also let u be the velocity of m, or n, about G; v, the velocity of G towards the horizon; w, its velocity in a direction parallel to the horizon; t, the time the rod has been in motion, while it touches AC; T, the time from the rod's ceasing to touch AC; d, the distance of G from AB, at the commencement of the motion; x, the distance of G from AB, at any time after the motion has commenced; $2a$, the length of the rod; $s = 16\frac{1}{12}$ feet; and $y = \sqrt{aa - xx}$.

Then will the absolute weight of the rod, the pressure against AB, and the pressure against AC, be as $2s$, $\frac{aA'}{3y}$, and $\frac{aA''}{3x}$ respectively: And $\frac{aA''}{3x} \times \frac{-\dot{x}}{v}$ will be $= \dot{w}$, $\overline{A' - A''} \times \frac{-\dot{x}}{v} = \dot{u}$, $\overline{2s - \frac{aA'}{3y}} \times \frac{-\dot{x}}{v} = \dot{v}$, $av = uy$, and $aw = ux$.

By means of which equations, we find $3w\dot{w} + u\dot{u} = -6s\dot{x} - 3v\dot{v}$, while the rod continues to touch AC.

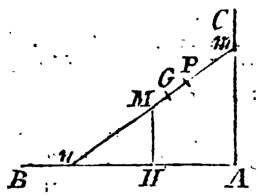
Hence, by taking the correct fluents and expunging u and w, we have $v = \frac{\sqrt{3ds - 3sx} \times y}{a} = \frac{\sqrt{3ds - 3sx} \times \sqrt{aa - xx}}{a}$.

Therefore w is $= \frac{\sqrt{3ds - 3sx} \times x}{a}$; whose fluxion is $= 0$, and consequently $x = \frac{2d}{3}$, when $A'' = 0$, i. e. when the rod ceases to touch AC. Moreover $\dot{t}$ is $= \frac{-a\dot{x}}{\sqrt{3ds - 3sx} \times \sqrt{aa - xx}}$; whose correct fluent, when x is $\frac{2d}{3}$, is the time required in the question.

After the rod has ceased to touch AC, (A'' being $= 0$, and w invariable) we shall have $A' \times \frac{-\dot{x}}{v} = \dot{u}$, $\overline{2s - \frac{aA'}{3y}} \times \frac{-\dot{x}}{v} = \dot{v}$, and $av = uy$. From which equation we get $u\dot{u} = -6s\dot{x} - 3v\dot{v}$: Hence, by taking the correct fluents expunging u, and putting $d - \frac{d^3}{9aa} = b$, we find $v =$

$\frac{2y\sqrt{3sh-3sx}}{\sqrt{aa+3yy}} = \frac{2\sqrt{aa-xx}\times\sqrt{3sh-3sx}}{\sqrt{4aa-3xx}}$. Consequently $\dot{T}$ is $= \frac{\dot{x}\sqrt{4aa-3xx}}{2\sqrt{aa-xx}\times\sqrt{3sh-3sx}}$; whereby the time from the rod's ceasing to touch AC, to its coincidence with AB, will be found; from whence and the given (invariable) velocity $\frac{2d\sqrt{ds}}{3a}$, of the center of gravity from the plane AC, during that time, the required distance of the end of the rod from AC will be obtained.

Mr. *O'Cavanah* has also obliged us with a solution to this very difficult problem, which agrees in every particular with that exhibited above. This gentleman, at the end of his solution, subjoins the following remark: 'If the figure of the rod 'should be such that the center 'of gravity (G) is not in the 'middle point M; then, putting '$h = nG$, and k ($= nP$) = the 'distance of the center of oscil-



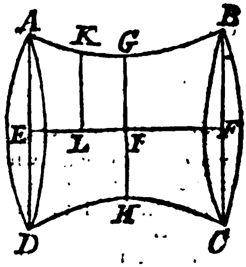
'lation (P) from the end n (considered as the point of suspension), it will appear, in like manner, that the celerity 'with which the said end recedes from the plane AC, will 'be $2y\sqrt{\frac{hf\times\overline{c-y}}{\frac{1}{4}dhk-\overline{h-d}\times yy}}$,* When this quantity is a 'maximum, the end (m) will quit the plane AC; after 'which the relative celerity wherewith n recedes from 'MH, will be $= 2y \times \sqrt{\frac{df\times\overline{r-y}}{ddk-hyy}}$: wherein r is a constant quantity, to be determined (like m above) from the 'value of y, when the end m ceases to to touch the plane 'AC.'

* Note, That, in this author's solution, $d = Mn$, $y = MH$, $c =$ the first value of y, and $f = 16\frac{1}{12}$ feet.

PRIZE

PRIZE QUESTION *answered by Mr.* E. Rollinson.

Let $ABCD$ represent the cask or vessel; put the given semi-length $EI = c$, and the given semi-diameter (AE) at the end $= b$; put also $IL = x$, and $LK = y$, supposing LK to be variable, and ∥ IG: Then, the fluxion of the curve surface being $3{\cdot}1416 \times 2y\sqrt{\dot{y}\dot{y} + \dot{x}\dot{x}}$, it follows, from the conditions of the problem, that the fluent of $\sqrt{\dot{y}\dot{y} + \dot{x}\dot{x}}$ ought to be a minimum, when that of $\dot{x}$ becomes equal to the given quantity IE.



Hence, by the general rule for the resolution of isoperimetrical problems (vid. p. 101 of Simpson's Miscellaneous Tracts) it appears that $\dfrac{y\dot{x}}{\sqrt{\dot{y}\dot{y} + \dot{x}\dot{x}}} - a = 0$; whence $\dot{x}$ is found $= \dfrac{a\dot{y}}{\sqrt{yy - aa}}$; and consequently $x = a \times$ hyp. log. $\dfrac{y + \sqrt{yy - aa}}{a}$, answering to the catenaria; in which $a = IG$ (because when $y = a$, x will be $= 0$).

The equation of the curve being thus known, the fluxion of the generated solid, $3{\cdot}1416 \times yy\dot{x}$, will be found $= 3{\cdot}1416a \times \dfrac{y^2\dot{y}}{\sqrt{yy - aa}}$, and consequently the solid itself $= 3{\cdot}1416a \times \frac{1}{2}y\sqrt{yy - aa} + \frac{1}{2}ax$; the double of which, when $x = c$, and $y = b$, will be the whole content of the vessel.

But now to find a (which is yet unknown), the general equation, when $y = b$, and $x = c$, will give $c = a \times$ hyp. log. $\dfrac{b + \sqrt{bb - aa}}{a}$, or $\dfrac{c}{a} =$ H. L. $\dfrac{b}{a} + \sqrt{\dfrac{bb}{aa} - 1}$; which, by putting $\dfrac{b}{a} = v$, will become hyp. log. $v + \sqrt{vv - 1} = \dfrac{cv}{b} = {\cdot}5v$ (in the present case); whence v is found $= 1{\cdot}1788$; and from thence $a = 31{\cdot}388$. Hence the content will come out

out 16121 cubic inches, or 69·79 wine gallons. And the curve surface will be less than that of the circumscribing cylinder, in the proportion of 95·54 to 100.

The solution by Mr. *Rollinson*, here given, being the only true one that came to hand before Candlemas-day, that gentleman is therefore intitled to a prize of 12 diaries, without the chance of drawing lots.

Of the Eclipses in the Year 1758.

There will this year be six eclipses, four of the sun, and two of the moon; whereof one of the moon only will be visible to the inhabitants of Great Britain.

The 1st is of the sun, January 9th, at 6 in the evening, invisible.

The 2d is a visible, and total eclipse of the moon, on January 24th, some calculations whereof are as follow:

Calculated by		Beg. of Eclip.	Beg. total Dark.	Mid. of Eclip.	End total Dark.	End of Eclip.	Digits eclipsed
	For	h. m.	h. m.	h. m.	h. m.	h. m.	° ′
Mr. S. Towndrow,	London	4 22	5 28	6 21	7 14	8 20	21 25
	Chesterfield Derbyshire	4 13	5 19	6 12	7 05	8 11	
Mr. W. Terrill,	London	4 21	5 27	6 20	7 13	8 19	21 30
	Rudruth, in Cornwall	4 1	5 7	6 0	6 53	7 59	
Mr. J. Gutteridge,	London	4 24	5 30	6 23	7 16	8 22	21 32
	Loughboro'	4 19	5 25	6 18	7 11	8 17	

The 3d is of the sun, February 8th, between 4 and 5 in the morning, invisible.

The 4th is of the sun, July 4th, at 9 in the morning, invisible.

The 5th is of the moon, July 20th, at 5 in the afternoon, invisible.

The sixth is of the sun, December 30th, between 7 and 8 in the morning, invisible.

*** Mr. *Charles Brent* observes (very justly) that those authors are mistaken who affirm, That there can never happen more than six eclipses in one year. Besides those mentioned above, he takes notice of a small solar defect of about half a digit, on August 3d, between 10 and 11 at night; which no other author has adverted to.—But whether the errors of the best tables extant, may not be sufficient to render the happening of *such* an eclipse doubtful, we do not undertake to decide.

New

New Questions.

I. QUESTION 434, *by Miss* T. S—e.

Addressed to Mr. U. T——r, *who took the Liberty to ask her the following Questions, viz. What age? What fortune? And what height she was?*

My height, sir, in inches, is three times my years;
My fortune their squares will both shew;
Put all these together, there then, sir, appears
The number expos'd to your view.* (*4494.
From which, sir, determine the things you requir'd;
And then, if more favours you want,
As lovers of science I always admir'd,
Those favours, perhaps, I may grant.

II. QUESTION 435, *by Mr.* P. O'Cavanah.

A tortoise once (or Æsop lies)
Run with a hare, and won the prize;
The hare, in the first minute's space,
Four furlongs run o'th' destin'd race,
While far behind her foe crept on,
And only crawl'd the fourth of one:
Puss, looking back, observes the case,
Disdains her foe, and bates her pace:
In the next minute she run o'er
But half the ground she run before;
And in the third was only reckon'd,
One half the space she went the second;
Decreasing still (as artists call)
In ratio geometrical:
Mean time the tortoise still crept on,
At the same rate as he began:
Now deign to shew, ye learned fair,
In what time he o'ertook the hare.

III. QUESTION 436, *by Mr.* John Brickland, *Teacher of the Mathematics in* Oxford.

What is the side of that equilateral triangle, whose area cost as much paving at 8d. a foot, as the pallisadoing the three sides did at a guinea a yard?

IV. QUESTION 437, *by Mr.* Thomas Moss.

From the vertical angle of a triangle (whose base is 70, and its two sides 40 and 50) to draw a line, terminating in the

the base, so as to be a mean proportional between the two segments of the base made thereby.

V. QUESTION 438, *by Mr.* Peter Walton.

To place three circles (whose radii are 10, 15, and 20) so that three right lines may be drawn each, to touch all the three circles.

VI. QUESTION 439, *by Mr.* W. Bevil.

The base of an isosceles triangle being given = 100, and each of the equal sides = 60; so to draw a right line from the vertex to the base, that the solid (or continual product) under it and the two segments of the base shall be the greatest possible.

VII. QUESTION 440, *by Mr.* J. Fellows, *of his Majesty's Ship* Captain.

Under the line (or very near)
NE, SW, two ports do bear,
Whose distance, if I rightly guess,
Is fifty leagues, no more nor less;
Between which ports a current goes,
SE, three knots, as I suppose:
The quere is, how must I sail
From northern port, in five-knot gale,
That I the southern port may make,
In the least time the thing can take?
And what that time will be, declare:
I'll do as much for you next year.

VIII. QUESTION 441, *by Mr.* Lionel Charlton.

It being a common practice among ship-chandlers to make their log-lines 42 feet long, and their half-minute glasses 28 vibrations of a pendulum, whose length is 38 inches and an half: Quere, how far a ship's reckoning will vary from the truth, that uses only these, and sails by mercator on one rhumb from the Lizard to the Capes of Virginia?

IX. QUESTION 442, *by Mr.* J. Wilson.

Given $\left\{\begin{array}{l} x^2z+z^2x=a \\ x^2z^7+z^2x^7=b \end{array}\right\}$ To find x and z, without having the root of any adfected equation to extract higher than a quadratic.

X. QUES-

X. QUESTION 443, *by Mr.* H. Watson.

From the equation $44000xx+1=zz$, to find both x and z in whole numbers.

XI. QUESTION 444, *by* Fr. Bell.

A gentleman having in his garden an elliptical fountain, whose greater diameter is 30, and the lesser 24 feet, orders a free-stone walk to be made round it, to be every where of an equal breadth, and to take up just the same quantity of ground as the fountain itself: Quere, what must the breadth of the walk be?

XII. QUESTION 445, *by* Plus-Minos.

One morning last summer, being in a gentleman's garden, I saw a very good horizontal sun-dial, which was not fixed down, but neatly let into the post on which it stood: Taking hold of the gnomon, I pulled it towards me, thereby elevating the south side of the dial, still keeping the gnomon in the plane of the meridian: Upon this, the shadow came forward from the hour-line of 7, till it marked 7 h. 40 m. and no nearer to the meridian, or 12 o'clock, would it come; for, if the dial was farther elevated, it turned back again: Now the day of the month (which I have forgot, and for private reasons should be glad to recollect) is here required.

XIII. QUESTION 446, *by Mr.* P. O'Cavanah, *of* Dublin.

How must I inclose an acre of land into a garden, with a fence of 80 poles in circumference, so as to be able to form therein the longest (straight) walk possible, and what will that length be, supposing the breadth of the walk to be 10 feet?

XIV. QUEST. 447, *by* Peter Walton, Discip. Landenii.

A chain, 10 yards long, consisting of indefinitely small equal links, being laid straight on an horizontal (perfectly polished) plane, except one part, a yard in length, which hangs down perpendicularly below the plane: In what time will the said chain (drawn by the gravity of the descending part) entirely quit the plane?

PRIZE QUESTION, *by Mr.* G. Witchell.

To determine the nearest distance of the orbit of the expected comet from that of the earth, together with the longitude of the earth and comet in that situation, supposing, 1. That the orbit of the earth is a circle. 2. That the trajectory

jectory of the comet is a parabola, whose focus is the center of the earth's orbit. 3. That the perihelion distance of the comet is to the radius of the circle, as 0·5868 to 1. 4. That the inclination of the planes is 17° 2′, the place of the perihelion ♒ 3° 30′, and that of the descending node ♍ 22° 13′.

1759.

Questions answered.

I. QUESTION 434 *answered by Mr.* Tho. Baker, *to Miss* T. S——e, *the Proposer.*

YOUR age, dear Miss, is twenty-one, your height is five feet three;
Forty-four hundred pounds and ten, will just your fortune be.

Algebraic Solution.

Let x represent the lady's age, then her height (in inches) will be $3x$, and her fortune (in pounds) $= 10xx$, by the conditions of the question; from whence we have also given $\underline{10xx + 3x + x = 4494}$; therefore $xx + 0{\cdot}4x = 449{\cdot}4$, and consequently $x = \sqrt{449{\cdot}44} - 2 = 21$. Hence the lady's age appears to be 21, her height five feet three inches, and her fortune 4410 pounds.

In this manner the answer is given by Mr. *W. Ashton*, Mr *G.* and *J. Atkinson*, Mr. *Bamfield*, Mr. *Barras*, Mr. *J. Brickland*, Mr. *Cha. Cave*, Mr. *J.* and *W. Chapman*, Mr. *T. Corbett*, Mr. *N. Cory*, Mr. *T. French*, Mr. *E. Hare*, Mr. *D. Hastings*, Mr. *R. Hodges*, Mr. *T. Jeffery*, Mr. *Sam. Koit*, Mr. *G. Langley*, Mr. *J. Lewin*, Mr. *B. Lydal*, *Little Conjurer*, Mr. *R. Marsh*, *Mathematicus*, Mr. *G. Nokes*, Mr. *R. Quant*, Mr. *Pet. Rogers*, Mr. *W. Swift*, Mr. *J. Thorne*, Mr. *Jos. Wastel*, Mr. *J. Writer*, and by a multitude of others.

II. QUESTION 435 *answered by Mr.* B. Lydal.

Since the spaces run by the hare, in the several succeeding minutes, are expressed by 4, 2, 1, ½, ¼, &c. respectively, it is plain, from addition only, that the whole space gone over

over from the beginning, will be always less than 8 furlongs by a space equal to that passed over in the last minute. Therefore since the space passed over in the last minute (after a continual decrease of one half every minute) must be extremely small, the whole space gone over by the hare, when the tortoise overtakes her, will amount to 8 furlongs, extremely near; which, at the rate of $\frac{1}{4}$ of a furlong in a minute, will take the tortoise 32 minutes to crawl over, in order to come up with the hare.

The same answered by Mr. W. Kingston, *of* Bath.

If the number of minutes be denoted by x, the space gone over by the tortoise will be $= \frac{x}{4}$, and that by the hare $= 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$ &c. $= 8 \times \overline{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}}$ &c. continued to x terms, whereof the last is evidently $= 8 \times \frac{1}{2^x}$ $(= \frac{8}{2^x})$. Whence the sum of the whole progression (being = the double of the first term *minus* the last, in all cases where the ratio is $\frac{1}{2}$) is had $= 8 - \frac{8}{2^x}$. Hence we have $8 - \frac{8}{2^x} = \frac{x}{4}$, or $32 - \frac{1}{2^{x-5}} = x$; where, if $\frac{1}{2^{x-5}}$ be rejected on account of its extreme smallness, we shall have $x = 32$. To correct this value, let $\frac{1}{2^{x-5}} = \frac{1}{134217728}$, be now subtracted from 32; whence $x = 31\frac{134217727}{134217728}$.

The same answered by Pettovius.

Supposing that the hare moved uniformly in each minute, the distance run by her was $= 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \overline{\tfrac{1}{2}}|^2 + \overline{\tfrac{1}{2}}|^3 +$ &c. ... $+ \overline{\tfrac{1}{2}}|^{27} \times x = 31 + \frac{67108863}{67108864} + \frac{x}{134217728}$, and by the tortoise $= 1 + 1 + 1 +$ &c. ... $+ x = 31 + x$; Therefore $x = \frac{67108863}{67108864} + \frac{x}{134217728}$; whence $x = \frac{134217726}{134217727}$, and therefore $31\frac{134217726}{134217727}$ minutes is the true time required.

According to some one or other of the three methods above given, the problem is also answered by Mess. *T.* and *W. Allen*, *W. Ashton*, *W. Bamfield*, *T. Barker*, *E. Barras*, *J. Beresford*, *T. Bosworth*, *Turner Boston*, *J. Buddles*, *T. Corbett*, *T. Crabtree*, *E. Ellis*, *R. Gibbons*, *H. Green*, *J. Hampson*, *T.* and *W. Harrison*, *Jos. Harrington*, *D. Hastings*, *G. Hicks*, *Malachi Hitchins*, *J. Honey*, *Abr. Horsfall*, *T. Hopkinson*, *T. Howe*, *Steph. King*, *R. Langley*, *W. Matthewson*, *Chr. Mesban*, *S. Pedley*, *R. Pitches*, *W. Penn*, *Alex. Rowe*, *J. Scott*, *W. Spicer*, *G. Stapley*, *W. Swift*, *J. Thompson*, *J. Thorne*, *Ja. Vicary*, *Matt. Ward*, *Jos. Wastel*, *R. Walton*, *T. Wilkin*, *J. Wilson*, and a great many others.

III. QUESTION 436 answered.

If the length, in feet, of each side of the triangular garden be denoted by x, then the pallisadoing of the three sides, at 21s. per yard, will amount to $21x$ shillings: Moreover, the perpendicular being $= \sqrt{xx - \frac{xx}{4}} = \frac{x\sqrt{3}}{2}$, the area of the garden, in square feet, will be $= \frac{xx\sqrt{3}}{4}$; whereof the paving, at $\frac{2}{3}$ of a shilling per foot, will come to $\frac{xx\sqrt{3}}{6}$ shillings. Hence, by the question, we have $\frac{xx\sqrt{3}}{6} = 21x$; and therefore $x = \frac{126}{\sqrt{3}} = 42\sqrt{3} = 72{\cdot}74615$, the required length of each side.*

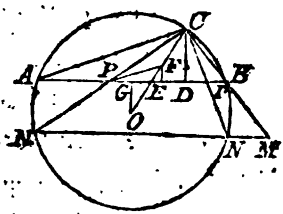
Thus the problem is resolved by Mr. *J. Brickland* (the proposer), Mr. *E. Barras*, Mr. *Turner Boston*, Mr. *J. Buddles*, Mr. *F. Butcher*, Mr. *C. Cave*, Mr. *W. Cooke*, Mr. *T. Crabtree*, Mr. *W. Harrison*, Mr. *Jos. Harrington*, Mr. *Malachi Hitchins*, Mr. *E. Houlston*, Mr. *T. Jeffery*, *Little Conjurer*, Mr. *W. Matthewson*, Mr. *Chr. Mesban*, Mr. *S. Pedley*, Mr. *W. Penn*, Mr. *R. Quant*, Mr. *Alex. Rowe*, Mr. *Oliver Shakespear*, Mr. *G. Stapley*, Mr. *R. Walton*, Mr. *Matt. Ward*, Mr. *Jos. Wastel*, Mr. *T. Wilkin*, and by upwards of 40 other persons.

IV. QUES-

* See this Question solved without Algebra p. 118 Hutton's Mensuration.

IV. QUESTION 437 *answered by Mr.* Rob. Shirtcliffe, *Author of the* Theory and Practice of Gauging.

About the given triangle ABC let a circle be circumscribed, and in the side CB, produced, take $BM = CB$; drawing MNN parallel to AB, intersecting the circle in N, N; then draw CN, cutting AB in P; so shall CP be a mean proportional between AP and BP. For, MN being parallel to AB (by construct.) and $BM = BC$, it is evident that PN is also $= PC$ (Euc. 2. 6.); and consequently $CP^2 = CP \times PN = AP \times BP$ (Euc. 35. 3.) or $AP : CP :: CP : BP$ (Euc. 16. 6.) *Q. E. D.*



Thus the problem is also constructed by Mr. *Moss* (the proposer), and Mr. *W. Spicer*.

The same constructed, and answered otherwise, by Mr. W. Davies.

Let a circle be described about the given $\triangle ABC$; and, having bisected the radius OC in F, from this point to AB apply $FP = FC$ (or FO); then will GP, when drawn, be the required mean proportional between AP and BP.

CALCULATION. There being given $BC = 40$, $AC = 50$, and $AG = BG = 35$, the perpendicular CD will be found $= \frac{80\sqrt{6}}{7}$, $GD = \frac{45}{7}$, and $GE = ED = \frac{45}{14}$: Then $\frac{AC \times BC}{2CD} = OC = \frac{175}{2\sqrt{6}}$, $\frac{1}{2}CD - \frac{1}{2}OG = FE = \frac{40\sqrt{6}}{7} - \frac{35}{4\sqrt{6}}$, and $\overline{FP^2 - FE^2}|^{\frac{1}{2}} = EP = \frac{5\sqrt{1649}}{14} = 14.5$: Whence $AP = 23.712$, and $BP = 46.288$.

In this manner the solution is also given by Mr. *Rob. Butler*, and Mr. *Henry Watson*.

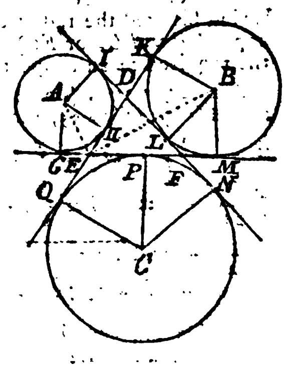
An Algebraical Solution to the same by Mr. Ed. Johnson.

All the sides of the triangle being given, the segments of the base are readily obtained, viz. $AD = 41\frac{3}{7}$, and $BD = 28\frac{4}{7}$. Putting therefote $AC = a$, $AD = b$, $BD = c$, and $DP = x$, we shall have $AP = b - x$, and $BP = c + x$; whence CP^2 ($= AP \times BP$ per quest.) $= \overline{b-x} \times \overline{c+x}$; But CP^2 is also $= CD^2 + DP^2 = aa - bb + xx$; consequently $aa - bb + xx = \overline{b-x} \times \overline{c+x}$; whence $xx - \frac{b-c}{2} \times x = \frac{bb+bc-aa}{2}$, and $x = \frac{b-c \pm \sqrt{\overline{3b+c}^2 - 8aa}}{4}$ $= 17{\cdot}71708$, or $-11{\cdot}28851$. From which $AP = 23{\cdot}71148$, or $52{\cdot}71708$.

In this last manner it is likewise answered by Mess. T. and *W. Allen, W. Ashton, T. Barker, Ja. Beresford, J. Buddles, F. Butcher, Tim. Drury, Edward Ellis, G. Godhelp, J. Goodhead, J. Hampson, D. Hastings, G. Hicks, J. Honey, Abr. Horsfall, T. Hopkinson, T. Howe, J.* and *R. Hudson, Stephen King, R. Langley, J. Lightfoot, B. Lydal. R. Mallock, J. Milbourn, J. Pearce, R. Pitches, Jos. Rennard, W. Spalton, J. Scott, Paul Sharp, J. Taylor, J. Vicary, Stephen West, J. Wilson, Geo. Witchell,* and some others.

V. QUESTION 438 *answered by Mr.* Henry Watson.

Let GPM, QHK, and NLI be the three right lines which each circle is to touch, (forming by their intersections the $\triangle EDF$); and from the centers A, B, C, to the points of contact, let radii be drawn. Because $EG = EH$, and $DI = DH$, it is evident that the sum of the two (equal) tangents FG and FI is equal to the sum of all the three sides of the triangle EDF. And, by the same argument, the sum of the tangents, $EM + EK$, or $DQ + DN$, must also be equal to the very same quantity, and consequently all these tangents equal among themselves. Now from $FG = EM$, let EF (common) to be taken away, and



there

there remains $EG = FM$; but $EH = EG$, and $FL = FM$; consequently $EH = FL$. In the very same manner, $DH = PF$, and $DL = EP$.

Now, per sim. fig. $\begin{cases} AH : EH :: CP : EP = \dfrac{EH \times CP}{AH} \\ BL : FL(EH) :: CP : FP = \dfrac{EH \times CP}{BL} \end{cases}$

Hence it appears, that the three sides EF, ED, DF, of the triangle DEF, are equal to $\frac{EH \times CP}{AH} + \frac{EH \times CP}{BL}$, $EH + \frac{EH \times CP}{BL}$, and $EH + \frac{EH \times CP}{AH}$, respectively; which, therefore, are to each other in the given ratio of $CP + \frac{AH \times CP}{BL}$, $AH + \frac{AH \times CP}{BL}$, and $AH + CP$: From whence the problem may be very easily constructed; and the angles, &c. all found by common trigonometry.

The same answered algebraically by Mr. O'Cavanah.

This gentleman, after proving EG and FM to be equal, (as in the solution above given by Mr. *Watson*) puts $AG = a$, $BM = b$, $CP = c$, and $EM = x$: Then (from the similarity of the triangles EBM, AEG) it will be $EM(x) : BM(b) :: AG(a) : EG = \frac{ab}{x}$. Again (by sim. fig.) $AG(a) : GE\left(\frac{ab}{x}\right) :: CP(c) : EP = \frac{bc}{x}$; and $BM(b) : FM(= EG = \frac{ab}{x}) :: CP(c) : FP = \frac{ac}{x}$: Therefore $\frac{bc}{x} + \frac{ac}{x} + \frac{ab}{x} (= EP + FP + FM = EM) = x$; and consequently $x = \sqrt{bc + ac + ab} = 25{\cdot}495098$: Whence every thing else is easily determined.

Mess. L. Charlton, R. Pitches, and Penovius have also given new and very neat algebraical solutions to this problem. The two latter have determined the distances of the centers A, B, C, in numbers, viz. $AB = 31{\cdot}274$, $AC = 34{\cdot}807$, and $BC = 37{\cdot}596$.

Penovius, making $m = \sqrt{ab + ac + bc}$ $(= EM)$, proves also, that the sides (DE, DF, EF) of the $\Delta\, DEF$ will be

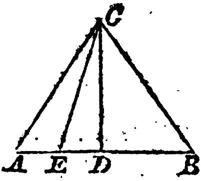
=

$= a \times \frac{b+c}{m}$, $b \times \frac{a+c}{m}$, and $c \times \frac{a+b}{m}$, respectively; and the radius of the circle inscribed therein $= \frac{abc}{ab+ac+bc}$.

Mess. *T. Allen*, *J. Hudson*, and *E. Johnson*, have likewise obliged us with the same elegant conclusions, from Ex. 19. p. 23. of Mr. Landen's Mathematical Lucubrations, to which they severally refer.

VI. QUESTION 439 *answered by Mess.* J. *and* R. Hudson.

Let CD be perpendicular to AB, and put AC $(= BC = 60) = a$, AD $(= BD = 50) = b$, and $x = CE$, the line required: Then $aa - bb = CD^2$, and $\sqrt{xx+bb-aa} = DE$ (Euc. 47. 1.): whence $AE = b - \sqrt{xx+bb-aa}$, and $BE = b + \sqrt{xx+bb-aa}$: Therefore $AE \times BE \times CE = \overline{aa - xx} \times x = a^2x - x^3$; which being a maximum, we have, in fluxions, $a^2\dot{x} - 3x^2\dot{x} = 0$; whence $x = \frac{a}{\sqrt{3}}$: Consequently $AE = 40$, and $BE = 60$.



The same answered by Mr. T. Hopkinson.

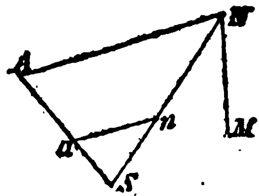
Making $CD \perp AB$, and putting $AC = a$, and $EC = x$, we have (by Euc. 5. 2.) $AE \times BE = AD^2 - ED^2 = AC^2 - EC^2 = aa - xx$; and consequently $AE \times BE \times CE = aax - x^3$: Which, in fluxions, gives $aa\dot{x} - 3x^2\dot{x} = 0$; whence $x = \sqrt{\frac{aa}{3}}$; therefore DE $(= \sqrt{AD^2 - \frac{2}{3}AC^2}) = 10$, $AE = 40$, and $BE = 60$.

From this same ingenious method and substitution, the answer is likewise brought out by Mr. *T. Barker*, Mr. *B. Lydal*, Mr. *J. Milbourn*, Mr. *Paul Sharp*, Mr. *J. Wilson*, Mr. *G. Witchell*, and a few others. Mr. *O'Cavanah* has given the construction and demonstration from geometrical principles independent of fluxions. But the scantiness of our room will not allow us to insert all that our most able and experienced correspondents favour us with.

VII. QUES-

VII. QUESTION 440 *answered by Mr.* J. Milbourn.

Let AS be the given direction of the current at the southermost of the two ports N, S: Take $Sa = 3$, (expressing the rate of the current's motion) and, from a to SN, apply $an = 5$, the rate of the ship's motion. Then draw NA parallel to na, which will give the course and distance required; because, by the composition of motions, the ship will then really move in the right line NS, directly towards her port.



CALCULATION. In the right-angled triangle Sna, we have given $\frac{Sa}{na} = 0·6$ = nat. sin. of Sna (or SNA) = 36° 52′, whence the angle of the course $ANM = 81° 52′$; also NA (the dist. run by the log) $= NS \times \frac{5}{4} = 62·5$; and consequently the time $= 37\frac{1}{2}$ hours.

Mess. *L. Charlton*, *W. Davies*, *T. Drury*, *T. Hopkinson*, and *G. Witchell*, solve it in the same manner.

The same answered by Mr. Richard Terry.

Let NM represent the meridian of the place sailed from, N and S the two points, AS the direction of the current, and NA the course steered by the compass; then, by the composition of motions, the ship's real direction will be in the line NS. Put now $NS = a$, $SA = 3x$, and $NA = 5x$: Then (by Euc. 47. 1.) $25xx - 9xx$ $(= NA^2 - SA^2 = NS^2) = aa$; whence $x = \frac{1}{4}a = 37·5$ miles: From which the course is found to be W. by S. 3° 7′ S. and the time required 37h. 30m.

Thus it is also answered by Mess. *T. Allen*, *Birchoverensis*, *D. Hastings*, *T. Howe*, *J. Goodhead*, *J.* and *R. Hudson*, *J. Pearce*, *Jos. Rennard*, and *W. Spicer*.

VIII. QUESTION 441 *answered by Mr.* Rich. Mallock, *of* Lyme Regis.

By the known rules for pendulums, $\sqrt{39·2} : \sqrt{38·5} :: 28 : 27·75$ = the number of seconds that the glass is running, which

which let be denoted by n; then it will be as 3600 (the number of seconds in an hour) is to n, so is 6120 (the feet in a nautical mile) to $1\cdot7n =$ the distance which the knots ought to be asunder, in order to measure truly the ship's way; which distance, in the present case, coming out 47·175 feet, (instead of 42 feet) we therefore have, as 42 to 47·175, so is 3100 miles (the whole given distance of the Lizard from the capes of Virginia) to 3482 miles, the distance run, by the ship's reckoning, when she arrives at her port; exceeding the truth by 382 miles.

Almost in the same manner the solution is also given by Mr. *Lionel Charlton* (the proposer), *Birchoverensis*. Mr. *Tim. Drury*, Mr. *T. Hopkinson*, Mr. *J. Hudson*, Mr. *Edward Johnson*, Mr. *W. Kingston*, Mr. *R. Terrey*, and Mr. *G. Witchell*.

IX. QUESTION 442 *answered*.

In the given equations ($xz \times \overline{x+z} = a$, and $xxzz \times \overline{z^5 + x^5} = b$) let there be assumed $x + z = s$, and $xz = p$; then will $z^5 + x^5 = s^5 - 5s^3p + 5sp^2$; and so, by substitution, there will be had $ps = a$, and $p^2 \times \overline{s^5 - 5s^3p + 5sp^2} = b$: In the latter of which equations let $\frac{a}{s}$, the value of p (as given by the former) be substituted; then will $\frac{aa}{ss} \times \overline{s^5 - 5s^3 \times \frac{a}{s} + 5s \times \frac{aa}{ss}} = b$; whence $s^6 - \overline{5a + \frac{b}{aa}} \times s^3 = -5aa$; and consequently $s =$

$$\sqrt[3]{\frac{5a}{2} + \frac{b}{2aa} \pm \sqrt{\overline{\frac{5a}{2} + \frac{b}{2aa}}\Big|^2 - 5aa}}$$: From which p $(= \frac{a}{s})$ will also be known; whence x and z are easily obtained.

Thus the answer is given by Mr. *L. Charlton*, Mr. *Rich. Harvey*, Mr. *J. Honey*, Mess. *J.* and *R. Hudson*, Mr. *E. Johnson*, Mr. *W. Kingston*, Mr. *R. Langley*, Mr. *W. Spalton*, Mr. *H. Watson*, Mr. *J. Wilson* (the proposer), and some others.

The

The same answered otherwise.

Put $y+v=x$, and $y-v=z$; then the two given equations will become $\overline{yy-vv}\times 2y=a$, and $\overline{yy-vv}^{2}\times\overline{2y^{5}+20y^{3}v^{2}+10yv^{4}}=b$; from the former whereof $vv=yy-\frac{a}{2y}$; which value substituted in the latter, gives (after proper reduction) $y^{6}-\frac{5a^{3}+b}{8aa}\times y^{3}=-\frac{5aa}{64}$; whence $y^{3}=\frac{5a^{3}+b\pm\sqrt{5a^{6}+10a^{3}b+bb}}{16aa}$.

In this manner the solution is given by Mr. *R. Butler*, *Birchoverensis*, Mr. *Jos. Harrington*, Mr. *Steph. King*, *Penovius*, Mr. *R. Pitches*, Mr. *W. Spicer*, and Mr. *J. Thompson*.

Mr. W. M. *of* Plymouth, *answers it thus.*

Let $c=\frac{1}{2}a$, $c-y=xxz$, and $c+y=zzx$; which values being wrote in the second equation, it becomes $\frac{\overline{c-y}^{4}}{c+y}+\frac{\overline{c+y}^{4}}{c-y}=b$; whence, by reduction, $y^{4}+2ccyy+\frac{byy}{10c}=\frac{bc-2c^{4}}{10}$; and consequently (by putting $d=2cc+\frac{b}{10c}$) $y=\overline{\sqrt{\frac{dd}{4}+\frac{bc-2c^{4}}{10}}-\frac{d}{2}}\Big|^{\frac{1}{2}}$: But $\frac{\overline{c-y}^{2}}{c+y}=x^{3}$, and $\frac{\overline{c+y}^{2}}{c-y}=z^{3}$; whence x and z are also known.

X. QUESTION 443 *answered by* Birchoverensis.

In the given equation $44000xx+1=zz$, let z be assumed $=210x-b$; then will $100xx-1=420xb-bb$: Here x being greater than $4b$, and less than $5b$, let $4b+c=x$, then will $79bb+1=380bc+100cc$. Here $b\sqsubset 5c$, and $\sqsupset 6c$; let therefore $5c+d=b$; then will $25cc-1=410cd+79dd$. Here c being $\sqsubset 16d$, and $\sqsupset 17d$, make $17d-e=c$; then will $216dd-1=440de-25cc$. From whence, by proceeding on in this manner, (making $2e+f=d$, $2f+g=e$, $4g-h=f$, $5h-i=g$, $17i-k=h$, $105k-l=i$, $17l-m=k$, $5m-n=l$, $4n+o=m$, $2o+p$

$+p=n$, $2p-q=o$, $17q+r=p$, and $5r+s=q$) you will, at last, come to the equation $100rr-1=380sr+79ss$; where $4s=r$, and $s=1=ss$; whence $r=4$, $q=21$, $p=361$, $o=701$, $n=1761$, &c. x coming out $=40482981221781$, and $z=8491781781142001$: From which two numbers, answering the conditions of the problem, an infinity of others may be found.

Mr. *H. Watson* (the proposer), Mr. *Edward Johnson*, Mr. *Stephen King*, and Mr. *R. Pitches*, solve it exactly the same way, and have brought out the very same numbers.

The same otherwise answered by Peter Walton, *in a new Method.*

First let $\frac{y}{20}$ be substituted, in the proposed equation ($44000xx+1=zz$), instead of x; then will $110yy+1$ be $=zz$. Now, z being greater than $10y$, suppose $z=10y+b$, and you will, by substitution and reduction, have $10yy-20by=bb-1$. In this last equation y is greater than $2b$; therefore suppose $y=2b+c$, and you will find $bb-20cb=10cc+1$; from whence is found $b=10c+\sqrt{110cc+1}$.

Suppose now $c=0$; then will you have $b=1$, $y=2$, $z=21$, and $x=\frac{2}{20}$; which last not being an integer, we must seek farther. To that end, take $c=2$ (the value of y just now found); then will $\sqrt{110cc+1}$ be $=21$ (the above-found value of z), and b will be $=41$, $y=84$, $z=881$, and $x=\frac{84}{20}$; which not being an integer, we must proceed farther, by taking $c=84$ (the last value of y); then will $\sqrt{110cc+1}$ be $=881$ (the last value of z); and b will be $=1721$, $y=3526$, $z=36981$, and $x=\frac{3526}{20}$; which not being an integer, we must proceed yet farther, by taking $c=3526$, &c. At length, by proceeding in that manner, we find $x=40482981221781$, and $z=8491781781142001$.

A third Method of Solution by Mr. Patrick O'Cavanah, *of* Dublin.

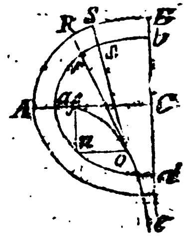
From the given equation ($44000xx+1=zz$) it is evident that z is nearly $=210x$. Putting therefore $\overline{210-y}\times x=z$, we

we thence get $420y - yy - 100 + \frac{1}{xx} = 0$; which, by making $v = \frac{y}{10}$ (to reduce the coefficients still lower) will become $42v - vv - 1 + \frac{1}{100xx} = 0$. Now, it may be observed, that as the term $\frac{1}{100xx}$ is very small, the true value of v, in this equation, must be an approximate value of v, in the equation $42v - vv - 1 = 0$ (where the said term $\frac{1}{100xx}$ is omitted). In order therefore to find such an approximate value of v, let a series of numbers, 1, 42, 1763, 74004, 3106405, &c. be so formed, that each new term may be equal to 42 times the last, *minus* the last but one (42 and -1 being the coefficients of v and vv in the equation here to be resolved): Then (by what is demonstrated at page 174 of Simpson's Algebra, 2d edit.) it will appear, that $\frac{1}{42}$, $\frac{42}{1763}$, $\frac{1763}{74004}$, $\frac{74004}{3106405}$, &c. are so many successive approximations to the value of v, each more exact than the preceding one. It will also appear (and might be easily demonstrated in a general manner, if room would permit) that the error, or quantity arising by substituting any one of these fractions for v, in the given equation $42v - vv - 1 + \frac{1}{100xx} = 0$, will be always expressed by $-\frac{1}{D^2} + \frac{1}{100xx}$, D denoting the denominator of the fraction so substituted. Therefore, in order that the error may entirely vanish, and a solution in integers be obtained, we have nothing to do but to continue the preceding series of numbers to five terms farther, so that the last of these (D) may be a multiple of 10, or so that x $(= \frac{D}{10})$ may be an integer; by which means D is found $= 40482981221781$0, $x = 40482981221781$, and $z = 849178178114200$1. *W. W. R.*

XI. QUES-

XI. Question 444 *answered by* Mr. C. Wildbore.

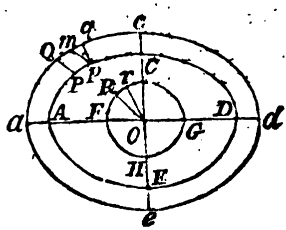
Let dab represent half the elliptic fountain, foe its evolute, ro and so two radii of curvature indefinitely near each other, and let $Aa = Rr = Ss = Bb$ be every where perpendicular both to the ellipsis and the curve $ARSB$, and then it is evident that this will represent the breadth of the required walk.



Put $ar = z$, $AR = v$, $Rr = b$, and $ro = r$; then $ro : \dot{z} :: Ro : \dot{v} = \frac{r+b \times \dot{z}}{r}$, which multiplied by $\frac{1}{2} Ro$, gives $\frac{\overline{r+b}^2 \times \dot{z}}{2r}$ for the area RoS; and in the same manner the area ros is found to be $\frac{r\dot{z}}{2}$; the difference of which area is $b\dot{z} + \frac{b^2}{2} \times \frac{\dot{z}}{r}$ the fluxion of the walk $AaRr$; the fluent of which is $bz + \frac{b^2}{2} \times \angle ron$ (to rad. unity): This, when AR becomes $= AB$, is $bz + \frac{b^2}{2} \times 1.5708$; which expression is general, let the curve arb be what it will, and, in the present case, must be equal to the area of the quarter of the ellipsis $abC = a$; this reduced (putting $1.5708 = c$) gives $b = \frac{\sqrt{2ac+zz} - z}{c} = \frac{\sqrt{282.744 \times 1.5708 + 21.25 \times 21.25} - 21.25}{1.5708}$ $= 5.5247$ feet, the breadth of the walk required.

The same answered by Mr. E. Rollinson.

Let $ACDE$ be the fountain, and PQ (pq, Aa, Cc, or Dd) the breadth of the walk surrounding it: Let pq be supposed indefinitely near to PQ, and pm parallel to PQ; moreover, supposing $OFRG$ to be a circle whose radius is $= PQ$, let OR be conceived parallel to PQ, and Or to pq. Then,



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the area mpq being $= ROr$, and $QPpm = PQ \times Pp = Aa \times Pp$, it is manifest that ($QPpq$) the whole increment of the area $APQa$, is every where equal to a rectangle under Aa, and the corresponding increment (Pp) of the arch AP, together with the increment (ROr) of the circular sector ROF; and, consequently, that the area $APQa = Aa \times AP + RO \times \frac{1}{2}FR = Aa \times \overline{AP + \frac{1}{2}FR}$. Hence the area of the whole walk is evidently $= Aa \times \overline{ACDEA + \frac{1}{2}FRGHF}$; which is general, let the curve $ACDEA$ be what it will. But, in the present case, $ACDEA$ being an ellipse (whose two axes are 30 and 24) the length thereof, by the known series for the periphery of an ellipsis, will be found $= 27{\cdot}0834 \times p$ (p being $\doteq 3{\cdot}14159$ &c.) Whence, if Aa be denoted by x, the area of the walk will here come out $x \times \overline{27{\cdot}0834p + px} = 30 \times 24 \times \frac{1}{4}p$ (by the quest.) From which $xx + 27{\cdot}0834x = 180$; and consequently $x = 5{\cdot}520566$, the required breadth of the walk.

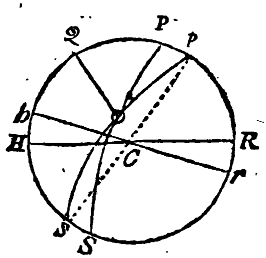
The same answered by Mr. T. Barker, *of* Westhall, *in* Suffolk.

Although the external boundary of the walk is not accurately an ellipsis, yet the deviation therefrom is so very little, that the considering of it as such, will be sufficient to give the solution very near the truth. Let therefore AO ($= 15$) $= a$, OC ($= 12$) $= b$, and Aa ($= Cc$) $= x$; then we shall have $\overline{a+x} \times \overline{b+x}$ ($Oa \times Oc$) $= 2ab$ ($2OA \times OC$) by the question; whence $xx + \overline{a+b} \times x = ab$, and consequently $x = \sqrt{ab + \frac{\overline{a+b}|^2}{4}} - \frac{a+b}{2} = 5{\cdot}5328 \doteq$ the required breadth of the walk, very near.

In this last manner it was also answered by Mess. *W. Allen, W. Ashton, Edward Barras, Ja. Beresford, T. Bosworth, Turner Boston, J. Buddles, J. Chapman, Charles Cave, T. Crabtree, Edward Ellis, G. Godhelp, J. Goodhead, H. Green, Jos. Harrington, W. Harrison, G. Hicks, R. Hodges, J. Honey, Abr. Horsfall, E. Houlston, B. Lydal, W. Matthewson, Chr. Meslyan, J. Milbourn, J. Pearce, S. Pedley, W. Penn, Jos. Rennard, Alex. Rowe, J. Scott, G. Stapley, H. Stephens, J. Taylor, J. Thompson, T. Wilkin,* and some others.

XII. QUESTION 445 *anſwered by Mr.* O'Cavanah.

Let *HPRS* be the meridian (or plane of the ſtyle), *P* the north pole, *HCR* the horizon, ☉*PH* the given hour angle from noon, *P*☉ the ſun's diſtance from the north pole, and ☉*Q* a great circle perpendicular to the meridian: Moreover, let *hCr* be ſuppoſed to repreſent the poſition of the plane of the dial, and *Cp* that of the gnomon, when the ſouth ſide of the dial *hC* is elevated (as by the queſtion) above its proper horizontal poſition *HC*.



It is evident, from the general conſtruction of all dials, that the hour from 12 marked out by the ſhadow (converted into degrees) will, in every poſſible poſition of any dial, be the true meaſure of the angle made by the plane of the ſtyle and another (imaginary) plane paſſing by the edge of the ſtyle thro' the ſun's center: Whence it appears that the ∠*Qp*☉, made (in the preſent caſe) by the plane of the ſtyle *HPRS* and the plane *p*☉*sC* paſſing by the edge of the ſtyle in the poſition *pC*, will be truly expreſſed by 65°, anſwering to 4h. 20m. the time from 12, given by the queſtion. Now, the ∠*Qp*☉ (here given) being (by the queſt.) leſs than it can, otherwiſe, be in any other poſition of *p* (or *Cp*) it is plain that, let ☉*Q* be what it will, *Qp* and ☉*p* muſt be archs of 90 degrees each; ſince, in this caſe, the meaſure of the ∠*Qp*☉, or *Qs*☉, (which in all other poſitions of *p* is greater) will become = ☉*Q*. Hence we have (per ſpherics) as ſin. ☉*PQ* (75°) : ſin. ☉*Q* (65°) : : rad. : ſin. *P*☉ = 69° 45′ 52″; whoſe complement 20° 14′ 8″, is the declination ſought: The time correſponding being May the 21ſt, or July the 22d.

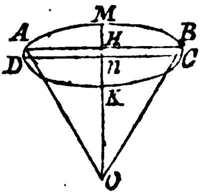
COROLLARY. In all latitudes, the ſhadow, while the plane of the dial is elevating, will proceed to the ſame hour before it returns back: Becauſe the true time (or hour) being ſuppoſed the ſame in all latitudes, ☉*Q* as well as *PQ* will remain invariable, and therefore *Pp* (the complement of *PQ*) invariable likewiſe.

In this manner the problem was alſo determined by Mr. *L. Charlton*, Mr. *W. Davies*, Mr. *E. Rollinſon*, and Mr. *C. Wildbore*.

XIII. QUES-

XIII. QUESTION 446 *answered by Mr.* L. Charlton.

Supposing $AMBCKD$ to represent the garden, and $ABCD$ the walk; it is evident, seeing the length AMB is given, that AB will be a maximum, when the excess of AMB above AB is the least quantity possible (the area being given, or supposed to remain the same; which will therefore be when AMB is an arch of a circle; because, the circle being the most capacious figure, any other curve drawn from A to B, to



contain with AB an area equal to $AMBHA$, must necessarily differ more from AB than AMB does, in this supposition. To determine, now, the radius of the circle (and from thence the length of the walk) put AM ($=325$ feet) $=a$, AD ($=10$) $=b$, the area $AMBCKD$ ($=43560$) $=c$, and AO ($=MO=BO$) $=r$. Then, the area AMH being $=\frac{1}{2}AM\times OM-\frac{1}{2}AH\times OH=\frac{1}{2}ar-\frac{1}{2}AH\times OH$, and the area $AHnD=b\times AH$, we thence get $2ar-2AH\times OH+2b\times AH=c$. From whence, by series, or any of the known methods of approximation, the required length AB of the walk may be determined, and will come out $=642{\cdot}312$ feet, or $38{\cdot}928$ furlongs.

Mess. *Goodhead*, *Hudson*, *Rollinson*, and *Wildbore*, have also given solutions to this problem, in a manner very little different.

XIV. QUESTION 447 *answered by Mr.* T. Allen, *of* Spalding.

Let the given length of the chain ($=30$ feet) $=a$; the part hanging down at the commencement of motion ($=3$ feet) $=c$; $s=32\frac{1}{6}$ feet, the velocity generated at the earth's surface in one second; x the space moved over by a particle in any variable time t; and $v=$ the corresponding velocity of the chain at that instant: Then it is evident, that $c+x$ is as the motive force acting on the chain; and therefore, $\frac{\dot{x}}{v}$ being $=\dot{t}$, $s\times\overline{c+x}\times\frac{\dot{x}}{v}$ will be $=a\dot{v}$, the fluxion of the quantity of motion generated in the time t; whence $s\times c\dot{x}+x\dot{x}=av\dot{v}$; and, taking the fluents, &c. $v=$

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$\sqrt{\frac{s}{a}} \times \overline{2cx+xx}$, which substituted in the above value of $\dot{t}$, gives $\dot{t} = \frac{\dot{x}}{\sqrt{\frac{s}{a}} \times \overline{2cx+xx}}$, whose correct fluent is $t = \sqrt{\frac{a}{s}} \times$ hyp. log. $\frac{c+x+\sqrt{2cx+xx}}{c}$, which, when x becomes $= a - c$, is $= 2{\cdot}890663$ seconds, the time required.

The same answered by Mr. Robert Butler.

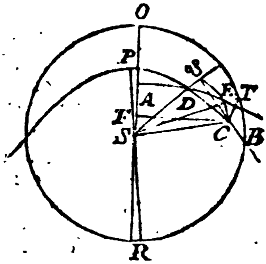
Let $a =$ the chain's length ($= 30$ feet), $d = 32\frac{1}{6}$ feet, the velocity generated by the uniform force of gravity in one second, $x =$ the length of the part of the chain disengaged from the plane at the end of any time t, and $v =$ the velocity acquired in that time. Then $a : x :: d : \frac{dx}{a}$ the velocity that would be acquired in one second by the uniform force of x length: Therefore, as 1 (second) is to $\frac{\dot{x}}{v}$ $(= \dot{t}) :: \frac{dx}{a} : \frac{dx\dot{x}}{av} = \dot{v}$ (the uniform increase of the velocity in the time $\dot{t}$). Hence $\frac{dx\dot{x}}{a} = v\dot{v}$; and therefore, by taking the correct fluents, $\frac{dxx}{a} - \frac{dbb}{a} = vv$, b being ($=$ 3 feet) $=$ the first or given value of x, when the motion commences. Now $\dot{t}$ $(= \frac{\dot{x}}{v})$ being given from hence $= \sqrt{\frac{a}{d}} \times \frac{\dot{x}}{\sqrt{xx-bb}}$, by taking again the correct fluents, we get $t = \sqrt{\frac{a}{d}} \times$ hyp. log. $\frac{x+\sqrt{xx-bb}}{b} = \sqrt{\frac{a}{d}} \times$ hyp. log. $\overline{10+\sqrt{99}}$ (when $x = a = 30$) $= 2{\cdot}89066$ seconds, the time required.

Peter Walton (the proposer), Mr. *O'Cavanah*, Mr. *Rollinson*, Mr. *Watson*, and Mr. *C. Wildbore*, answer this question in the same manner.——*Tyro* has given a very pretty solution to it upon different principles.

PRIZE

PRIZE QUESTION *answered by Mr.* G. Witchell, *the Proposer.*

Let OTR be the earth's orbit, PDC the comet's trajectory, ADB the orthographic projection of it on the plane of the ecliptic, D the descending node, and P the place of the perihelion. From any point C of the parabola let fall CE perpendicular to the plane of the ecliptic; through S and E draw ST and join C, T, it is evident that T is the nearest point in the circle to C. Now put $SC = 1 - x$, $PS = \frac{a}{4}$, $n = \frac{a}{2} - 1$, $m = a - \frac{aa}{4}$, $t =$ sin. of $E☋C$ (the greatest inclin.) $b =$ the sine and $d =$ the cosine of $PS☋$; then, by the nature of the parabola, we shall have $SF = n + x$ and $FC = \sqrt{m - ax}$; therefore $1 - x : 1 :: \sqrt{m - ax} : \frac{\sqrt{m - nx}}{1 - x} :: n + x : \frac{n + x}{1 - x} =$ sin. and cos. of PSC; but sin. $PSC - PS☋ =$ sin. $PSC \times d -$ cos. $PSC \times b = \frac{d\sqrt{m - ax} - b \times \overline{n + x}}{1 - x} =$ sin. $☋SC$; then $1 : 1 - x :: \frac{d\sqrt{m - ax} - b \times \overline{n + x}}{1 - x} : d\sqrt{m - ax} - b \times \overline{n + x} = ☋C$, and $1 : d\sqrt{m - ax} - b \times \overline{n + x} :: t : td\sqrt{m - ax} - tb \times \overline{n + x} = EC$; which being squared and subtracted from $\overline{1 - x}|^2 = SC^2$, we have $\overline{1 - x}|^2 - ttdd \times \overline{m - ax} - ttbb \times \overline{n + x}|^2 + 2ttdb \times \overline{n + x} \times \sqrt{m - ax} = SE^2$: This being resolved into a series (and A, B, C, &c. subst. for the known coefficients) we shall get $A^{\frac{1}{2}} \times \overline{1 - \frac{B}{2A} \times x - \frac{B^2 + 4AC}{8A^2} \times xx + \frac{B^3 - 4ABC + 8A^2D}{16A^3} \times x^3 \text{ \&c.}} = SE$: But $1 + SC^2 - 2SE = CT^2$; whence we have $2 - 2A^{\frac{1}{2}} - \overline{2 + \frac{B}{A^{\frac{1}{2}}}} \times x + \overline{1 + \frac{B^2 - 4AC}{4A^{\frac{3}{2}}}} \times xx -$



$\frac{B^3-4ABC+8A^2D}{8A^{\frac{5}{2}}} \times x^3$ &c. $= CT^2$; which, when CT is a minimum, will give $2 + \frac{B}{A^{\frac{1}{2}}} = \overline{\frac{B^2-4AC}{2A^{\frac{3}{2}}}} + 2xx - \frac{B^3-4ABC+8A^2D}{8A^{\frac{5}{2}}} \times 3xx$ &c. Whence we get $x = 0{\cdot}0146$, and $CT = 0{\cdot}041974$; which (taking $ST = 80000000$) is $= 3357920$ miles. Moreover, the cos. of PSC being $= \frac{n+x}{1-x} =$ cos. 78° 59′, we shall have the long. of the comet in its orbit ♏ 14° 31′, and its arg. of latitude 7° 42′; from which, and the given inclination, its longitude in the ecliptic will be found ♏ 14° 51′; which place the earth transits, May 4th.

Hence it appears, that we need not be under any apprehensions from the return of this comet, notwithstanding the prediction of a certain experimental philosopher; for if it should return at the time he mentioned, it would be distant from the earth (by his own scheme) 9000000 miles.

In a manner equally concise and ingenious, the solution to this problem is also given by Mr. *O'Cavanah*, and Mr. *Wildbore*; but the prize belongs to Mr. *Witchell*, his solution being the only true one that came to hand before Candlemas-day.

The Eclipses calculated for 1759.

There will this year be three eclipses, one of the moon, and two of the sun; whereof that of the moon, only, will be visible to the inhabitants of Great Britain: Some calculations of which are as follow.

☽ Ecl. Jan. 13, in the morning.

Calculated by		Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig.
Mr. G. Witchell, from Mayer's Tables	for London	6 37	7 57	9 18	2 41	6 29
Mr. Gutteridge,	for London	6 38	7 55	9 21	2 43	6 39
	Loughbor.	6 33	7 54	9 16		
Mr. E. Greenstead,	for London	6 38	8 0	9 21	2 43	6 39
Mr. T. Hopkinson,	for London	6 39	7 55	9 18	2 46	6 41
Mr. R. Langley,	for London	6 34	7 53	9 15	2 44	6 35
Mr. T. Allen, from Halley's Tables	Spalding	6 43	8 1	9 19	2 46	6 24

The

The second eclipse is of the sun, June 24th, about 5 in the afternoon.

The third is also of the sun, Dec. 19th, about 2 in the afternoon.——These two eclipses are both invisible in England, but very large in the West Indies.

New Questions.

I. Question 448, *by* Richard *of the* Vale.

Teach me, fair artists, how to find,
When my dear Nancy will be kind;
The charming maid my love allows,
Yet is averse to crown my vows;
In mystic terms * prescribes her will,
Which to disclose, exceeds my skill.
T' unfold the knot, * ye fair, descend,
And serve, for once, an unlearn'd friend.

* $\left\{\begin{array}{l} xx + yy - 5x + 9y = 968 \\ 8yy + xy - 30x + 54y = 5808 \end{array}\right\}$; x being (she says) the months, and y the days, before she can consent to compleat my happiness.

II. Question 449 *by Mr.* Paul Sharp.

The difference of the two legs of a right-angled triangle, whose area is double to that of its inscribed circle, being given $(=6)$; to find all the sides, and the radius of the circle.

III. Question 450, *by Mr.* Richard Gibbons.

Somewhere, on fam'd Europa's strand,
A lofty cone aspiring stands,
O'erlooking far the watry plain,
A guide to those who plow the main;
Whose ample bulk contains compleat
One fifteenth of two million feet.
It chanc'd, that from the top, a ball
Down by its side to th' ground did fall,
Which reach'd, in five half-second's space,
The utmost limit of the base.
From hence, fam'd artists, let be shown
The heigh of this stupendous cone.

IV. QUESTION 451, *by Mr.* T. Barker, *of* Westhall, *in* Suffolk.

Two lines drawn to bisect and terminate in the opposite sides of any trapezium, will also bisect each other: A demonstration of this is required.

V. QUESTION 452, *by Mr.* Christopher Mason.

A farmer proposes to have a bushel to contain three quarts above statute (being the customary measure of the neighbourhood) to be 1-5th of an inch in thickness, of hammered brass; the depth and diameter to be such as will require the least metal; he will likewise allow 16 d. per lb. and desires to know the expence at that rate.

VI. QUESTION 453, *by Mr.* J. Hudson, *of* Louth.

Having given the lengths (14 and 20) of two right lines drawn from the same point to make a given angle (60°) with each other; so to draw another right line through the point of concourse, that two perpendiculars being let fall thereon from the extremes of the first lines, the sum of the two triangles, thereby formed, shall be the greatest possible.

VII. QUESTION 454, *by Mr.* William Toft.

A ship, plying to windward, then at N. N. E. sails, with her starboard tacks on board, 60 miles; she then tacks, and, after having run 45 miles farther, finds, by an observation, that her whole difference of latitude, on both tacks, is just 40 miles: Hence you are desired to determine how near to the wind the ship made good her way.

VIII. QUESTION 455, *by Mr.* Tho. Moss.

To divide a given trapezium into two equal parts (geometrically) by a right line cutting off from opposite sides two segments, adjacent to the base, which shall also be equal to each other.

IX. QUESTION 456, *by Mr.* Mor. M'Roy.

In a right-angled triangular field there are three trees, viz. one in each fence: The distance of the tree in the base from that in the hypothenuse, and from the acute angle adjacent, and of the tree in the perpendicular from the right angle, are all equal, and given; and if lines be drawn from the

the tree in the base to the other two, those lines will form a right angle: The perpendicular of the proposed triangle is known to be the least of its kind (or that the data will admit of): Hence you are desired to find the sides of the triangle, and to construct the same geometrically.

X. Question 457, *by Mr.* G. Witchell.

The poet Hesiod tells us, That, in his time Arcturus rose at the setting of the sun 60 days after the winter solstice. It is required to find from hence, and the subjoined data*, collected from astronomical observations, at what time that author flourished.

* Lat. of the place 38° 54′ N. long. of the star, at the beg. 1690, ♎ 19° 53′ 52″, its lat. 30° 57′ N. annual precession of the equinox 50·3″.

XI. Question 458, *by Mr.* Edw. Johnson.

On May 12th, 1758, in latitude 53° 40′ N. a person sets out at six in the morning, and walks uniformly in the direction of his own shadow, till 50 minutes past ten; when he arrives at a place known to lie 12 miles west of the meridian of that from whence he set out: The question is to find how far, and at what rate he travelled in going from one place to the other.

XII. Question 459, *by Mr.* Hugh Brown.

Suppose a cistern, whose length, breadth, and depth are 60, 28, and 32 inches, to be supplied with water by a cock, running uniformly into it, at the rate of 2 gallons per minute; and that the discharge of water by a cock at the bottom, when the cistern is full, is at the rate of 5 gallons per minute. Now I desire to know, supposing the vessel to be full, and both cocks opened together, in what time 150 gallons may be drawn off, at the evacuating cock.

XIII. Question 460, *by Mr.* Edw. Rollinson.

To determine the curve in which a body must move, so as to continue always at the same invariable distance from another body moving uniformly in a right line; the velocity of the former body being also uniform, and exceeding that of the latter, in any given ratio.

XIV. QUEST. 361, *by* Peter Walton, *Discip. Landenii.*

To assign the sum of the series $1 - \frac{1^2}{2^2} + \frac{1^2 . 3^2}{2^2 . 4^2} - \frac{1^2 . 3^2 . 5^2}{2^2 . 4^2 . 6^2}$ + &c. by means of circular and elliptic arcs.

The PRIZE QUESTION, *by Mr.* O'Cavanah *of Dublin.*

To determine on what day of the year, in the lat. of London, the length of the afternoon exceeds that of the forenoon by the greatest difference possible, reckoning the day to begin at sun-rising, and to end at sun-setting.

1760.

Questions answered.

I. QUESTION 448 *answered by Mr.* G. Witchell.

AH! Richard, take a friend's advice,
Or you'll be ruin'd in a trice,
Don't wed the artful fair;
When she says Aye, do you say No:
Believe me, ev'ry scene of woe
Attend the ill-match'd pair.
Her head is turn'd with algebra;
She will not rise before 'tis day,
Nor trudge to milk your kine:
But, if no reason can persuade
You to refuse the yielding maid,
Behold your quarentine.*

* The first equation being multiplied by 6, and compared with the second, there results $yy + \frac{xy}{2} = 3xx$; whence, by compleating the the square, $y = \sqrt{3xx + \frac{xx}{16}} - \frac{x}{4} = \frac{3x}{2}$; which value substituted in the first equation, gives $\frac{13xx}{4} + \frac{17x}{2} = 968$, whence $x = 16$ months, and $y = 24$ days.

In

In this manner the solution is also given by Mr. *T. Barker*, Mr. *E. Batten*, Mr. *J. Beresford*, *Birchoverensis*, Mr. *Turner Boston*, Mr. *J. Buddles*, Mr. *R. Butler*, Mr. *W. Fowler*, Mr. *P. George*, Mr. *Nath. Gerrad*, Mr. *R. Gibbons*, Mr. *J. Hampson*, Mr. *Jos. Harrington*, Mr. *G. Hicks*, Mr. *Abr. Horsfall*, Mr. *E. Houlston*, Mr. *Steph. King*, Mr. *W. Kingston*, Mr. *W. Litson*, Mr. *R. Mallock*, Mr. *Chr. Mesban*, Mr. *J. Milbourn*, Mr. *Alex. Rowe*, Mr. *Jos. Ruston*, Mr. *W. Spicer*, Mr. *W. Stoker*, Mr. *W. Toms*, Mr. *Jos. Wastel*, and several others.

II. QUESTION 449 *answered by Mr.* T. Barker, *of* Westhall, Suffolk.

Put $\frac{1}{2}$ the sum of the two legs $= x$, and $\frac{1}{2}$ their difference $= a$; then $x - a =$ the less leg, and $x + a =$ the greater; and consequently $\sqrt{2xx + 2aa} =$ the hypothenuse. Also, by a well-known theorem, $2x - \sqrt{2xx + 2aa}$ (= the sum of the legs minus the hyp.) = the circle's diameter; whence its periphery (putting $n = 3{\cdot}1416$) will be $= 2nx - n\sqrt{2xx + 2aa}$; the double of which (by another well-known theorem, and the conditions of the question) must be equal to the perimeter of the triangle; that is, $4nx - 2n\sqrt{2xx + 2aa} = 2x + \sqrt{2xx + 2aa}$; whence x is found $= a\sqrt{\dfrac{(2n+1)^2}{2 \times (2n-1)^2 - (2n+1)^2}} = 13{\cdot}1$; therefore the sides are $10{\cdot}1$, $16{\cdot}1$, 19, and the diameter $= 7{\cdot}19$.

Thus (or in a manner very little different) the answer is also given by Mr. *T. Allen*, Mr. *E. Batten*, Mr. *T. Baxtenden*, *Birchoverensis*, Mr. *R. Butler*, Mr. *G. Brown*, Mr. *W. Chapman*, Mr. *Chr. Cave*, Mr. *Tim. Drury*, Mr. *P. George*, Mr. *Nath. Gerrad*, Mr. *R. Gibbons*, Mr. *J. Goodhead*, Mr. *T. Harris*, Mr. *J. Hampson*, Mr. *Abr. Horsfall*, Mr. *R. Mallock*, Mr. *Chr. Mesban*, *Penovius*, Mr. *W. Spicer*, Mr. *Abr. Stone*, Mr. *W. Stoker*, Mr. *Jos. Wastell*, Mr. *G. Witchell*, and some others.

III. QUESTION 450 *answered.*

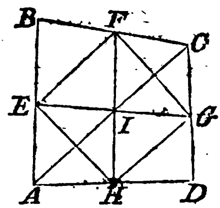
Put the content of the cone $(= 133333\frac{1}{3}) = c$, and its altitude $= x$, also put $p = 3{\cdot}1416$, and let $a =$ diameter of a sphere described about the cone; which (by the laws of descent of heavy bodies) is given $= 100\frac{25}{48} =$ the perp. descent in

in the given time in which the chord or side of the cone is described. Now, by the property of the circle, $x \times \overline{a - x}$ is $=$ the square of the rad. of the cone's diam. at the base; and consequently $\frac{1}{3}\, pxx \times \overline{a - x} = c'$ ($=$ the cone's content): From which equation $x = 81{\cdot}219$, or $x = 50{\cdot}408$.

Thus the problem is resolved by Mr. *W. Banfield*, Mr. *T. Barker*, Mr. *T. Baxtonden*, *Birchoverensis*, Mr. *Turner Boston*, Mr. *G. Brown*, Mr. *J. Buddles*, Mr. *R. Butler*, Mr. *W. Chapman*, Mr. *Chris. Cave*, Mr. *W. Davies*, Mr. *Tim. Drury*, Mr. *P. George*, Mr. *Ja. Giles*, Mr. *R. Gibbons*, Mr. *J. Goodhead*, Mr. *T. Harris*, Mr. *Abr. Horsfall*, Mr. *Steph. King*, Mr. *W. Kingston*, Mr. *W. Litson*, Mr. *R. Mallock*, Mr. *J. Milbourn*, *Penovius*, Mr. *Alex. Rowe*, Mr. *Jos. Ruston*, Mr. *W. Spicer*, Mr. *Abr. Stone*, Mr. *W. Toms*, and Mr. *G. Witchell*.

IV. QUESTION 451 *answered.*

Let the bisecting points be joined by the lines EF, FG, GH, and HE, and let AC be drawn: Then, because the sides of the triangles ABC and ADC are divided proportionally, both EF and HG will be parallel to AC (Euc. 2. 6.); and, therefore, parallel to each other; and in the very same manner is EH parallel to FG; and so, the triangles EIF and HIG being equiangular (Euc. 29. 1.) and EF being $= HG$, thence will $EI = GI$, and $FI = HI$. Q.E.D.



In this manner the demonstration is given by Mess. *Allen*, *Barker*, *Baxtonden*, *Butler*, *Brown*, *Davies*, *George*, *Goodhead*, *Mallock*, *Milbourn*, *Penovius*, *Spicer*, *Wildbore*, and *Witchell*; to none of whom, in particular, we have a right to ascribe the solution above exhibited in preference to the others

V. QUESTION 452 *answered.*

Put $\cdot 7854 = p$, the content of the bushel in cubic inches $= a$, and the internal diameter $= x$; then will the area of the base $= pxx$, the circumference of the base $= 4px$; the depth

depth of the bushel $= \frac{a}{pxx}$, the concave surface $= 4px \times \frac{a}{pxx} = \frac{4a}{x}$, and the whole internal surface (including the base) $= \frac{4a}{x} + pxx$: Which being a minimum (by the quest.) we thence have $-\frac{4a\dot{x}}{xx} + 2px\dot{x} = 0$; whence $x = \sqrt[3]{\frac{2a}{p}} = 18{\cdot}1604$; and the depth $(\frac{a}{pxx} = \frac{x}{2}) = 9{\cdot}0802$: Hence $158{\cdot}85 =$ the quantity of the metal; which, at 16d. per pound (supposing a cubic foot of hammered brass to weigh 8349 ounces, as by the tab. in Misc. Cur.) comes to 3l. 3s. 11d.

Thus the solution is given by Mess. *Barker*, *Beresford*, *Birchoverensis*, *Boston*, *Buddles*, *Chapman*, *Drury*, *Hastings*, *Gibbons*, *Giles*, *Goodhead*, *Harris*, *Hicks*, *Horsfall*, *Houlston*, *King*, *Kingston*, *Rowe*, *Toms*, and some others.

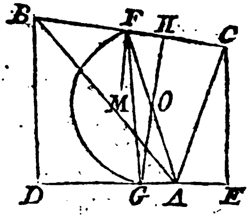
The same answered otherwise.

Let x and y denote the internal diameter and depth, and a the thickness of the metal; then the external diameter and depth being $x + 2a$ and $y + a$, we shall have $pyxx = c$ (= the given content) and $p \times \overline{y+a} \times \overline{x+2a}^2 - c$ a minimum (by the question): Whence, in fluxions, $\dot{y}xx + 2yx\dot{x} = 0$, and $\dot{y} \times \overline{x+2a}^2 + \overline{y+a} \times \overline{x+2a} \times 2\dot{x} = 0$; hence $\dot{y} = -\frac{2y\dot{x}}{x} = -\frac{\overline{y+a} \times 2\dot{x}}{x+2a}$, $\frac{y}{x} = \frac{y+a}{x+2a}$, and $x = 2y$: Whence also $\frac{1}{2}px^3$ $(= px^2y) = c$; from which x is given $= \sqrt[3]{\frac{2c}{p}} = 18{\cdot}1604$, and $y = 9{\cdot}0802$; hence the quantity of metal $(= p \times \overline{y+a} \times \overline{x+2a}^2 - p \times yx^2 = 4pa \times \overline{3yy + 3ay + aa})$ comes out $= 158{\cdot}85$ cubic inches, and the value thereof 3l. 3s. 11d.

Thus the problem is resolved by Mr. *T. Baxtonden*, Mr. *R. Butler*, Mr. *W. Davies*, Mr. *R. Mallock*, *Philarithmus*, *Penovius*, *P. M.* Mr. *W. Spicer*, and Mr. *G. Witchell*.—— From this last method it appears, that, let the thickness of the metal be ever so great, the external diameter and depth (as well as the internal) will be, accurately, in the ratio of two to one.

VI. QUESTION 453 *answered by Mr.* Thomas Moss.

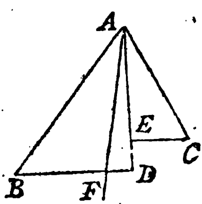
Let BC, joining the extremes of the given lines AB and AC, be bisected by AF; upon which conceive a semicircle FGA to be described, and conceive FG to be perpendicular to ED, and GH to BC: Then it is evident, that $DE \times FG$, or $BC \times GH$, will be $=$ the area $BCED$; which (as BAC is given) must be a maximum (by the quest.), and therefore GH (as BC is invariable) must also be a maximum; and this, since G is always in the circumference of the circle, must necessarily happen when HG passes through the center O; in which case the angle $OFG = \frac{1}{2}FOH = \frac{1}{2}OFM$ (FM being supposed perpendicular to BC): Whence this *Construction*. Having joined A, F, and made FM perpendicular to BC, draw FG to bisect the angle AFM; then, through A, draw DE perpendicular to FG, and the thing is done.——The numerical calculation is, from hence, very easy; whence the angle DAB comes out $= 54° 24\frac{1}{2}'$, and the sum of the two triangles ($ADB + ACE$) $= 131.53$.



P. M. (of *Durham*) has also favoured us with a very neat geometrical construction of this problem, which does not essentially differ from that above exhibited.

An algebraical Solution to the same, by Philarithmus.

Let AD be the line sought, and let AF bisect the given angle BAC; call $2BAF$ (60°) A; $2FAD$, V; tab. rad. 1; AB (20), a; AC (14), b; sin. A, s; cos. A, c; sin. V, x; cos. V, y; then by trigonometry $\frac{2BD \times DA}{aa} =$ sin. $2BAD$ (or s. $\overline{A+V}$) $= sy + cx$, and $\frac{2CE \times EA}{bb}$ $=$ s. $2CAE$ (or s. $\overline{A-V}$) $= sy - cx$;



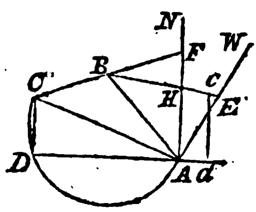
therefore $\frac{BD \times DA}{2}$, or the $\triangle ABD = \overline{sy + cx} \times \frac{aa}{4}$, and

and $\frac{CE\times EA}{2}$, or the $\triangle\ ACE = \overline{sy-cx}\times\frac{bb}{4}$, and their sum $= sy\times\frac{aa+bb}{4}+cx\times\frac{aa-bb}{4}$, a minimum; therefore $s\dot{y}\times\overline{aa+bb}+c\dot{x}\times\overline{aa-bb}=0$; whence, substituting $-\frac{x\dot{x}}{y}$ for $\dot{y}$, and dividing by $-s\dot{x}$, and transposing $\frac{x}{y}\times\overline{aa+bb}=\frac{c}{s}\times\overline{aa-bb}$; or $aa+bb : aa-bb$ :: cot. A : tan. V; whence V is given $= 11^\circ\ 10'7'$, and FAD, its half, $= 5^\circ\ 35'35'$, and the greatest sum of the triangles 131·532.

Thus (or in a manner very little different) the answer is also given by Messrs. *Barker*, *Baxtonden*, *Bircboverensis*, *Brown*, *Butler*, *Davies*, *Fowler*, *George*, *Kingston*, *Penovius*, *Spicer*, and *Witchell*.

VII. QUESTION 454 *answered by Mr.* O'Cavanah.

Draw AB and BC equal to the two given distances (60 and 45), so that the supplement (ABF) of the angle contained by them, shall be equal to (45°) double the given angle included between the wind and the meridian; join AC, and let a semicircle be described thereon, in which apply CD (= 40) = the given difference of latitudes; draw AN parallel thereto; then, if NAW be made = 22° 30', I say that BAW will be the angle sought, which the ship makes with the wind.

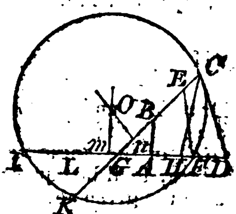


For, having produced CB to meet AN in F, let $BHcE$ be so drawn, as to make the $\angle BHA = BCD = BFN$, in which take $Bc = BC$, and let cd be parallel (and consequently equal) to CD; then AB, Bc, and cd, being (by construction) equal to the three quantities given, it only remains to prove, that the $\angle BEA$, which the ship makes with the wind, on her second course (Bc), is equal to the $\angle BAE$, on her first course (AB). Now (by construction) $BFN = BHA$; but $BFN - FBA$ ($2NAW$) $= BAH$; whence $BFN - NAW = BAE$: But BHA (BFN) $-$ $NAW = BEA$; therefore $BAE = BEA$.

CALCULATION. In the triangle ABC we have given AB ($=60$), BC ($=45$), and the angle ABC ($=135°$), to find BAC ($=19° 7'$), and AC ($=97\cdot178$); from which last, and CD ($=40$), CAD is found $=24° 18'$: Whence BAD $=43° 25'$, and $BAW = 69° 5'$.

Mr. *W. Davies*, and Mr. *Tho. Moss*, have likewise constructed this problem: And several very neat algebraical solutions to it, have been received from Mr. *T. Allen*, Mr. *T. Baxtonden*, Mr. *Geo. Brown*, Mr. *W. Kingston*, Mr. *W. Spicer*, Mr. *W. Stoker*, Mr. *G. Witchell*, and others; which, for want of room, we are obliged to omit.

VIII. QUESTION 455 *answered by* P. M. *of* Durham.



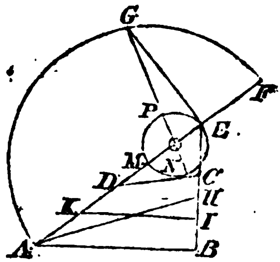
From the angle C draw CH to bisect the given trapezium $ABCD$ (p. 4. 6. Simpson's Elem. Geom.); produce BC, AD indefinitely, cutting each other in G, from which point, on CG produced, take $GK = GH$, and, on DG produced, take $GL = GB$; draw two right lines bisecting BG, LA perpendicularly in n and m, and intersecting each other in O; round which point O as a center, with the distance OC or OK, describe a circle cutting AD (produced both ways if needful) in F and I; from G take $GE = GI$; join E, F, and the thing is done.

DEMONSTRATION. mL and mA are equal (by hypoth.) and also mI, and mF (3 Euc. 3.); therefore the remainders IL, AF will be equal; but since GE, GI, and GB, GL are equal (by hypoth.) IL, BE, and consequently AF, BE will be equal. But I say moreover, that EF bisects the trapezium, for $CG : GF :: GE\ (GI) : GH\ (GK)$ (35 Euc. 3.) therefore the triangles CGH, EGF are equal (15 Euc. 6.); and, taking away the common triangle AGB, there will remain the quadrilateral $BH = BF$; therefore, because CH bisects the given trapezium BD (by hypoth.) EF will bisect it also. *Q. E. D.*

The

The same constructed otherwise, by Mr. W. Davies.

Draw AH so as to make the triangle ABH equal to half the given trapezium $ABCD$ (p. 4, 6. of Simps. Geom.); produce AD and BC to meet in E, and, in the former of them, take $AM = EB$, and $EF = EH$; then take EG a mean proportional between AE and EF; and from G, to the middle of EM, draw GO; make $OK = OG$, $BI = AK$, and draw KI, which will divide the given trapezium $ABCD$ into two equal parts.

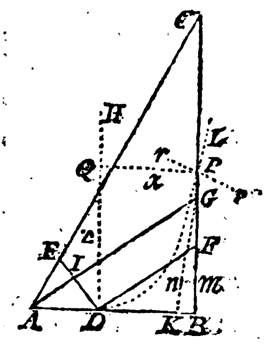


For, if GO be conceived to cut, in P and N, a circle described on the diameter ME, then will $KE \times KM$ $(= GN \times GP) = GE^2$ (Euc. 36. 3.) $= AE \times EF = AE \times EH$; but, because $AM = EB$ and $AK = BI$, thence is $KM = EI$; therefore $KE \times EI$ $(= KE \times KM) = AE \times EH$; and so, the triangle EKI being $= EAH$ (Euc. 15. 6.), the trapezium $ABIK$ is also $= ABH = \frac{1}{2} ABCD$. *Q.E.D.*

In the two solutions here exhibited, it will be observed, that the authors have considered different cases: For, of the two equal trapeziums, into which the given one is to be divided, the opposite sides of either the one or the other, may, by the question, be taken as equal; according as the intersection of the other opposite sides is supposed to fall above or below the base.

IX. QUESTION 456 *answered by Mr.* H. Watson.

Let ABC represent the field, and D, E, F the places of the trees; and let AIG be parallel to DF: Then AIE being $= EDF =$ a right angle, EAI (or CAG) is $(=$ comp. $AEI =$ comp. $EAD)$ $= C$; whence $CG = AG$, and $BC = BG + AG$. From which, as many points as you please, in the curve bounding the perpendicular (BC) may be determined.



But to determine the least value of BC (calling BF, a; and DB, (x), we have DB (x) : $BF + DF$

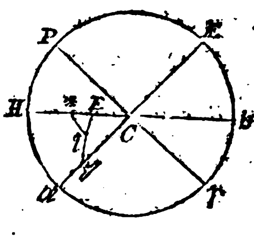
 $(a +$

$(a+\sqrt{xx+aa}::\overline{AB}\ (a+x):BG+AG\ (=BC)$ $=\frac{a+\sqrt{aa+xx}\times a+x}{x}$. Whence, by taking the fluxion, &c. $x^4=2a^3x+a^4$. To construct this equation, put $az=xx$, and let the parabola DPL answering to $az=xx$ be described, having its axis DH perpendicular to AD; then if there be taken $DK=a$, Kn (parallel to DH) $=\frac{1}{2}a$, and from the center n, with the radius $\frac{3a}{2}$, an arch rPr of a circle be described cutting the parabola in P; the perpendicular sought will pass through this point P; and, if in the perpendicular, so drawn, there be taken $BF=AD$, and the length of AG (parallel to DF) be set off from G to C, then shall ABC (when AC is drawn) be the triangle required to be constructed.

Mr. *M'Roy* (the proposer) has also given an elegant solution of this problem, with some very ingenious observations thereon; the whole of which (though very masterly) is rather too long for our scanty limits, to be given here.—— Mr. *O'Cavanah*, Mr. *Rollinson*, and Mr. *Witchell* have also constructed this problem; and several correspondents have gone through part of the solution.

X. QUESTION 457 *answered by* Philarithmus.

Let E be the point of the ecliptic opposite to the sun's center, and rising with Arcturus; $*l$ an arc of the circle of longitude passing through the star; then (by carrying back the perihelion) the sun's distance from the winter solsticial point in 60 days, about the year 850 A. C. (the middle of the century in which Hesiod is usually placed) will be 60° 10′; whence assuming ♎E (= its complement) 29° 50′, in the triangle ♎EC there are



also given the angles at ♎ and C, whence ♎EC is given (= 109° 12′); therefore in the triangle $*lE$ the angle at E being given, and $*l$ (= Arcturus's latitude), El is found = 12° 3′, and ♎l = 17° 47′, which, added to 19° 53′ 52″, gives the precession of the equinox from Hesiod's time to 1609, equal to 37° 30′ 52″, answering to the year 997 A. C. or rather 1002 A. C.

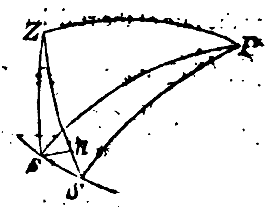
But

But the arc $\simeq E$ ought to be shortened at least 1° for the refractions of the sun and Arcturus, and the sun's semidiameter: Now 1° in the arc $\simeq E$ (or an error of a day in Hesiod's account) lessens the arc $\simeq l$, 50′, and consequently the time 60 years. So that Arcturus's image was in the horizon 60 days after the winter solstice, just when the sun disappeared about the year 940 A. C. But how far the data are to be depended upon as accurate, I am not able to say, considering that Hesiod lived before astronomy was cultivated among the Greeks as a science, and these observations of the stars were used to mark the returns of the seasons in the room of an artificial year, which they wanted. There is one thing indeed might be alledged in favour of those who place him rather later than earlier, that supposing the observations made with all the accuracy they were capable of at first, yet it might have been done before Hesiod's time, and retained for the sake of a round number, whilst its deviations from truth were not very great, or the cause of the alteration known.

Mr. *G. Witchell*, the proposer of this question, proceeds in the solution of it exactly in the same manner. He makes the year sought to be ant. Chr. 1007; which he thinks is near a century and half earlier than the truth; but observes, that, if the sun had been set about 8¼′ before the appearing of the star, the result would give ann. ant. Chr. 870, as Sir Isaac Newton has it in his chronology. Hence he infers, that chronologers ought not to lay any great stress, or too much rest their systems on such observations as these.

XI. QUESTION 458 *answered by Mr.* G. Witchell.

Let P represent the north pole, Z the zenith, and $S s$ a very small part of the sun's parallel of declination described in a given particle of time; supposing PS, Ps, ZS, Zs to be drawn, and also sn perpendicular to ZS. It is evident, that the celerity with which the man (walking uniformly in the direction of his own shadow) leaves the meridian, is always proportional to the sine of the sun's azimuth PZS; but the sine of PZS is to the sine of PSZ (or Ssn) in the constant ratio of the sine of PS to the sine of PZ: And the sine of Ssn is, again, to Sn (the increase of the sun's altitude) in the given ratio of the sine Sns to Ss. Therefore

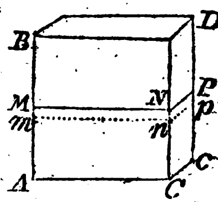


fore the celerity aforesaid is every where in a constant ratio to that with which the sun's altitude increases: And consequently the whole increase of the sun's altitude, from six, to 50 m. after 10 o'clock (found by calculation to be 37° 27′) must be to 12 m. (the whole of the man's westing during that time) in that same constant proportion. But the increase of the sun's altitude when due east (at which time the man leaves the meridian with his absolute motion) is at the rate of 8° 53¼′ per hour, or at the rate of 42° 57′ in 4 h. 50 m. (the whole time of walking): Therefore it will be, as 37° 27′ : 42° 57′ :: 12 miles : 13·76 = whole distance travelled; being at the rate of 2·85 miles per hour. *Q. E. I.*

Thus also (without the least variation in principles) the solution is given by Mess. *O'Cavanah*, *Philarithmus*, *Rollinson*, and *Wildbore*.

XII. QUESTION 459 *answered by Mr.* Hugh Brown (*the Proposer*).

Put $a =$ the depth (AB) of the cistern, and $b = 190{\cdot}638$ = the content, in ale gallons: Then, denoting the three given quantities 2, 5, and 150, by m, n; and p, and calling AM, x; Mm, $-x$, and the time of the descent through BM, t; we shall have $n\sqrt{\frac{x}{a}} =$ the rate (per min.) at which the water runs out; and consequently $n\sqrt{\frac{x}{a}} - m$



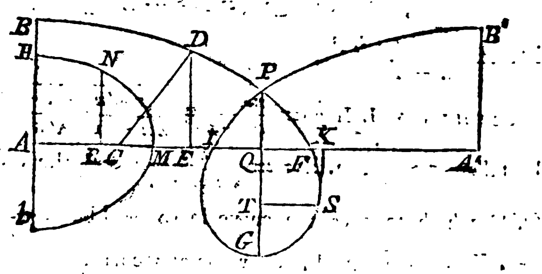
= the rate of the real decrease. Hence, as $1 : \dot{t} :: n\sqrt{\frac{x}{a}} - m : -\frac{b\dot{x}}{a}$ (= the decrease $MNPpnm$, in the time $\dot{t}$); whence $\dot{t} = -\frac{\dot{x}}{a} \times \frac{b}{n\sqrt{\frac{x}{a}} - m}$: Also (by the quest.) we have $mt + b \times \frac{a-x}{a} = p$. But to find the value of t from these equations, let y be assumed $= \frac{n\sqrt{\frac{x}{a}} - m}{n - m}$ (= the ratio

of

of the decrease at M to that at B) so shall $\frac{x}{a}=\frac{\overline{\overline{n-m}\times y+m}^2}{nn}$, and $\dot{t}=-\frac{2b}{nn}\times\overline{\overline{n-m}\cdot\dot{y}+\frac{m\dot{y}}{y}}$; and consequently $t=\frac{2b}{nn}\times\overline{\overline{n-m}\times\overline{1-y}-m\times\text{h. log. }y}$. But, by the other equation, t is $=\frac{p}{m}-\frac{b}{m}+\frac{b}{m}\times\frac{\overline{\overline{n-m}\times y+m}^2}{nn}$: Which two values being compared together, we get $\overline{\frac{n}{m}-1}\times 2y+\overline{\frac{n}{m}-1}^2\times\frac{yy}{2}+\text{h. log. }y=\overline{\frac{n}{m}-1}\times\overline{\frac{n}{2m}+\frac{3}{2}}-\frac{nnp}{2mmb}$: Which, in the present case, becomes $3y+\frac{9yy}{8}+\text{h. log. }y=1{\cdot}666$; whence y is found $=\cdot 9954$; and from thence $t=34{\cdot}33=34$ m. 20 s. the time required.

In a manner very little different, this problem is also answered by Mr. *G. Brown*, Mr. *R. Butler*, *Philarithmus*, and Mr. *Cha. Wildbore*; whose solutions are equally concise and elegant.

XIII. QUESTION 460 *answered by Mr.* O'Cavanah, *of* Dublin.



Let $BDPFGIPB'$ be the required curve, described (in a whole revolution) by the Body D moving uniformly therein, whilst the other proposed body C moves uniformly over the right line AA': Conceive CD, and also the ordinate DE, to be drawn; putting CD ($=AB$) $=a$, $CE=z$, $AC=x$, $AE=y$, and the given ratio of BD to AC, as n to 1. Then

Then will $DE = \sqrt{aa-zz}$, $AE = x+z$, and $BD = nx$; whereof the fluxions being $\frac{-z\dot{z}}{\sqrt{aa-zz}}$, $\dot{x}+\dot{z}$, and $n\dot{x}$, we therefore have $\frac{z^2\dot{z}^2}{aa-zz} + \overline{\dot{x}+\dot{z}}|^2 = n^2\dot{x}^2$; whence $\dot{x} = \frac{\dot{z}}{nn-1} + \frac{\dot{z}\sqrt{nnaa-zz}}{\overline{nn-1}\times\sqrt{aa-zz}}$, and $\dot{y}\ (=\dot{x}+\dot{z}) = \frac{nn\dot{z}}{nn-1} + \frac{\dot{z}\sqrt{nnaa-zz}}{\overline{nn-1}\times\sqrt{aa-zz}}$. To assign the fluent hereof, let HMh be a semi-ellipsis, having its greater semi-axis $AM = AB$, and its lesser (AH) in proportion thereto, as $\sqrt{nn-1}$ is to n; then take $AR = z\ (= CE)$ and make RN parallel to AB; so shall the arch $HN =$ the fluent of $\frac{\dot{z}\sqrt{nnaa-zz}}{n\sqrt{aa-zz}}$; and we shall therefore have $y = \frac{nn\times AR+n\times HN}{nn-1}$; whence the ordinate DE is also known.

COROLLARY I. When z becomes $= a$, then y is $= AF = \frac{nn\times AM+n\times HNM}{nn-1}$: But, when the two bodies are arrived at Q and G, and z is (again) $= 0$, then $y = AQ = \frac{n\times HMh}{nn-1}$; whence $QF\ (= AF - AQ)$ is given $= \frac{nn\times AM - n\times HM}{nn-1}$.

COROLLARY II. If $\dot{y}\ \left(= \frac{nn\dot{z}}{nn-1} + \frac{\dot{z}\sqrt{nnaa-zz}}{nn-1\times\sqrt{aa-zz}}\right)$ be taken $= 0$, then will $z = \frac{na}{\sqrt{nn+1}}$; and the ordinate (KS) which touches the curve in (S) will be $= \frac{a}{\sqrt{nn+1}}$ $(=\sqrt{aa-zz})$. And if the value of y corresponding hereunto be found (by the general equation) and from it AQ be subtracted, we shall thence get the semi-breadth QK (or ST) of the nodus $PSGIP$.—As to the value of PQ it may be also obtained from the same equation, by which $\frac{nn\times AR+n\times HN}{nn-1}\ (=y)$ is given $= \frac{n\times HMh}{nn-1}\ (=AQ)$: Where,

Where, if the value of n be such, that $AR = AM$, then will $n \times AM = HM$, or $n : 1 :: HM : AM$; in which case the intersection P will fall in the line AA': But if the value of n be less than that determinable from hence (but still greater than unity) the point P will then fall below the line AA', but never by a distance greater than $\frac{2884}{10000} \times AB$; nor can ST be ever less than $\frac{266}{1000} \times AB$.

When n is supposed less than unity, the radical quantity $\sqrt{nnaa - zz}$ will be no longer possible than till z becomes $= na$; after which time the conditions of the problem can no longer be fulfilled.

Mr. *Tho. Allen*, Mr. *E. Rollinson* (the proposer), Peter Walton, *Mr. Cha. Wildbore*, and *Philarithmus*, have also favoured us with very elegant solutions to this problem. The last of these gentlemen (who is a new correspondent, and whose judgment and accuracy we greatly approve) has to his solution subjoined some curious observations, differing very little from those above exhibited.

XIV. QUEST. 461 *answered by* (*the Proposer*) P. Walton.

$\frac{\dot{x}}{\sqrt{1 - x^4}}$ being $= \frac{\dot{x}}{\sqrt{1 - x^2} \times \sqrt{1 + x^2}} = \frac{\dot{x}}{\sqrt{1 - x^2}} \times \overline{1 - \frac{1}{2}x^2 + \frac{1.3}{2.4}x^4 - \frac{1.3.5}{2.4.6}x^6 + \&c.}$ it appears, by art. 286 of Mr. Simpson's Fluxions, that the whole fluent of $\frac{\dot{x}}{\sqrt{1 - x^4}}$ is $= f \times \overline{1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \&c.}$ f being the whole fluent of $\frac{\dot{x}}{\sqrt{1 - x^2}}$; which, it is well known, is $\frac{1}{4}$ of the periphery of a circle whose radius is 1.

Now, if x be supposed $= y^{\frac{1}{2}}$, $\frac{\dot{x}}{\sqrt{1 - x^4}}$ will be $= \frac{\frac{1}{2}y^{-\frac{1}{2}}\dot{y}}{\sqrt{1 - y^2}}$ of which fluxion (by Landen's Math. Lucubrations, p. 146) the whole fluent is $\frac{e + \sqrt{ee - 2f}}{2}$, e denoting $\frac{1}{4}$ of the periphery of an ellipsis, whose semi-axes are $\sqrt{2}$ and 1.

It

It follows therefore, that the series $1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \&c.$ is $= \frac{e + \sqrt{ee - 2f}}{2f}$.

The same answered by Mr. E. Rollinson.

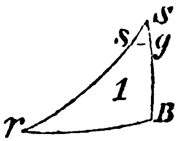
It is well known that $Q \times : 1 - \frac{1}{2}e + \frac{1.3}{2.4}f - \frac{1.3.5}{2.4.6}g +$ &c. is the fluent of $\frac{\dot{x}}{\sqrt{1-x^2}} \times : 1 - ex^2 + fx^4 - gx^6 +$ &c. when $x = 1$; Q being the fluent of the first term, and $= \frac{1}{4}$ of the periphery of the circle whose radius is unity. Hence, if e be taken $= \frac{1}{2}$, $f = \frac{1.3}{2.4}$, $g = \frac{1.3.5}{2.4.6}$, &c. then $Q \times : 1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2} + \&c. =$ flue. $\frac{\dot{x}}{\sqrt{1-xx}} \times : 1 - \frac{1}{2}x + \frac{1.3}{2.4}x^2$ &c. $=$ flu. $\frac{\dot{x}}{\sqrt{1-xx}} \times \overline{1+xx}\,^{-\frac{1}{2}} =$ flu. $\frac{\dot{x}}{\sqrt{1-x^4}}$; and consequently $1 - \frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} - \frac{1^2.3^2.5^2}{2^2.4^2.6^2} = \frac{1}{Q} \times$ flu. $\frac{\dot{x}}{\sqrt{1-x^4}}$ (when $x = 1$). But $\frac{\dot{x}}{\sqrt{1-x^4}} + \frac{x^2\dot{x}}{\sqrt{1-x^4}}$ is known to express the fluxion of an elliptical arch (R), whose two semi-axes are 1 and $\sqrt{2}$; and it may be easily demonstrated (by the method laid down at p. 76. in Simpson's Laws of Chance) that, if A and B be assumed to denote the fluents of $\frac{x^{pn-1}\dot{x}}{\sqrt{1-x^n}}$ and $\frac{x^{pn+\frac{1}{2}n-1}\dot{x}}{\sqrt{1-x^n}}$ (p and n being any positive numbers whatever), the product AB (of these fluents) will (when $x = 1$) be $= \frac{2Q}{pnn}$. Whence, by taking $n = 4$, and $p = \frac{1}{4}$, so that A and B may become the respective fluents of $\frac{\dot{x}}{\sqrt{1-x^4}}$ and $\frac{x^2\dot{x}}{\sqrt{1-x^4}}$ (whereof the sum is given above $= R$), we have AB, in this case, $= \frac{1}{2}Q$: From whence, and the equation $A + B = R$, A is found $= \frac{R}{2} + \frac{1}{2}\sqrt{R^2 - 2Q}$; and $\frac{A}{Q} = \frac{R}{2Q} + \frac{1}{2}\sqrt{\frac{RR}{QQ} - \frac{2}{Q}} =$ the series proposed.

Some

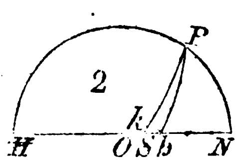
Some correspondents have objected against this question, as an unfair one; "because there can be no reasoning from the data, to arrive at a conclusion."——We shall not take upon us to decide this point; but shall not, however, scruple to allow, that problems of a very abstracted and intricate nature, are less eligible in a work like this, than some others of a different kind.

The PRIZE QUESTION *answered by* Κυβερνήτης.

Let sS represent the increase of the sun's longitude (Υs in the space of six hours (or $\frac{1}{4}$ of a day); and let Sg be the increase of declination corresponding thereto: Putting $s =$ sin. Υ, $x =$ sin. ΥS, and $u = sS$: Then will $sx =$ sin. dec. BS, and $\sqrt{1-ssxx}$ $=$ its cosine: And so, again (per spherics) tang. ΥS $\left(\frac{x}{\sqrt{1-xx}}\right)$: tang. BS



$\left(\frac{sx}{\sqrt{1-ssxx}}\right)$: : rad. : cos. S : : $sS\,(u)$: $Sg = \frac{su\sqrt{1-xx}}{\sqrt{1-ssxx}}$ $=$ the increase of decl. in $\frac{1}{4}$ of a day. Therefore, if m be now put for $\frac{1}{4}$ of the whole periphery of the circle (the rad. being unity) and z for the arch, expressing the sun's ascensional difference, we shall have $m : m+z : : \frac{su\sqrt{1-xx}}{\sqrt{1-ssxx}}$: $\frac{su\times\overline{m+z}\sqrt{1-xx}}{m\sqrt{1-ssxx}} =$ the alteration of decl. from noon to sun-setting. Let this (*in Fig.* 2.) be denoted by Sk (supposing PS to be the sun's polar distance, and S his place in the horizon HON, at setting) and let b, c, and d be taken to express the sine, cosine, and tangent of PN the pole's elevation. Then, the sine of S being $= \frac{b}{\sqrt{1-ssxx}}$, its cosine will



be $= \frac{\sqrt{cc-ssxx}}{\sqrt{1-ssxx}}$; whence $bk = \frac{b}{\sqrt{cc-ssxx}} \times (Sk)$; and consequently sin. kPb $\left(= \frac{\text{rad}}{\text{sin.}\,PS} \times (bk)\right. =$

$$\frac{b\times(Sk)}{\sqrt{1-ssxx}\times\sqrt{cc-ssxx}}=\frac{bsu\times\overline{m+z}.\sqrt{1-xx}}{m\times\overline{1-ssxx}\times\sqrt{cc-ssxx}};$$

which, by the question, is to be a maximum; and consequently its log. ($=$ log. $\frac{bs}{m}$ + log. u + log. $\overline{m+z}$ + $\frac{1}{2}$ log. $\overline{1-xx}$ − log. $\overline{1-ssxx}$ − $\frac{1}{2}$ log. $\overline{cc-ssxx}$) a maximum also: Whence, in fluxions (putting $t=\frac{s}{c}$) we have $\frac{\dot{u}}{u}+\frac{\dot{z}}{m+z}$ $-\frac{x\dot{x}}{1-xx}+\frac{2ssx\dot{x}}{1-ssxx}+\frac{ttx\dot{x}}{1-ttxx}=0$; but the sine of z being $\frac{bsx}{c\sqrt{1-ssxx}}$, we have $\dot{z}=\frac{bs\dot{x}}{\overline{1-ssxx}.\sqrt{cc-ssxx}}$ $=\frac{ds\dot{x}}{\overline{1-ssxx}.\sqrt{1-ttxx}}$. And, to determine u and $\dot{u}$ also, let e ($=$·017) denote the eccentricity of the earth's orbit, and y the sine complement of the sun's (or earth's) true anomoly: Then $\overline{1-ey}|^2$ (as is well known) will exprefs the rate of the increafe of this anomoly (or of the fun's true longitude): Which quantity being therefore proportional to u, we have $\frac{\dot{u}}{u}=\frac{-2e\dot{y}}{1-ey}$. But the difference of the two angles, whofe fines are y and x, being given ($=9°\ 14'$) $=$ the diftance of the aphelion from the folftitial point, we fhall, by putting the fin. of $9°\ 14'=p$, and its cofine $=q$, have $y=qx-p\sqrt{1-xx}$; and confequently $\frac{\dot{u}}{u}$ ($=\frac{-2e\dot{y}}{1-ey}$) $=-\frac{2eq\dot{x}\sqrt{1-xx}-2epx\dot{x}}{\sqrt{1-xx}\times\overline{1-eqx+ep\sqrt{1-xx}}}=-2e\dot{x}$, very near (becaufe the terms both in the numerator and denominator, after the two firft, may, on account of their fmallnefs, be neglected). This value, therefore, with that of $\dot{z}$, being fubftituted in the general equation, we at length get

$$-2e+\frac{ds}{\overline{1-ssxx}\sqrt{1-ttxx}\times\overline{m+z}}-\frac{x}{1-xx}+\frac{2ssx}{1-ssxx}$$
$$+\frac{ttx}{1-ttxx}=0$$

: From whence, (either by refolving the whole into a feries, or by any other of the known methods of approximation) the value of x may be found, and will come out $=$·493 $=$ fin. of $29°\ 32'$ $=$ fun's longitude fought; anfwering to the 19th day of April; the afternoon of th[illegible] day exceeding the forenoon [illegible]

The Eclipses calculated for 1760, by *Mr.* George Witchell.

There will, in the course of this year, be four eclipses, two of the sun, and as many of the moon; three of which will be visible to the inhabitants of Great Britain: The calculations whereof from Mayer's Tables, for the meridian and latitude of London, are as follow.

The first is a very small eclipse of the moon, May 29th, in the evening; beginning at 9 h. 14 m. the middle at 9 h. 34 m. the end at 9 h. 54 m. apparent time; the duration being only 40 m. and the quantity eclipsed $\frac{1}{3}$ of a digit.

The second is a solar eclipse, June 13th, in the morning; beginning at 6 h. 33 m. 58 s. the greatest obscuration at 7 h. 25 m. 26 s. and the end at 8 h. 12 m. 30 s. apparent time; the duration 1 h. 32 m. 32 s. and the digits eclipsed 4° 33'.—— This eclipse will be total in the Lesser Asia; and the central shade will nearly traverse the same route with that celebrated eclipse of antiquity, May 18th, an. ante Chr. 603, which frightned the Medes and Lydians into a peace, after a five years obstinate war.

The third is an eclipse of the moon, Nov. 22d, in the evening; beginning at 7 h. 43 m. the middle at 8 h. 56 m. and the end at 10 h. 3 m. duration 2 h. 25 m. digits eclipsed 6° 10'.

The fourth is an eclipse of the sun, Dec. 7th, about two in the afternoon, but invisible to all these northern parts, by reason of the moon's great south latitude.

Some other calculations of the solar eclipse (June 13th) by other correspondents, from different tables, and for different parts of the kingdom, are here subjoined.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig.
Mr. T. Harris, from { London	6 37	7 23	8 10	1 33	4 33
Brent's Tab. for { Bugbrook	6 33	7 17	8 4	1 31	4 12
Mr. T. Allen, for Spalding	6 38	7 26	8 17	1 39	5 10
Mr. J. Goodhead, for Nottingham	6 35	7 16	8 8	1 33	4 24

Other calculations of this eclipse have been received, but so very incorrect, and wide of the truth, that it would do the authors no credit to insert them.

New Questions.

I. QUESTION 462, *by Miss* Ann Nichols.

Two partners, in a venture made,
Gain twice two hundred pounds in trade:
The stock of *A*, when they began,
Exceeded *B*'s by eighty-one;
Twice ninety-five *B* gain'd in all.
Now for the stock of both I call.

II. QUESTION 463, *by Miss* S. T.

A drover bought in as many sheep, of different sorts, as cost him 48 l. one-third of which he sold again at 20s. a-piece; one-fourth at 18s. and the rest at 16s. a-piece; and found his gain, upon the whole, to be 5 l. 10s. What number of sheep did he buy and sell?

III. QUESTION 464, *by Mr.* Tho. Harris.

Near Ouse's verdant banks, in Bedfordshire,
Stands Carlton, blest with a sagacious fair,
In whom at once Minerva's wit is seen,
Diana's chastness, and the graces' mien.
Would you the name of this fair charmer know,
First solve th' equations which you'll find below:

* $x+y+z+v=56$, $xx+yy+zz+vv=910$, $xv+2yy-zz=6$, $z=2y$; in which the values of x, y, z, and v denote the places in the alphabet of the four letters that compose this amiable maiden's name.

IV. QUESTION 465, *by Mr.* Geo. Brown.

From two given points, to draw two lines to meet in the circumference of a circle given in position and magnitude, so that the sum of their squares shall be the least possible.

V. QUESTION 466, *by Mr.* W. Chapman.

There are three circles whose diameters are 30, 20, and 25 inches, having their centers all placed in the same right line; whereof the distance of the first from the second is 36, and of the second from the third 28 inches; now you are desired to determine the diameter and position of a fourth circle, that shall touch all the three given ones.

VI. QUES-

VI. Question 467, *by Mr.* W. Spicer.

Surveying of a field, I found the four ſides thereof to be 10, 9, 7, and 6 chains, in a ſucceſſive order; I likewiſe, at the two extremes of the longeſt ſide, took the bearings of the oppoſite angles, which were N. E. by E. and W. S. W. Hence the content of the field is required.

VII. Question 468, *by Mr.* Tho. Barker.

Suppoſing two ſides of a triangle to be given (equal to 25 and 16 feet), whereof the greater is parallel to the horizon; I deſire to know what the length of the third ſide muſt be, ſo that the time of the deſcent of a heavy body along the ſame, ſhall be a minimum?

VIII. Question 469, *by* P. M. *of* Durham.

To deſcribe the circumference of a circle through two given points, and which ſhall cut off from a given circle an arc equal to an arc given.

IX. Question 470, *by Mr.* Patrick O'Cavanah.

On one of the banks of a certain river, ſtand four wind-mills, all in the ſame right line; whereof the diſtance of the firſt from the ſecond is known to be 150 yards, of the ſecond from the third 180 yards, and of the third from the fourth 200 yards: Being on the oppoſite ſhore, I found by obſervation, that the two middlemoſt of them ſubtended an angle, at my eye, of 15 degrees; and that the angles ſubtended by the firſt and ſecond, and by the third and fourth, were equal, the one to the other. Hence I would know (by means of a geometrical conſtruction) not only the breadth of the river, but alſo my particular diſtance from each of the four objects, in that ſtation.

X. Question 471, *by Mr.* E. Rollinſon.

A gentleman bought an eſtate in houſes for 1500l. which, being let, brought him in 120l. per annum, clear of all expences and deductions: At the end of ten years (moſt of the houſes being out of repair, and he not chooſing to be at the expence of fitting them up) he ſold the whole eſtate again for 800l. The queſtion is, to find what intereſt he made of his money.

XI. QUESTION 472, *by Mr.* W. Bevil.

Out of a semi-circular piece of ground, the radius of which is 12 chains, I am to take a garden with a fence of 10 chains (terminating in the semi-circle) so as to contain the most land possible; how many acres will fall to my lot.

XII. QUESTION 473, *by Mr.* Cha. Wildbore.

'Tis proposed to divide a given number into so many parts, and so proportioned among themselves, that the continual product of the first, the square of the second, the cube of the third, the biquadrate of the fourth, &c. shall be a maximum.

XIII. QUESTION 474, *by Mr.* G. Witchell.

Supposing the moon's declination, as well as the latitude of the place to be given, I desire to know, at what elevation her parallax in altitude increases the quickest.

PRIZE QUESTION, *by Mr.* O'Cavanah.

Two persons, *A* and *B*, whose chances for winning a single game are as 3 to 2, the former having seven guineas, and the latter four, enter into play together, on condition that *A* every game shall stake two guineas to *B*'s one; and that the play between them shall continue till one of them, either by having lost his whole stock, or for want of a complete stake, is obliged to give out. The question now is, to find the gain of *B* and the disadvantage of *A* in this agreement; and to point out a general method for the resolution of all questions of this nature.

Note, M. *De Moivre*'s method will not give a true solution; nor has there been any rule or method yet laid down by which such questions can be resolved.

The question by the very ingenious *Peter Walton*, requiring the sum of the series $x + \frac{x^2}{4} + \frac{x^3}{9} + \frac{x^4}{16}$ &c, by means of circular arcs and logarithms, when $x = \frac{1}{2}$; is of a limited, and very intricate nature, and what we could not venture to give in form among the others, as some of our correspondents, we are convinced, would not allow it to be a fair one. It is worthy, however, to be observed, that it is only in the case here proposed (where $x = \frac{1}{2}$) and in that where

where $x = 1$ (x being positive), in which the series seems possible to be summed in the manner specified. With respect to the latter case, it is pretty well known, at this time, that the sum is expressed by two-thirds of the square of the semi-circumference of the circle whose diameter is unity. And if from the half of this sum, half the square of the h. log. of the number 2 be subtracted, the remainder will be the sum in the other case, where $x = \frac{1}{2}$. How this is derived, is left to be considered (agreeably to the design of the proposer) by those who have a taste for these subtile and abstracted speculations.

1761.

Questions answered.

I. QUESTION 462 *answered.*

SINCE, according to the question, *A*'s stock exceeded *B*'s by 81 l. and his gain was greater than *B*'s by 20 l. Say therefore as 20 : 81 : : 210 (*A*'s whole gain) 850 l. 10 s. the stock of *A*; whence that of *B* is found = 769 l. 10 s.

Thus the solution is given by *Amaryllis*, Mr. *S. Burt*, Mr. *J. Collins*, Mr. *W. Hervey*, Mr. *E. D. Hudson*, Mr. *W. Ingram*, *Lesbia*, Mr. *W. Litson*, Mr. *R. Locke*, Mr. *R. Marsh*, Mr. *R. Miles*, Mr. *R. Morris*, Mr. *G. Nokes*, *Mall Ormishaw*, Mr. *Sim. Pedley*, Mr. *T. Sandling*, Mr. *W. Snaith*, Mr. *R. Spencer*, Mr. *Mat. Ward*, Mr. *T. Wilkin*, and others. ——A great number of algebraical solutions to this question have likewise been received from Mr. *W. Beer*, Mr. *T. Bromhall*, Mr. *E. Hare*, Mr. *T. Jones*, Miss *A. Nicholls* (the proposer), Mr. *J. Lyon*, Mr. *T. Sadler*, Mr. *J. Scholfield*, Mrs. *Eleanor Suggett*, Mr. *T. Wilson*, and others.

II. QUESTION 463 *answered.*

Let $12x$ = the number of sheep; then, by the question, $4x \times 20 + 3x \times 18 + 5x \times 16 = 1070$, or $214x = 1070$: Therefore $x = 5$, and $12x = 60$, the number of sheep required.

In this manner it is resolved by Mr. *T. Bromhall*, Miss *S. Fenwick*, Mr. *E. Hare*, Mr. *Jos. Harrington*, Mr. *D. Hastings*, Mr. *E. Houlston*, Mr. *G. Langley*, Mr. *R. Mallock*, Mr. *Chr. Mesban*, Miss *Nicholls*, Mr. *T. Roe*, Mr. *Alex. Rowe*, Mrs. *E. Suggett*, Mr. *T. Walker*, and some others. ——Mr. *Jonath. Ashton*, *Amaryllis*, Mr. *J. Chapman*, Mr. *W. Comport*, Mr. *Nath. Cory*, Mr. *T. Crump*, Mr. *R. Dening*, Mr. *J. Hitchcock*, *Lesbia*, Mr. *E. Lowe*, Mr. *W. Penn*, *Mall Ormishaw*, Mr. *J. Salter*, Mr. *R. Spencer*, Mr. *W. Wells*, Mr. *T. Wilkin*, and Mr. *E. Wright*, likewise obliged us with solutions to this problem.

III. QUESTION 464 *answered by Mr.* Edw. Nott, *of* Stamford.

By writing $2y$ for z, the equations become $x+3y+v$ $(=36)=a$, $xx+5yy+vv$ $(=910)=b$, $vx-2yy$ $(=6)$ $=c$: To the second of which add twice the third, and we shall have $xx+2vx+vv$, or $\overline{x+v}^2=b+2c-yy=\overline{a-3y}^2$ (by the first); whence $10yy-6ay=b+2c-aa$; from which y will be found $=9$, $x=8$, $z=18$, and $v=21$: And the letters of the alphabet corresponding, whereby the young lady's name is formed, are, *I, H, S, W*.

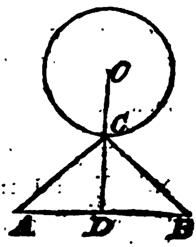
I WISH I cou'd such a fair charmer disclose,
I'd gladly another equation propose,
Which the best algebraist should never untie,
Tho' he puzzled for ever with v, x, and y.

In this same manner the solution is also given by Mr. *T. Allen*, Mr. *T. Atkinson*, Mr. *T. Baker*, Mr. *T. Barker*, Mr. *E. Barras*, Mr. *E. Batten*, Mr. *T. Bramley*, Mr. *J. Buddles*, Mr. *W. Burtoft*, Mr. *Chr. Cave*, Mr. *J. Collins*, Mr. *T. Cooke*, Mr. *W. Embleton*, Miss *S. Fenwick*, Mr. *W. Terrill*, Mr. *N. Gerrad*, Mr. *R. Gibbons*, Mr. *J. Hampson*, Mr. *Jos. Harrington*, Mr. *D. Hastings*, Mr. *J. Hicks*, Mr. *Malachy Hitchins*, Mr. *J. Honey*, Mr. *E. Houlston*, Mr. *S. Kemp*, Mr. *E. Lowe*, Mr. *R. Mallock*, Mr. *Chr. Mesban*, *Nosnibctub*, Mr. *J. Orchard*, Mr. *T. Roe*, Mr. *A. Rowe*, Mr. *Jos. Ruston*, Mr. *T. Sadler*, Mr. *T. Walker*, Mr. *R. Walton*, and Miss *M. Williams*.

IV. QUES-

IV. QUESTION 465 *answered by Mr.* Davies, *of* Newent, Gloucestershire.

Let O be the center of the given circle, and let A and B be the two given points; draw the line AB and bisect it in D; also draw DO cutting the circle in the point C. So shall AC and BC, when drawn, be the two lines required.

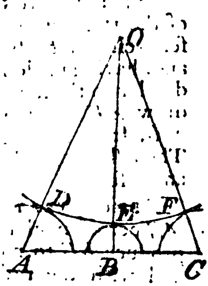


DEMONSTRATION. Because DC is a minimum, or the shortest line that can possibly be drawn from D to the circumference of the circle, $2DC^2$ must therefore be a minimum. But, by a known theorem, $AC^2 + BC^2 = 2AD^2 + 2DC^2$; therefore ($2AD^2$ being constant) $AC^2 + BC^2$ must be a minimum. Q. E. D.

In the very same manner the problem was constructed by Mr. *Jos. Brampton*, Mr. *T. Moss*, and Mr. *G. Witchell*. ——Mess. *T. Allen*, *T. Barker*, *J. Boddles*, *W. Chapman*, *W. Terrill*, *J. Hampson*, *T. Harris*. *E. Nott*, *Nosnibetuh*, *Steph. Ogle*, *Paul Sharp*, *W. Spicer*, and some others, have given neat algebraic solutions to it.

V. QUESTION 466 *answered by Mr.* E. Batten.

Let A, B, and C be the centers of the three given circles, and O that of the required one. Put $a = AB = 30$, $b = BC = 28$; $c = AC = 58$, $x = OB$, $x + m\ (x + 5) = OA$, $x + n\ (x + 2\frac{1}{2}) = OC$. Now (per lemma, p. 128 of Mr. Simpson's Select Exercises) $\overline{x+m}^2 \times b + \overline{x+n}^2 \times a = \overline{xx+ab} \times c$; that is, $\overline{b+a}.xx + \overline{2bm+2an}.x + \overline{bmm+ann} = cxx + abc$; or, $\overline{2bm+2an}.x + bmm + ann = abc$; whence $x = \frac{abc - bmm - ann}{2bm + 2an} = 111\frac{41}{172}$; and consequently the radius OE of the required circle $= 101\frac{41}{172} = 101{\cdot}238$, &c.

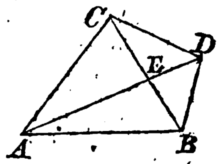


In

In the same, or in a manner very little different, the answer is also brought out by Mess. *T. Allen, W. Chapman, L. Charlton, W. Davies, J. Hampson, T. Harris, S. Kemp, W. Kingston, E. Nott, P. Sharp, W. Spicer*, and *T. Walker*.*

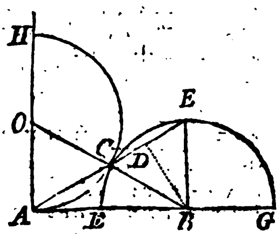
VI. QUESTION 467 *answered.*

The best solutions that have been received to this question are derived in virtue of a theorem in the Diary for 1758, where it is demonstrated, that the quadruple of the area of any trapezium *ABCD* is equal to $AB^2 + CD^2 - BC^2 - AD^2 \times \frac{\text{tang. } E}{\text{rad.}}$: From whence (seeing the angle *E*, as well as the sides of the trapezium, are all given, by the question) the area will be found: And in this manner is actually determined by Mr. *W. Davies*, Mr. *D. Hastings*, Mr. *T. Harris*, Mr. *Malachy Hitchins*, Mr. *W. Kingston*, Mr. *W. Spicer*; and is found to be $= 8 \times$ tang. $11^\circ\ 15' =$ 1·5912992 square chains.—If the second bearing were to be W. N. W. instead of W. S. W. (as it was misprinted) the angle made by the two diagonals would then be 5 points (instead of one); and the area of the trapezium 11·9728 square chains.



VII. QUESTION 468 *answered by Mr.* T. Barker, *the Proposer.*

CONSTRUCTION. With a radius *BE* equal to the lesser of the two given sides, let a semi-circle *EFG* be described; and, having made *BF* perpendicular to the longer side *AB*, draw *AF*, cutting the circle in *C*, from whence draw *CB*: Then shall *ABC* be the triangle sought.



DEMONSTRATION. Make *AH* perpendicular to *AB*, and produce *BC* to meet it in *O*; and with the radius *OA* or

OC

* A *Construction* of this prob. may be seen in Lawson's Apollonius on Tangencies.

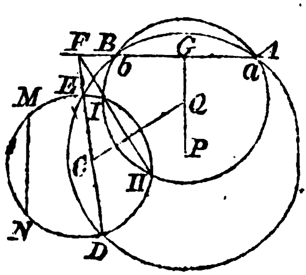
OC (for they are equal, becauſe BF and BC are equal) let the ſemi-circle ACH be deſcribed. Then the time of deſcent along the chord CA will, it is well known, be equal to the time of deſcent along the diameter HA; which will be the ſhorteſt poſſible, becauſe the ſemi-circle ACH only touches the given one $ECFG$, and has therefore its diameter leſs than any other ſemi-circle that can be deſcribed on AH produced, to cut the given ſemi-circle $ECFG$. Q. E. D.

CALCULATION. In the right-angled triangle ABF, are given $AB = 25$, and $BF = 16$; whence the angle $BAF = 32° 27'$: Then in the triangle ACB are given AB, BC, and the angle A; whence $AC = 12{\cdot}425$.

But the required ſide AC may be otherwiſe found, independent of trigonometry: For, if BD be made perpendicular to AF, then, by ſimilar triangles, $FD = \frac{BF^2}{AF}$; and conſequently $AC\ (= AF - 2FD) = \frac{AF^2 - 2BF^2}{AF} = \frac{AB^2 - BF^2}{\sqrt{AB^2 + BF^2}} = 12{\cdot}425$.

In the very ſame manner the problem is conſtructed, and determined, by Mr. *J. Brampton*, Mr. *W. Davies*, Mr. *R. Gibbons*, Mr. *J. Hicks*, Mr. *T. Harris*, and Mr. *G. Witchell*. ——Very neat algebraic ſolutions to it have alſo been received from Mr. *T. Allen*, Mr. *J. Buddles*, Mr. *L. Charlton*, Mr. *T. Cooke*, Mr. *W. Embleton*, Miſs *Sally Fenwick*, Mr. *D. Haſtings*, Mr. *J. Honey*, Mr. *S. Kemp*, Mr. *E. Nott*, Mr. *P. Sharp*, Mr. *W. Spicer*, and others.

VIII. QUESTION 469 *anſwered by Mr.* Lionel Charlton, *of* Whitby.



Having drawn GP to biſect, at right angles, the line AB joining the given points, from any point in it, as P, let the circumference of a circle be deſcribed through A and B, to cut the given circle $EMND$ in any two points D and E; through which points draw DEF to meet AB produced in F; then draw FH at an equal diſtance from the center C

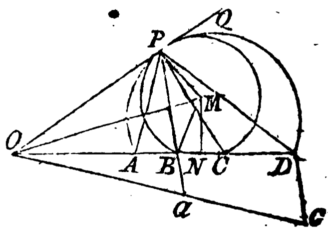
with

with the chord MN of the arch given; and, perpendicular thereto, draw CQ, meeting GP in Q: So shall Q be the center of the circle required.

DEMONSTRATION. From the center Q, through H and I, let the circumference of a circle be described, cutting AF in b and a. Then (by Euc. 36. 3.) $Fa \times Fb = FH \times FI = FD \times FE = FA \times FB$: But (by Euc. 6. 2.) $Fa \times Fb = FG^2 - bG^2$, and $FA \times FB = FG^2 - BG^2$; therefore $FG^2 - bG^2 = FG^2 - BG^2$; and so, bG being $= BG$, the points b, B coincide, and the circumference of the circle described from the center Q, through H and I, likewise passes through B and A. Q.E.D.

IX. QUESTION 470 *answered by* Mr. E. Rollinson.

Let A, B, C, D be the places of the four objects; and upon BC let a segment of a circle BPC be described to contain the given angle of 15°: Make Ba and Dc parallel to each other, taking the former $= BA$, and the latter $= DC$; and then draw caO to meet DA, produced, in O; from whence draw OQ to touch the circle in P, which is the point sought.



DEMONSTRATION. Conceive the circumference of a circle to be described through A, P, and D. By construction and sim. △s, Ba (BA) : OB :: Dc (DC) : OD; whence, by division, $OA : OB :: OC : OD$, and consequently $OA \times OD = OB \times OC = OP^2$ (Euc. 36. 3.) whence it is evident, that OQ also touches the circle APD in the point P: But (by Euc. 32. 3.) the angle $APO = ADP$, and $BPO = BCP$; and consequently APB $(BPO - APO = BCP - ADP) = CPD$. Q.E.D.

CALCULATION. Having, from the center M, let fall the perpendicular MN, and drawn MB, MP, MO, &c. it will be as $Dc - Ba$ (50) : BD (380) :: Ba (150) : $BO = 1140$; whence $NO = 1230$. Also as NB (90) : NO (1230) :: tang. NMB (15°) : tang. $NMO = 74° \; 43' \frac{1}{2}$; whence $NOM = 15° \; 16\frac{1}{2}$, and $BMO = 59° \; 43\frac{1}{2}$. Again, as sin. MBN (75°) : rad. (:: MN : MB (MP) :: sin. NOM : sin. $POM =$

$15^\circ\ 49\frac{1}{2}'$; whence $PMO = 74^\circ\ 10\frac{1}{2}'$, PMB ($= PMO + BMO$) $= 133^\circ\ 54'$, and BCP ($= \frac{1}{2}PMB$) $= 66^\circ\ 57'$: From which the rest are readily found, viz. $PB = 639.93$, $PC = 688.61$, APB (or CPB) $= 13^\circ\ 29\frac{1}{2}'$, $PA = 636.49$, $PD = 788.66$, and the perpendicular distance of P from AD ($=$ breadth of the river) $= 633.62$.

Mr. *W. Davies*, Mr. *D. Hastings*, Mr. *W. Kingston*, Mr. *E. Nott*, and Mr. *G. Witchell* have constructed this problem in the same manner.——*P. M.* of *Durham* has also obliged us with exceeding neat constructions of this, and of the 4th, 7th, and 8th problems; which, had they come to hand in due time, would have obtained a place suitable to their merit.

X. QUESTION 471 *answered by Mr.* J. Honey, *of* Redruth, Cornwall.

Let $a = 1500$l. $b = 800$l. $c = 120$l. and x the required rate of interest: Then will $ax =$ principal and interest, and $ax - c =$ amount after the first payment is deducted. And in the same manner we have $ax^{10} - cx^9 - cx^8 - cx^7 - cx^6 - cx^5 - cx^4 - cx^3 - cx^2 - cx - c = b$ (per quest.) or $ax^{10} - c \times \frac{x^{10} - 1}{x - 1} = b$; therefore $ax^{11} - \overline{a - c} \times x^{10} - bx + b + c = 0$. Solved, $x = 1.04142$, &c. and the rate of interest required 4l. 2s. 10d. per cent.

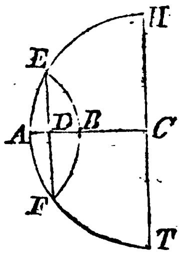
The same answered by Mr. W. Kingston, *of* Bath.

Let x be the rate: Then the amount of 1500l. capital, at the end of 10 years, will be $1500x^{10}$; and an annuity of 120l. forborn the same time, will amount to $120 \times \frac{x^{10} - 1}{x - 1}$. Therefore, we have $1500x^{10} = 120 \times \frac{x^{10} - 1}{x - 1} + 800$ (by the question), or $x^{10} - .08 \times \frac{x^{10} - 1}{x - 1} = \frac{8}{15}$. Put $x = 1 + v$; then x^{10} ($= \overline{1 + v}|^{10}$) being $= 1 + 10v + 45v^2 + 120v^3 + 210v^4$, &c. our equation, by proper substitution, will become $6.4v + 35.4v^2 + 113.2v^3$, &c. $= \frac{7}{15}$; whence, by reverting the series, v is found $= .04142$: And the rate of interest sought is 4l. 2s. 10d. per cent.

Mr. *T. Barker*, Mr. *W. Chapman*, Mr. *R. Dening*, Mr. *T. Harris*, and some others, sent solutions to this problem.

XI. QUESTION 472 *answered by Mr.* G. Witchell.

It is demonstrable, even by common geometry (see Simpson's Elem. p. 207, 2d edit.) that the given length, or boundary EBF (whatever the length the right line EF is to have) must form the arch of a circle. In order therefore to the resolution of this, and other problems of the like nature, it will be very convenient to have ready at hand, a proper series, or near approximation for the area of a circular segment (EBF) expressed in terms of the arch BE (a), and its sine DE (s), independent of the radius. Such an approximation is the following one, viz.



$as\sqrt{\frac{2}{3}a\times\overline{a-s}} = EBF$: From whence, by making s variable (and $a = 5$, as given by the quest.) we have $\frac{\frac{4}{3}aa\dot{s}-2as\dot{s}}{\sqrt{\frac{2}{3}a\times\overline{a-s}}}$ = flux. of the segment EBF. But the fluxion of the other segment EAF, if the radius CA ($=12$) be denoted by r, will, it is well known, be $= \frac{2s^2\dot{s}}{\sqrt{rr-ss}}$. Therefore, when the sum of both segments is a maximum, we shall have $\frac{\frac{4}{3}aa\dot{s}-2as\dot{s}}{\sqrt{\frac{2}{3}a\times\overline{a-s}}} + \frac{2s^2\dot{s}}{\sqrt{rr-ss}} = 0$; and consequently $\frac{3a}{2}\times\overline{s-\frac{2}{3}a}^2\times\overline{rr-ss} - s^4\times\overline{a-s} = 0$: Whence s is found $= 3{\cdot}844$; from which the area of the segment EBF comes out $= 15{\cdot}09$, and that of the segment $EAF = 3{\cdot}26$: And consequently the whole of both $= 18{\cdot}35$ square chains.*

XII. QUESTION 473 *answered by Lieut.* Henry Watson.

Let a represent the quantity given, and x the required number of parts into which it is to be divided; and let the first (or least) part be denoted by y. Then it is very easy to demonstrate

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* Although approximating theorems ought not generally to be used when they are to be put into fluxions, yet they will sometimes answer pretty well, as is the case here.

demonstrate, that the 2d part will be $=2y$, the 3d part $=3y$, &c. Therefore we have $y+2y+3y+4y\ldots\ldots+xy$ (or $x\times\frac{x+1}{2}\times y$) $=a$: And consequently $\frac{2a}{x\times\overline{x+1}}$. Now in order to find such an integer for the value of x, we shall make $y\times\overline{2y}|^{2}\times\overline{3y}|^{3}\ldots\times\overline{xy}|^{x}$, or its equal, $2^2\times3^3\times4^4$

$\ldots\ldots\times x^x\times\overline{\frac{2a}{x.\overline{x+1}}}\Big|^{\frac{x.\overline{x+1}}{2}}$, a maximum, let $z-1$, and

z be wrote successively therein, instead of x, and let the two quantities thus arising, be made equal to each other: So

shall $2^2\times3^3\times4^4\ldots\ldots\overline{z-1}|^{z-1}\times\overline{\frac{2a}{\overline{z-1}.z}}\Big|^{\overline{z-1}.\frac{1}{2}z}$

$=2^2\times3^3\times4^4\ldots\ldots z^z\times\overline{\frac{2a}{z.\overline{z+1}}}\Big|^{\frac{1}{2}z.\overline{z+1}}$; which, if

the whole be divided by $2^2\times3^3\times4^4\ldots\ldots\times\overline{z-1}|^{z-1}$, will

will give $\overline{\frac{2a}{\overline{z-1}.z}}\Big|^{\overline{z-1}.\frac{1}{2}z}=z^z\times\overline{\frac{2a}{z.\overline{z+1}}}\Big|^{\frac{1}{2}z.\overline{z+1}}$;

And, if the indices be now divided by $\frac{1}{2}z$, we shall have

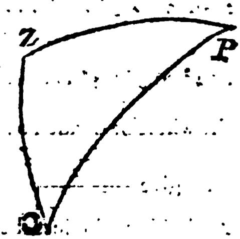
$\overline{\frac{2a}{\overline{z-1}.z}}\Big|^{z-1}=z^2\times\overline{\frac{2a}{z\times\overline{z+1}}}\Big|^{z+1}$; whence, by reduction, $\frac{\overline{z+1}|^{z+1}}{\overline{z-1}|^{z-1}}=4aa$, or $2\log. z+\overline{z+1}\log.\overline{1+\frac{1}{z}}$

$-\overline{z-1}\log.\overline{1-\frac{1}{z}}=2\log. 2a$; from either of which equations the value of z may be found: And the next inferior integer thereunto will be the number of parts required.

COROLLARY. When a is a large number, z being also large, $\overline{z+1}\times\log.\overline{1+\frac{1}{z}}-\overline{z-1}\times\log.\overline{1-\frac{1}{z}}$ will then be $=2$, very near: And consequently $2\log.\overline{z}+2=2\log. 2a$; or, $\log.\overline{z}+1=\log. 2a$; or, by putting $e=(2.71828$, &c. the number whose h. log. is 1), we have $\log. z+\log. e=\log. 2a$: Therefore $ez=2a$, and $z=\frac{2a}{e}$.

A very curious and explicit solution to this problem has been received from Mr. *Rich. Holding*; which we omit (with regret) as being rather too long for our scanty limits. This gentleman's farther correspondence will be very acceptable.——Mr. *Witchell*'s solution does not essentially differ from that given above. The proposer (Mr. *Wildbore*) has given a fluxionary solution: Which are all the true answers that have been received to this problem.

XIII. QUESTION 474 *answered by Mr.* T. Allen, *of* Spalding.



It is evident that the parallax increases the quickest, when its fluxion bears the greatest ratio possible to the fluxion of the angle at the pole *P*.

Put a and b for the sine and cosine of P☽ the moon's polar distance, c and d equal to the sine and cosine of PZ (the comp. of lat.) n = horizontal parallax, and x = cosine of P; then will $\sqrt{1-\overline{bd+acx}^2}$ = sine Z☽ the moon's zenith distance, and $n\sqrt{1-\overline{bd+acx}^2}$ = her parallax in altitude; whose fluxion $\frac{-nac\dot{x}\times\overline{bd+acx}}{\sqrt{1-\overline{bd+acx}^2}}$ divided by $\frac{-\dot{x}}{\sqrt{1-xx}}$ (the fluxion of the angle P) gives $\frac{nac\sqrt{1-xx}\times\overline{bd+acx}}{\sqrt{1-\overline{bd+acx}^2}}$, a max. The fluxion of the log. of which being made = 0 (making at the same time $r = bd$, and $s = ac$) we have $-\frac{x}{1-xx}+\frac{s}{r+sx}+s\times\frac{r+sx}{1-\overline{r+sx}^2}$ = 0: From whence the value of x (and consequently the moon's altitude) may be found.

The same answered by Mr. Witchell, *the Proposer.*

This gentleman substitutes x for the cosine of the moon's zenith distance; and then (the notation of the known quantities being here made to agree with that in the solution above

above given, by Mr. *Allen*) he finds (by a known theorem) the cosine of the angle at the pole to be $= \frac{x-r}{s}$; and, consequently its sine $= \frac{\sqrt{ss-rr+2rx-xx}}{s}$; whence (by dividing the fluxion of the one by the other) the fluxion of the angle itself is found $= \frac{-\dot{x}}{\sqrt{ss-rr+2rx-xx}}$; which (by the quest.) being to $\frac{-n\dot{x}x}{\sqrt{1-xx}}$ (the flux. of the parallax $n\sqrt{1-xx}$) in the least ratio possible, $\frac{nnxx \times \overline{ss-rr+2rx-xx}}{1-xx}$ must therefore be a maximum; whence, by taking the fluxion, &c. we have $x^4 - rx^3 - 2x^2 + 3rx + ss - rr = 0$; from which x will be known.

PRIZE QUESTION *answered by the Proposer.*

Let the number of the pieces staked at each game by *A* and *B*, be denoted by r and s; and let their chances for winning each game, be in the ratio of a to b: Moreover, let Δ denote the expectation of *A*, when (in the course of the play) he has any number (q) of pieces in his possession; and let Δ', or $\bar{\Delta}$ be his expectation, after one more game, when he will have either $q+s$, or $q-r$ pieces, according as he wins or loses. If he wins the game (whereof the probability is $\frac{a}{a+b}$) he will have $q+s$ pieces, and his expectation will then be Δ', which therefore, multiplied by $\frac{a}{a+b}$; gives $\frac{a}{a+b} \times \Delta'$ for his present expectation on that event. But if he loses the game (whereof the probability is $\frac{b}{a+b}$) he will only have $q-r$ pieces, and his expectation being then denoted by $\bar{\Delta}$, his present expectation on this event will therefore be $= \frac{b}{a+b} \times \bar{\Delta}$: And consequently $\frac{a}{a+b} \times \Delta' + \frac{b}{a+b} \times \bar{\Delta}$ will be $= \Delta$ his whole (present) expectation. Whence, putting $n = \frac{b}{a}$ and $m = n+1$, we have $\Delta' = m\Delta - n\bar{\Delta}$.

Let now, 1, C, D, E, F, G, &c. denote the respective expectations of A, in the case proposed, when he has 1, 2, 3, 4, 5, 6, &c. pieces in possession: Then, r being in this case $=2$, and $s=1$, it is manifest, from the equation $\Delta' = m\Delta - n\Delta_r$: that the value of each new term of the series 1, C, D, E, F, G, &c. will be equal to the last (or preceding) term multiplied by m, minus the last but two drawn into n. And so these values are derived, one from another, as in the annexed scheme.

1	$1=$	1
2	$C=$	C
3	$D=$	mC
4	$E=$	m^2C-n
5	$F=$	$\overline{m^3-n}\times C-mn$
6	$G=$	$\overline{m^4-2mn}\times C-m^2n$
7	$H=$	$\overline{m^5-3m^2n}\times C-m^3n+nn$
8	$I=$	$\overline{m^6-4m^3n+n^2}\times C-m^4n+2mnn$

&c. &c.

And, universally, if r be taken to denote the number of any term of this series, reckoned from the beginning, the term itself will be truly expressed by $C\times \overline{m^{r-2}-\overline{r-4}.m^{r-5}n+\frac{r-6}{1}.\frac{r-7}{2}m^{r-8}nn-\frac{r-8}{1}.\frac{r-9}{2}.\frac{r-10}{3}m^{r-11}n^3,\text{\&c.}}$ $-m^{r-4}n+\overline{r-6}.m^{r-7}nn-\frac{r-8}{1}.\frac{r-9}{2}m^{r-10}n^3+\frac{r-10}{1}.\frac{r-11}{2}.\frac{r-12}{3}m^{r-13}n^4$, &c. Which, when r is taken $=11$ (the whole number of pieces of both A and B) will become $C\times\overline{m^9-7m^6n+10m^3n^2-n^3}-m^7n+5m^4n^2-3mn^3=11$; because, if A should have all the 11 pieces in possession, the play will then be at an end, and he will then have $\frac{1}{4}$ that sum, certain. Now, if in this equation there be wrote $\frac{3}{5}$ and $\frac{2}{5}$ instead of their equals m and n, the value of C will be found $=\frac{376923}{383543}$.

But the required expectation of A, when he has 7 pieces in possession, is found above to be $\overline{m^5-3m^2n}\times C-m^3n+nn=\frac{1775C-642}{243}=\frac{1739933}{383543}=4.5364$, &c. Therefore his required loss, or disadvantage, will be $=7-4.5364=2.4636$, &c. $=$ 2l. 11s. $3\frac{1}{4}$d. *Q. E. I.*

After the same manner the loss of A and the gain of B may be determined in any other case: But it must be ob-

ferved,

ſerved, that, when B, as well as A, ſtakes more than one piece at a time, the values of the ſeveral terms in the propoſed ſeries of expectations will then be expreſſed by means of as many of the leading terms, as there are units in (s) the ſtake of B. And if the s laſt terms of the ſeries ſo expreſſed, be made reſpectively equal to the numbers which they anſwer to (or ſtand againſt), then as many (ſimple) equations will from thence be obtained, as there are unknown quantities C, D, E, &c. to be determined.

To exemplify the proceſs here pointed out, would, I apprehend, take up too much room: For which reaſon I am alſo obliged to omit the invention of a general approximation for the reſolution of all problems of this kind; which comes exceeding near the truth in thoſe caſes where it is moſt wanted, that is, where the number of ſtakes is great.

The Eclipſes calculated for 1761.

There will, in the courſe of this year, be ſix eclipſes, four of the ſun, and two of the moon; whereof one of the latter only will be viſible to the inhabitants of Great Britain.

The firſt is a very ſmall eclipſe of the ſun, May 4th, between five and ſix in the afternoon, viſible only in great ſouth latitudes.

The ſecond is a viſible and total eclipſe of the moon, May 18th, of which ſome calculations, from different tables, and by different correſpondents, are here ſubjoined.

Calculated by	For	Beg. Ecl. h. m.	Beg. T.D. h. m.	Mid. Eclip. h. m.	End T.D. h. m.	End Eclip.
Mr. G. Witchell, from Mayer's Tables,	London	8 22	9 29	10 16	11 2	12 9
Mr. J. Metcalfe, from Brent's Tables,	London	8 24	9 31	10 20	11 8	12 45
	Wentworth	8 20	9 27	10 16	11 4	12 11
Mr. T. Harris, from Brent, by 3 equ. only	London	8 32	9 38	10 27	11 16	12 23
	Bugbrook	8 28	9 34	10 23	11 12	12 19
Mr. T. Allen,	Spalding	8 24	9 31	10 17	11 3	12 9
Mr. W. Terrill, from Leadbet. Tables	London	8 21	———	10 15	———	12 9
	Redruth	8 1	———	9 55	———	11 49
Mr. W. Chapman, from Dunthorn's Tables	Leiceſter	8 26	9 33	10 21	11 9	12 16
Mr. W. Reeves, from Halley's Tables	Burton on the Water	8 17	9 24	10 10	10 56	12 8

The

The third is a very ſmall ſolar eclipſe, June 3d, between 1 and 2 in the morning; viſible only within the artic circle.

The fourth is another of the ſame ſort, Oct. 27th, between 10 and 11 at night.

The fifth is a total eclipſe of the moon, Nov. 12th, about noon, and conſequently inviſible to us.

The ſixth and laſt is of the ſun, Nov. 26th, about 2 in the afternoon; but inviſible here, becauſe of the moon's great ſouth latitude.

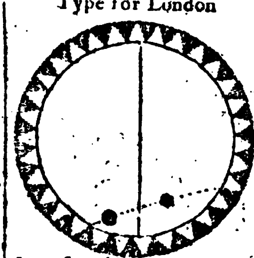
Beſides theſe eclipſes of the two great luminaries, there will happen on June 6th, the long-talked-of, and much-expected tranſit of the planet Venus over the ſun's diſk: Some calculations of which are as follow.

Calculated by	Begin. Tranſ. h. m.	Tot. Im. h. m.	Middle Tranſ. h. m.	Beg. Im. h. m	End Tra. h. m	N. Ap. ♀ to ☉'s Cent. ′ ″
Mr. G. Witchell	2 26	2 45	5 47	8 41	9 7	9 37
M. La Caille	1 37	2 0	5 4	7 50	8 32	8 57
Mr. T. Walker, from Hal. Tab.	2 12		5 27		8 41	
Mr. T. Allen	2 24		5 35		8 46	
Mr. T. Harris	2 4	2 27	5 23	8 20	8 43	9 39

Mr. *Witchell* (whoſe ſkill and great exactneſs in calculations of this ſort we cannot enough commend) takes notice, that in his computations (inſerted in the preceding page) no regard is had to parallaxes. He has however, obliged us with another calculation (and a type) for the city of London, on ſuppoſition that the ſun's horizontal parallax is 10 ſeconds; which is as follows.

Eq. time h.	′	″	App. Diſt. Cent. ′	″	
4	0	0	12	15	
5	0	0	10	28	
5	48	58	9	58	leaſt diſt. cent.
6	0	0	9	59	
7	0	0	11	2	
8	0	0	13	14	
8	41	31	15	12	begin. of emerſion
9	0	0	16	11	
	4	13	16	24	end
			31	36	diam. of ♀
			1	12	diam. of ☉

Type for London



from ſun-riſing to the end.

These

These numbers are the result of a calculation, in which the sun's place was computed by Mayer's tables (as agreeing best with observation) and that of ♀ from Halley's, except in the place of her node: For, by comparing Horrox's observation, Nov. 24th, 1639, with one taken at Paris by M. La Caille, Dec. 21st, 1746, it appeared that the motion of her nodes from the equinox, in 100 julian years, was 52′ 47″; whereas in Halley's tables it is only 51′ 40″. Upon this authority I have ventured to add 2′ 9″ to the place of the node; which is all the correction that the theory of ♀ in these tables seems to want.—It may not perhaps be improper to add here a short theorem, whereby the sun's parallax may be found by an observation of this transit, made in one place only; leaving it to the gentlemen who are conversant in observations to determine how far it is practicable. Observe the apparent motion of ♀ ☌ ☉ during the time of the transit, and let a denote the difference between that and the true motion for the same time, found by theory; let also the apparent distance of ♀ from the zenith, and the angle made by her path with the vertical circle passing through her center, be determined both at the beginning and end of the observation, and put b = sine of the 1st zenith distance, c = sine of the 2d, d = cosine of the 1st vertical angle, e = cosine of the 2d, and let the given ratio of the distances of the earth and Venus from the sun be as f to g: Then will $\frac{a \times \overline{f-g}}{g \times \overline{bd-ce}}$ = sun's horizontal parallax.

G. Witchell.

New Questions.

I. QUESTION 475, by *Miss* Ann Nicholls.

Old John, who had in credit liv'd,
Tho' now reduc'd, a sum receiv'd:
This lucky hit's no sooner found,
Than clam'rous duns come swarming round:
To th' landlord,—baker,—many more,
John paid in all pounds ninety-four.
Half what remain'd—a friend he lent—
On Joan and self, one-fifth he spent;
And when of all these sums bereft,
One-tenth o'th' sum receiv'd had left.
—Now shew your skill, ye learned fair,
And in your next that sum declare.

II. QUES-

II. QUESTION 476 *by Mr.* J. Hampson.

To find that number, which being any how divided into two unequal parts, the greater part added to the square of the lesser, shall be equal to the lesser part added to the square of the greater.

III. QUESTION 477, *by Mr.* Tho. Barker.

Near me two lovely maids reside,
Blest with each grace—their sex's pride;
But should such charms fail to engage,
Without the gold, in this wise age,
The two equations plac'd below,*
Will, when resolv'd, their fortunes shew.

* viz. $\left\{\begin{array}{l} xy = 125x + 300y \\ yy - xx = 90000 \end{array}\right\}$ to be solved by a quadratic.

IV. QUESTION 478, *by Mr.* W. Chapman.

Having, at a certain (unknown) distance, taken the angle of elevation of a steeple, I advanced 60 yards nearer (upon level ground) and then observed the elevation to be the complement of the former to a right angle: Advancing 20 yards still nearer, the elevation now appeared to be just the double of the first. Hence the steeple's height is required.

V. QUESTION 479, *by Mr.* S. Kemp.

Having given the, the vertical angle of a triangle (104°) and also the length (24) of a line dividing it in the given ratio of 5 to 4, and terminating in the opposite side; to determine the triangle so that the area thereof shall be a minimum.

VI. QUESTION 480, *by Mr.* W. Spicer.

The perimeter of a triangle being given (120 feet) and the vertical angle (70°); to determine all the sides thereof, so that the triangle itself shall be the greatest possible.

VII. QUESTION 481, *by* P. M. *of* Durham.

Two right lines, and also a third of any order, being given in position, to draw another right line intercepted between the line of the indeterminate order and one of the given right lines, also cutting the other, so as to make given angles at the intersection, and have its segments made thereby in a given ratio.

VIII. QUES-

VIII. QUESTION 482, *by Mr.* Rich. Mallock.

The area of a triangle being given = 126; the sum of its three sides = 54, and the sum of their squares = 1010; to determine the triangle.

IX. QUESTION 483, *by Mr.* J. Brampton.

There is a pond in the form of a right-angled triangle, which is intended by the owner for a decoy. Going to survey it, I found it so surrounded by bushes, to a considerable distance, that the following measures were all that I could take: On the base produced, I measured from the acute angle, 4 chains: Here I could see a tall poplar, which grew on the bank of the hypothenuse; I took its bearing from the chain-line 20°: When I got to the tree I could not see my former station, but found that the perpendicular of the triangle subtended a right angle there: Then I measured from the tree to the angle opposite the base, 5 chains. 'Tis required from these measures to plot the triangle.

X. QUESTION 484, *by Mr.* Rich. Gibbons.

A shell being thrown from a mortar, at an elevation of 30°, the report of its fall was heard at the mortar, just 20 seconds after the explosion: Hence to find the length of the range.

XI. QUESTION 485, *by Mr.* Hugh Brown.

A and *B* borrow 400l. each, for a certain stated time: At the expiration of which, *A*, who agreed to allow compound interest, had 463l. 1s. to pay; but the debt of *B* (who was to pay simple interest only) amounted but to 460l. The time and rate of interest are required.

XII. QUESTION 486, *by Mr.* T. Harris.

In sixty-six,* the time declare,
When day and twilight equal are.

* *Degrees of north latitude.*

XIII. QUEST. 487, *by Miss* Ann Nicholls, *of* Hadham.

Two places, *A* and *B*, are known to lie both under the same meridian: And it is observed, that on Jan. 20, the sun rises 36 minutes sooner at *A* than at *B*; and that on May 30, the

he rises 40 minutes earlier at *B* than at *A*: From which data, I demand the latitude of both places.

XIV. QUESTION 488, *by Mr.* G. Witchell.

To determine the equation of the curve, whose subtangent is every where, equal to the sum of the absciss and ordinate corresponding; and to find the area thereof, when the two quantities last mentioned are equal, and given.

The PRIZE QUESTION, *by Mr.* E. Rollinson.

To determine the orbit that a planet will describe, when, besides its proper gravitation to the sun, it is urged in the direction of the radius vector by a perturbating force, which is every where in proportion to the sun's attraction, as the cosine of the angle described about his center, from the commencement of the motion, to any given multiple of the radius.

1762.

Questions answered.

I. QUESTION 475 *answered by Mr.* Tho. Sadler.

THE sum receiv'd by John and Joan—
One hundred forty pounds and one;
And how dispos'd on you will find,
From the solution here subjoin'd.

If x be supposed = the whole sum John received, then will $\frac{x-94}{2} + \frac{x-94}{5} + \frac{x}{10} + 94$ or $\frac{8x+282}{10} = x$, per the conditions of the question; whence $x = 141$ l. the sum required.

Mr. *George Salmon*, late of Mr. *B. Donn*'s school, and several others, observe, that this question is ambiguous, it being doubtful whether $\frac{1}{5}$ of the whole sum received) or only $\frac{1}{5}$ of what remained after the 94 l. was paid, was the part thereof spent on Joan and himself, and accordingly find the whole

whole sum received to be either 141 or 235 pounds? and agreeably to one or other of these two meanings it has been answered by Messrs. *J. Askew*, *Tho. Atkinson*, *T. Baker*, *Tho. Barker*, *Richard Bassel*, *E. Batten*, *John* and *Wm. Bell*, *Wm. Beer*, *Birchoverensis*, *T. Bromhall*, *John Buddle*, *R. Butler*, *Wm. Chapman*, *John Collins*, *James Croom*, *Richard Dening*, *John Endon*, *Wm. Embleton*, *Humphrey Fry*, *Richard Gibbons*, *John Goodhead*, *Char. Green*, *John Hampson*, *Wm. Hardy*, *Thomas Harris*, *Wm. Harvey*, *Da. Hastings*, *Geo. Hicks*, *John Hitchcock*, *Richard Holden*, *Samuel Kemp*, *Edward Kimpton*, *Wm. Kingston*, *James Launders*, *John Lyon*, *Rich. Mallock*, *Robert Marsh*, *Cha. Mesban*, *R. Miles*, *James Milner*, *John Newland*, *Geo. Nokes*, *Nosniketub*, *James Paty*, *Sim. Pedley*, *Wm. Penn*, *John Potter*, *Geo. Reed*, *Tho. Robinson*, *Tho. Sandling*, *Rich. Spencer*, *Wm. Spicer*, *Wm. Stoker*, *Eleanor Suggett*, *John Swan*, *Wm. Toms*, *Tho. Walker*, *W. Wells*, and *Others*.

II. QUESTION 476 *answered by* Birchoverensis.

Let a and b represent any two given unequal numbers, and suppose ax denotes the greater, and bx the lesser part of the number required; then will be had $aaxx + bx = bbxx + ax$, by the question; whence $x = \frac{a-b}{aa-bb} = \frac{1}{a+b}$, and consequently $\frac{a}{a+b}$ and $\frac{b}{a+b}$ will be the two unequal parts, whose sum will always be $= 1$, the number required.

Mr *Tho. Barker* puts $x+y$ for the greater, and $x-y$ for the lesser part of the required number, and thence forms the equation $\overline{x+y}|^2 + x - y = \overline{x-y}|^2 + x + y$, according to the nature of the question; whence he finds $x = \frac{1}{2}$, and consequently the sum of the two parts $x+y$ and $x-y$ equal to 1, the number required; the same as before.

True solutions to this question have likewise been received from Mr. *Tho. Atkinson*, Mr. *T. Baker*, Mr. *Rich. Bassell*, Mr. *E. Batten*, Mr. *T. Bromhall*, Mr. *John Buddle*, Mr. *R. Butler*, Mr. *Tho. Bosworth*, Mr. *Wm. Chapman*, Mr. *John Collins*, Mr. *James Croom*, Mr. *Rich. Dening*, Mr. *Wm. Embleton*, Mr. *Humphrey Fry*, Mr. *Richard Gibbons*, Mr. *John Goodhead*, Mr. *Cha. Green*, Mr. *John Hampson*, Mr. *Wm Hardy*, Mr. *Tho. Harris*, Mr. *Da.*

Hastings, Mr. *Geo. Hicks*, Mr. *Rich. Holden*, Mr *Sam. Kemp*, Mr. *Edw. Kinipton*, Mr. *Wm. Kingston*, Mr. *James Launders*, Mr. *Cha. Mesban*, Mr. *Rich. Miles*, *Nosnihctub*, Mr. *Steph. Ogle*, Mr. *Wm. Penn*, Mr. *John Potter*, Mr. *Tho. Robinson*, Mr. *Geo. Salmon*, Mr. *Wm Spicer*, Mr. *Wm. Stoker*, Mrs. *Eleanor Suggett*, Mr. *Wm. Toms*, Mr. *Tho. Walker*, Mr. *W. Wells*, and several others.

III. QUESTION 477 *answered by Mr.* Hugh Brown, *of* Woolwich.

Put $300 = a$; and then $xy = 125x + 300y = \frac{5}{12}ax + ay$, and $yy - xx = 90000 = aa$. Whence, substituting $\frac{5ax}{12 \times \overline{x-a}}$ for y, we shall have $144x^4 - 288ax^3 + 263aaxx - 288a^3x + 144a^4 = 0$, or $x^4 - 2ax^3 + \frac{263}{144}aaxx - 2a^3x + a^4 = 0$; and, dividing every term by $aaxx$, and putting $z = \frac{2x}{a} + \frac{2a}{x}$ (according to directions given on p. 156 of Simpson's Algebra, 2d edition) we shall have $\frac{z^2}{4} - 2 - z + \frac{263}{144} = 0$, or $z^2 - 4z = \frac{100}{144}$; whence z will be found $= 4\frac{1}{6} = \frac{2x}{a} + \frac{2a}{x} = \frac{2x}{300} + \frac{2 \times 300}{x}$; and consequently $x^2 - 2\frac{1}{12} \times 300x = -90000$; from the resolution of which quadratic equation x comes out $= 400$; and from thence y will be found $= 500$.

The same answered by P. M. *of* Durham.

By transposition $x \times \overline{y - 125} = 300y$, and $x^2 \times \overline{y-125}|^2 = 300y^2$; but $x^2 = y^2 - 300^2$; therefore, by substitution, $\overline{y^2 - 300^2} \times \overline{y - 125}|^2 = 300y^2$; which, expanded, gives $y^4 - 250y^3 - 2.\ 300^2\ y^2 + 125^2\ y^2 + 2.\ 125.\ 300^2\ y - 125^2.\ 300^2 = 0 = \overline{y^2 - 125y - 300^2}|^2 - 300^4 - 125^2.\ 300^2$; whence $y^2 - 125y - 300^2 = 300 \times \sqrt{300^2 + 125^2} = 300 \times 325$; and $y^2 - 125y = 300 \times 625$; which quadratic equation solved, gives $y = 500$, and consequently $x = 400$.

The

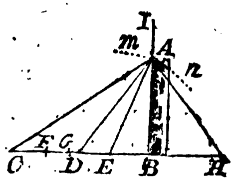
The same answered by Mr. Wm. Embleton.

Put $300 = a$, and $125 = b$; then will $xy = bx + ay$, and $yy - xx = aa$; whence $y = \frac{bx}{x-a} = \sqrt{xx+aa}$, and $bbxx = \overline{x-a}|^2 \times \overline{xx+aa} = x^4 - 2ax^3 + 2aaxx - 2a^3x + a^4$: Whence, adding $aaxx$ to each side of this equation, and extracting the square root, there will result $x\sqrt{aa+bb} = xx - ax + aa$; from which quadratic equation x and y will readily be found equal to 400 and 500 respectively.——Mr. *John Goodhead* answered it exactly in the same manner as this.——We have likewise received three other very different solutions to it, by quadratic equations only, from Mr. *Tho. Atkinson*, Mr. *Cha. Green*, and Mr. *Geo. Salmon*, which we are sorry our narrow limits will not permit us to insert.

We have also received true and concise answers to this question from Mr. *T. Baker*, Mr. *Tho. Barker* the proposer), *Birchoverensis*, Mr. *John Buddle*, Mr. *Robert Butler*, Mr. *Wm. Chapman*, Mr. *John Eadon*, Mr. *Rich. Gibbons*, Mr. *Tho. Harris*, Mr. *Geo. Hicks*, Mr. *Edward Kimpton*, Mr. *James Launders*, Mr. *John Lyon*, Mr. *Richard Mallock*, Mr. *Chr. Meshan*, *Nosnihctut*, Mr. *John Potter*, Mr. *J. Randles*, Mr. *Geo. Reed*, Mr. *Tho. Robinson*, Mr. *W. Spicer*, Mrs. *Eleanor Suggett*, Mr. *Wm. Toms*, Mr. *Tho. Walker*, and others.

IV. QUESTION 478 *answered by* P. M. *of* Durham.

Imagine the thing done, and BA to be the steeple whose height is sought, and C, D, E, the three given stations; and suppose the points C, A; D, A; and E, A, to be joined.



Then, since the $\angle AEB$ is double the $\angle ACB$ (by hypoth.) and equal to the $\angle$s ACB, CAE (32 Euc., 1.), they must therefore be equal to each other, and consequently the sides AE, CE, subtending them equal. Moreover, since the right-angled triangles ABC, ABD, have the acute $\angle$s BAC, ADB, equal (by hypth.) they will be similar, and so $CB : AB :: AB : BD$; whence $CB \times BD (= AB^2 = AE^2 - EB^2 = CE^2 - EB^2 =) CB \times CE - CB \times BE$; whence $BD = CE - BE$, take away

DE,

DE, which is common, and there will remain $BE = CD - BE$, or the double of $BE = CD$; whence is derived the following

CONSTRUCTION. Bisect CD in F, and FE in G, and produce CE till $GB = GC$; and about the center E, with the radius EC, let the arc of a circle be described, and at the point B erect a perpendicular cutting the said arc in A, and BA will represent the height of the steeple required.—— From this construction BA is readily found $= 74{\cdot}162$, &c. yards.

Much after the same manner this question was constructed by Mr. *Wm. Embleton*.

The same constructed otherwise, by Mr. Da. Hastings.

The three given stations being supposed to be at C, D, and E, as before, produce CE till EH becomes equal thereto, and bisect DH with the perpendicular BI, and from the center E, with the radius EC (EH), let the arc mn be described cutting BI in A, and BA will be the height of the steeple required.

DEMONSTRATION. Let the points C, A; D, A; E, A; and H, A, be joined. Then, CE being $= EA = EH$, by construction, a semi-circle, described about the diameter CH, will pass through the point A, and consequently CAH being a right angle, the $\angle ADH$ will be the complement of the $\angle C$ to a right angle; and the $\angle AEB$ being $=$ the $\angle C +$ the $\angle CAE$, will be $=$ twice the angle C, the triangle CEA being isosceles.

Mr. *Robert Butler* constructed it according to this last method, nearly.

The same answered (algebraically) by Mr. R. Mallock, Writing-master *in* Lyme-Regis, Dorset.

Let CE (EA) $= a$, $EB = z$, and x and $y =$ the sine and cosine of the $\angle ACB$ (rad. 1.); then will $2xy$ and $yy - xx$ be the sine and cosine of the $\angle AEB$; whence, by trigonometry, will be readily found $BA = \frac{x}{y} \times \overline{z + a} = \frac{y}{x} \times z + \frac{a}{4} = \frac{2xyz}{yy - xx}$; and consequently $z = \frac{3a}{8} = 30$, and from thence BA (the steeple's height) is directly found $= 74{\cdot}16198$ yards.

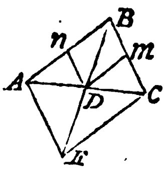
Mess.

Mess. *W. Chapman*, *Birchoverensis*, *Tho. Harris*, *Wm. Kingston*, *Paul Sharp*, and *W. Spicer*, substitute exactly the same as above, and, by an easy and very short process, find $2xx = \frac{5}{8} = 0.625 =$ the versed sine of 67° 58′ 32″ (double the $\angle ACB$); from whence they determine the steeple's height the same as above.

This question was likewise answered by Mr. *Tho. Atkinson*, Mr. *Tho. Barker*, Mr. *E. Batten*, Mr. *Sam. Beeken*, Mr. *Tho. Bosworth*, Mr. *John Buddle*, Mr. *John Eadon*, Mr. *Rich. Gibbons*, Mr. *John Goodhead*, Mr. *Cha. Green*, Mr. *John Hampson*, Mr. *Wm. Hardy*, Mr. *Rich. Holden*, Mr. *Sam. Kemp*, Mr. *Edw. Kimpton*, Mr. *Ja. Launders*, Mr. *R. Miles*, Mr. *Ja. Milner*, Mr. *J. Randles*, Mr. *Geo. Reed*, Mr. *Tho. Robinson*, Mr. *Wm. Stoker*, Mrs. *Eleanor Suggett*, Mr. *John Swan*, Mr. *Wm. Tons*, and several others.

V. QUEST. 479 *answered by Mr.* Rich. Gibbons.

CONSTRUCTION. Constitute the angle $ABC =$ the given vertical angle (104°), and divide it in the given ratio of 5 to 4 by the line $BE =$ double the given dividing line (24), and draw EA, EC, parallel to BC and BA respectively, and meeting them in the points A and C, and then ABC will be the triangle required.



DEMONSTRATION. $ABCE$ being a parallelogram, by construction, the diagonals BE, AC, will bisect each other (by theor. 12. 2d of Simpson's Geom. 2d edition), and consequently BD will be the given dividing line; and, DA being $= DC$, the triangle ABC will be a minimum (by theor. 8. p. 199 and 200 of Simpson's Elem. of Geom. aforesaid). *Q.E.D.*

The same constructed otherwise by Mr. Wm. Embleton.

The angle ABC being made $=$ the given vertical angle, and BD the given line dividing it in the given ratio of 5 to 4, as before, draw Dm parallel to BA meeting BC in m, and take $BA =$ twice Dm; then, through the points A and D, let the line AC be drawn meeting BC in C, and ABC will be the triangle required.——For Dm being $= \frac{1}{2} AB$ (by construction) CD will be $= \frac{1}{2} CA$, by the sim. triangles CDm and CAB, and consequently the triangle ABC a minimum (by theor. 8. p. 199 and 200 of Simpson's Elem. of Geom.) 2d edit.)

N 3 According

According to this laſt method, nearly, it was alſo conſtructed by Mr. *Da. Haſtings*.

We have likewiſe received neat and elegant geometrical conſtructions to this queſtion from *P. M.* of *Durham*, Mr. *Rob. Butler*, Mr. *Cha. Green*, and ſeveral others.*

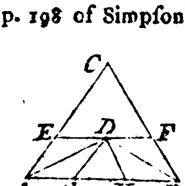
An algebraic Solution to the ſame by Mr. T. Bromhall, *at Mr.* Allen'*s School, at* Spalding, *in* Lincolnſhire.

This young gentleman puts $Dm = a$, Dn (parallel to BC) $= b$, $Cm = x$, and $s =$ ſine of vertical angle ABC; then he readily finds $AB = a + \frac{ba}{x}$, and $\overline{2b + x + \frac{bb}{x}} \times \frac{1}{4}as =$ area of the required triangle, which is to be a minimum: In fluxions, $x^2\dot{x} - b^2\dot{x} = 0$; whence $x = b$, or $Bn = An$, and conſequently $AD = DC$, the ſame as demonſtrated above. ——Much after the ſame manner it was anſwered by Mr. *John Eadon*, Writing-maſter, at the free ſchool in *Sheffield*, *Yorkſhire*.

Algebraic ſolutions to this queſtion have likewiſe been received from Mr. *Tho. Barker*, Mr. *E. Batten*, *Birchoverenſis*, Mr. *Tho. Boſworth*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *John Goodhead*, Mr. *John Hampſon*, Mr. *Wm. Hardy*, Mr. *Tho. Harris*, Mr. *Rich. Holden*, Mr. *Sam. Kemp*, Mr. *Wm. Kingſton*, Mr. *James Launders*, Mr. *Rich. Mallock*, Mr. *Geo. Reed*, Mr. *Tho. Robinſon*, Mr. *Paul Sharp*, Mr. *W. Spicer*, Mr. *Wm. Stoker*, Mr. *Wm. Toms*, Mr. *Tho. Walker*, and others.

VI. QUESTION 480 *conſtructed by Mr.* Da. Haſtings.

It evidently appears, from theor. 6. p. 198 of Simpſon's Elem. of Geom. 2d edition, that the triangle required will be iſoſceles; therefore upon AB, equal to the given perimeter, or ſum of the three ſides, conſtitute an iſoſceles triangle with the given vertical angle ACB ($= 70°$), and biſect the angles CAB, CBA, with the right lines AD, BD, meeting each other in D, and let the points A, D



and

* The above cannot be eſteemed geometrical conſtructions, as the method of dividing an angle in the ratio of 4 to 5, is not known.

and B, D be joined; then draw DG and DH parallel to CA and CB respectively, and CDH will be the triangle required; which is too evident to need any further demonstration.

P. M. of *Durham*, observes, that it is very easy to demonstrate, geometrically, that the two sides comprehending the given vertical angle will be equal; and therefore, premising that, he determines the triangle by the following

CONSTRUCTION. Make $ACB =$ the given vertical angle, and CA and CB each $=$ half the given perimeter; join A, B; and bisect the angles CAB, CBA, by right lines meeting in D; through the point D draw EF parallel to AB, meeting the sides CA, CB, in the points E, F; and CEF will be the triangle required.——For the $\angle EDA (= DAB$, 29 E. 1.) $= \angle EAD$ (by hyp.) therefore $AE = DE$ (6 Euc. 1.) and consequently $CA = CE + ED$; and in the same manner CB will be proved $= CF + FD$; therefore $CE + CF + EF = CA + CB$, *i. e.* equal to the given perimeter; whence, &c.

The same solved algebraically by Birchoverensis.

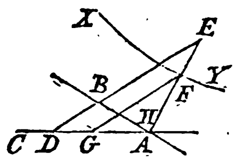
Let $s =$ sine of the given vertical angle ECF (70°), $m =$ 120 (the given perimeter), $a + x$ and $a - x$ the two sides comprehending the vertical angle; then will $m - 2a =$ the third side EF, and $\overline{aa - xx} \times \frac{1}{2}s =$ the area of the required triangle, which it is evident by inspection, will be the greatest when $x = 0$, or the triangle ECF is isosceles; and hence the remaining part of the question is easily solved by plain trigonometry, it being as the sum of the natural sines of the three angles of any plain triangle is to its perimeter, so is the natural sine of any one of those angles to the side corresponding or opposite thereto; whence the sides EC (CF) and EF are readily found $= 38{\cdot}1297$, &c. and $43{\cdot}74059$, respectively.——Mr. *E. Batten* and Mr. *Sam. Kemp* answer it exactly according to this method.

Mr. *R. Butler*, Mr. *Wm. Embleton*, Mr. *Francis King*, and several others, have given neat constructions to this question; and Mr. *Tho. Barker*, Mr. *Tho. Bosworth*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *John Eadon*, Mr. *Rich. Gibbons*, Mr. *John Goodhead*, Mr. *Cha. Green*, Mr. *John Hampson*, Mr. *Wm. Hardy*, Mr. *Tho. Harris*, Mr. *Rich. Holden*, Mr. *Wm. Kingston*, Mr. *James Launders*, Mr. *Rich. Mallock*, *Nosnibctuh*, Mr. *Geo. Reed*, Mr. *Paul Sharp*, Mr. *W. Spicer*, Mr. *Wm. Stoker*, &c. have sent neat and concise algebraic solutions to it.

VII. QUES-

VII. QUESTION 481 *constructed by* P. M. *of* Durham (*the Proposer.*)

Let *XY* be the line of the indeterminate order; *AB*, *AC*, the two right lines given by position and meeting in *A*; in one of them *AB*, with which the right line, required, is to make given angles, assume a point *B*, through which draw a right line, making the given angles with *AB*, and meeting *AC* in *D*; produce *DB* to *E*, so that *DB* may be to *BE* in the given ratio; through *E*, *A*, draw a right line (produced if necessary) to cut *XY* in *F*; and through *F* draw a line parallel to *ED*, cutting *AB* and *AC* in *H* and *G*; and the thing is done.



For the right line *AB* falling upon the parallels *DE*, *GF*, intercepted by the same right lines *AD*, *AE*, will cut them similarly in *B* and *H* (2 Euc. 6.) and make equal angles with each (29 Euc. 1.); but it makes the given angles with *DE*, and cuts it in the given ratio in *B*; therefore also *GF*, intercepted by *XY*, and one of the given right lines *AC*, is cut by the other *AB*, in *H*, in the given ratio, and makes given angles therewith at the intersection.

NUM. CALC. will vary according to the property of the indeterminate line *XY*.

Mr. *R. Butler*'s construction is nearly the same as the above.

VIII. QUESTION 482 *answered by Mr.* R. Butler.

Let $2p = 54$; $s = 1010$; $a = 126$; and x, y, and z denote the three sides of the triangle required. Then, by a known theor. $p \times \overline{p - x} \times \overline{p - y} \times \overline{p - z} = aa$; whence, putting $2p - x - y$ for its equal z, we shall have $\overline{x - p} \times yy + \overline{2pp + xx - 3px} \times y + 2ppx - pxx - p^3 = \frac{aa}{p}$. Moreover, $\overline{2p - x - y}|^2 + xx + yy = 4pp - 4px - 4py + 2xy + 2xx + 2yy$ being $= s$, per the question, we get $yy = \frac{1}{2}s - 2pp + 2px - xx + \overline{2p - x} \times y$; which, substituted for yy in the preceding equation, gives $\frac{1}{2}s - 2pp + 2px -$

$$xx + \overline{2p - x} \times y = \frac{2ppx - pxx - p^3 - \frac{aa}{p}}{p - x} + \frac{2pp - 3px + xx}{p - x} \times y;$$

$\times y$; but $\frac{2pp-3px+xx}{p-x}\times y$ is $=\overline{2p-x}\times y$; and therefore our equation becomes $\frac{1}{2}s-2pp+2px-xx=$
$\frac{2ppx-pxx-p^3-\frac{aa}{p}}{p-x}$, or $x^3-54x^2+953x=5460$;
the three roots of which cubic equation, viz. 13, 20, and 21, will express the three sides of the triangle required, as x stands indifferently for e'er a one of them.

P. M. of *Durham*, answered this question much in the same manner as the above; as likewise did Mr. *J. Hampson*, Mr. *Wm. Chapman*, Mr. *Wm. Spicer*, and some others.

The same answered by Mr. Paul Sharp.

This gentleman puts $a=126$, $2b=1010$, $2c=54$, $x=\frac{1}{2}$ sum of any two sides of the required triangle, and $y=\frac{1}{2}$ their difference; then will $x+y$, $x-y$, and $2c-2x$, represent the three sides themselves, and $6xx-8cx+2yy+4cc=2b$ (per quest.): Whence $yy=b-2cc+4cx-3xx$. Moreover, $\overline{c-x+y}\times\overline{c-x-y}\times\overline{2x-c}\times c$ is $=aa$; whence
$$yy=\frac{\frac{aa}{c}+c^3-2x^3+5cx^2-4c^2x}{c-2x}=b-2cc+4cx$$
$-3xx$; which, reduced and converted into numbers, becomes $x^3-54x^2+967{\cdot}25x=5750{\cdot}25$; whence x will be found $=17$, and $y=4$, and the sides of the required triangle equal to 13, 20, and 21.

According to this last method of solution this question was also answered by Mr. *Tho. Atkinson*, Mr. *Tho. Barker*, Mr. *E. Batten*, *Birchoveronsis*, Mr. *John Buddle*, Mr. *Wm. Embleton*, Mr. *R. Gibbons*, Mr. *John Goodhead*, Mr. *Tho. Harris*, Mr. *Geo. Hicks*, Mr. *Rich. Holden*, Mr. *Sam. Kemp*, Mr. *Wm. Kingston*, Mr. *James Launders*, Mr. *John Potter*, Mr. *Tho. Robinson*, Mrs. *Eleanor Suggett*, Mr. *Tho. Walker*, Mr. *James Wollward*, and several others.

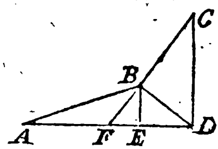
The same answered by Nosnihctuh.

The area of any plane triangle being = a rectangle under half its perimeter and the radius of its inscribed circle, it follows, that $\frac{126}{27}=4{\cdot}6$ will be the radius of the inscribed circle, in the case of this question, the square of which put $=b$;

$= b$; $1010 = n$; $27 = c$; and let x denote half the sum, and y half the difference of the two segments of any one of the sides made by the radius of the inscribed circle, drawn to the point of contact thereof, and then will the three sides of the required triangle be represented by $c + y - x$, $2x$, and $c - x - y$; whence (by a theorem on page 116 of Simpson's Select Exercises) $\frac{\overline{x+y} \times \overline{x-y} \times \overline{c-2x}}{x+y+x-y+c-2x} = \frac{cx^2 - cy^2 - 2x^3 + 2xy^2}{c} = b$, and consequently $y^2 = \frac{cb - cx^2 + 2x^3}{2x - c}$. But $\overline{c+y-x}|^2 + \overline{2x}|^2 + \overline{c-x-y}|^2 = n$ (per the quest.); whence $y^2 = \frac{n}{2} - c^2 + 2cx - 3x^2 = \frac{cb - cx^2 + 2x^3}{2x - c}$; hence, in numbers, we derive the equation $x^3 - 27x^2 + 235{\cdot}25x - 682{\cdot}5 = 0$; from which (by either of the two first methods explained in Sect. 12. of Simpson's Algebra) x will be found $= 10$, and $y = 4$, and the three sides of the triangle required 13, 20 and 21.

IX. QUESTION 483 *answered by Mr.* John Hampson.

Let FDC represent the triangular pond required; in which DB is perpendicular upon FC, and AF and BC equal 4 and 5 chains respectively, and the angle $BAF = 20°$.



Put $a = AF = 4$; $b = BC = 5$; m and n equal the sine and co-sine of the angle BAF (20°); $x =$ perpendicular CD, and $y = BF$.

Then, per similarity of triangles, will be found $BE = \frac{by}{x}$, $FE = \frac{y}{x}\sqrt{xx - bb}$, and consequently $\frac{ax + y\sqrt{yy - bb}}{x} = AE$; whence, by Trigonom. $m : \frac{by}{x} :: n : \frac{ax + y\sqrt{xx - bb}}{x}$; from whence is had $bny = amx + my\sqrt{xx - bb}$; but, from the similarity of the right angled triangles FDC, CDB, will be found $by + bb = xx$; from the resolution of which two equations x will be found $= 6{\cdot}0634$; and consequently FD and FC equal to $4{\cdot}15947$ and $7{\cdot}35296$, &c. respectively.

The

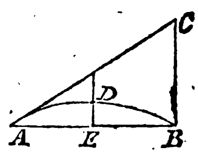
The same answered by Mr. Rich. Gibbons.

First, supposing the triangles ABD. AFB, to be similar, will be found the angle ABF ($= \angle ADB$) $= 35°$, and from thence the logarithm of BC too little by 0·0119032; next, supposing the same angle $ABF = 36°$. it gives the logarithm of BC too much by 0·0099480: Whence, as the sum of the errors is to 60′, the difference of the suppositions, so is the first error to 32′ 42″; which being added to 35° (the first supposition) gives 35° 32′ 42″ = the true measure of the angle ABF; whence AF, BC, and all the angles being given, and now become known, the triangular pond FDC may from thence be easily plotted and determined.

This question has likewise been answered by Mr. *Tho. Barker*, *Birchoverensis*, Mr. *Rob. Butler*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *John Goodhead*, Mr. *Tho. Harris*, Mr. *Edw. Kimpton*, Mr. *Rich. Mallock*, Mr. *Tho. Robinson*, Mr. *Paul Sharp*, Mr. *W. Spicer*, Mrs. *Eleanor Suggett*, Mr. *Tho. Walker*, Mr. *W. Wells*, and others.

X. QUESTION 484 *answered by Mr.* Thomas Allen, *of* Spalding.

Put t = tang. of the angle of elevation BAC ($= 30°$); $a = 1142$ feet (the velocity of sound per second); $s = 16\frac{1}{12}$ feet (the distance a falling body will descend in a second); and $x = AB$, the horizontal range required.



Then will $tx = BC$, and $\sqrt{\frac{tx}{s}}$ = the time of flight; and $\frac{x}{a}$ = the time of sounds moving from B to A: Therefore $\frac{x}{a} + \sqrt{\frac{tx}{s}} =$ 20″ (by the question); whence x is found = 6035·42 feet, the range required.

If p, (the parameter of the parabolic curve ADB) = 10450·7, and $\frac{1}{2}x = y$; then will $\frac{2y}{p}\sqrt{\frac{1}{4}pp + yy} + \frac{p}{2} \times$ hyp. log. $\frac{y + \sqrt{\frac{1}{4}pp + yy}}{\frac{1}{2}a}$ = 6353·4 feet, the length of the track ADB.

The

The same answered by Mr. W. Chapman.

Let $s = 16\frac{1}{12}$ feet (the distance a falling body descends in a second); $b = 1142$ feet (the space passed through by sound in a second); p and q equal the sine and cosine of 30° (the angle of elevation), and $x =$ the amplitude, or horizontal range required.

Then will $\frac{px}{q} = BC$; and $s : 1''^2 :: \frac{px}{q} : \frac{px}{sq} =$ the square of the time of flight; whence $c - \sqrt{\frac{px}{sq}}$ will be $=$ the time of the return of the sound (putting $20'' = c$); and consequently $1'' : b :: c - \sqrt{\frac{px}{sq}} : x$; whence $x = bc - b\sqrt{\frac{px}{sq}}$; from which equation x will be found $= 6033{\cdot}44$ feet, the amplitude required.——Much after the same manner this question was answered by *P. M.* of *Durham*.

Messrs. *Tho. Barker*, *Rob. Butler*, *Birchoverensis*, *John Buddle*, *Wm. Embleton*, *John Goodhead*, *Charles Green*, *Wm. Hardy*, *Tho. Harris*, *Rich. Holden*, *Edw. Kimpton*, *Wm. Kingston*, *Nosnihctuh*, *Stephen Ogle*, *Paul Sharp*, *Rich. Spencer*, *Wm. Spicer*, and several others have likewise answered this question, by methods not greatly different from those above.

XI. QUESTION 485 *answered by Mr.* Tho. Barker.

Let $x =$ amount of 1 l. in one year, and $y =$ the number of years required; then will $400x^y = 463{\cdot}05$ l. and $400y \times \overline{x-1} = 60$ l. (by the quest. and the nature of compound and simple interest), whence $y = \frac{3}{\overline{x-1} \times 20}$; and consequently

$x^{\frac{3}{\overline{x-1}\times 20}} = \frac{463{\cdot}05}{400} = 1{\cdot}15762$, &c. from which equation x will be found $= 1{\cdot}05$; and hence it appears, that 5 per cent. per annum, and three years, are the rate of interest and time required.

The same answered by Mr. Tho. Harris.

Put $P = 400$ l. $A = 463{\cdot}05$ l. $a = 460$ l. $t =$ time, and $r =$ rate

rate of interest required; then will $trP + P = a$ (by the nature of simple interest); whence $t = \frac{a - P}{rP} = \frac{b}{r}$ (putting $b = \frac{a - P}{P} = \cdot 15$): Whence, by compound interest, we have $P \times \overline{1+r}|^{\frac{b}{r}} = A$, the debt of A; from which equation, by the help of logarithms, r will be found $= 0\cdot05$: Whence the time required appears to be three years, and the rate 5 per cent. per annum.

The same answered by Mr. Rich. Holden.

Let $P = 400$ l. $A = 463\cdot05$ l. $a = 460$ l. $r =$ the amount of 1 l. in one year, and $t =$ the time required; then will $PR^t = A$, per compound interest, and $P + Pt \times \overline{R - 1}$ $= a$, per simple interest: Whence will be found $R = \overline{\frac{A}{P}}|^{\frac{1}{t}} = \frac{a - P}{Pt} + 1$; and hence, by a table of logarithms and a few trials, t is found = three years, and the rate 5 per cent.

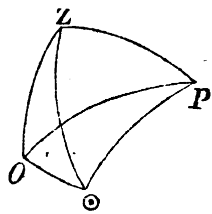
P. M. of *Durham*, and Mr. *Thomas Allen* of *Spalding*, put $a = 463\cdot05$ l. $b = 460$ l. $p = 400$ l. $t =$ number of years or time required, and $r =$ the required rate of interest of 1 l. for one year; and then they find $p \times \overline{1 + r}|^t = a$, and $p \times \overline{1 + tr} = b$ (by the nature of compound and simple interest): Whence $\frac{\mathrm{l}.a - \mathrm{l}.p}{\mathrm{l}.\overline{1 + r}} = (t =) \frac{b - p}{pr}$, and consequently $\overline{\mathrm{l}.a - \mathrm{l}.p} \times \frac{pr}{b - p} = \mathrm{l}.\overline{1 + r}$ $(= \frac{1}{m} \times r - \frac{r^2}{2} + \frac{r^3}{3} - \&c.$ m being the modulus); and hence, either by a table of logarithms, or reversion, or approximation of series, the value of r will be found $= 0\cdot05$. Therefore the rate of interest appears to be 5 l. per cent. and the time of the loan = three years.

True answers to this question have likewise been received from *Birchoverensis*, Mr. *Wm. Beer*, Mr. *Tho. Bosworth*, Mr. *John Buddle*, Mr. *Rob. Butler*, Mr. *Wm. Chapman*, Mr. *John Collins*, Mr. *Rich. Denning*, Mr. *John Eadon*, Mr. *Wm. Embleton*, Mr. *Cha. Green*, Mr. *Tho. Harris*, Mr.

Geo. Hicks, Mr. *John Hitchcock*, Mr. *Sam. Kemp*, Mr. *Wm. Kingston*, Mr. *Ja. Launders*, Mr. *Rich. Miles*, Mr. *Wm. Penn*, Mr. *G. Reed*, Mr. *T. Robinson*, Mr. *G. Salmon*, Mr. *P. Sharp*, Mr. *W. Spicer*, Mrs. *Eleanor Sugget*, Mr. *Tho. Walker*, Mr. *W. Wells*, and many others.

XII, QUESTION 486 *answered by Mr.* Charles Green, *at* Greenwich Observatory, *and Mr.* Thomas Harris, *of* Bugbrook, *near* Northampton.

Let x and $-y$ equal the sine and cosine of $P\odot$ (PO) the sun's dist. from the north pole P; s and c equal the sine and cosine of ZP ($=24°$) the complement of the given latitude; and $-d=$ the cosine of $Z\odot$ ($=108°$); then by a well-known theorem in spherics and the nature of the question, we shall have $\frac{cy}{sx}$ and $\frac{cy-d}{sx}=$ to the cosines of the angle ZPO and $ZP\odot$ respectively; but the angle $ZP\odot$ is double the angle ZPO (by the question), and therefore $\frac{2ccyy}{ssxx}-1$ (the cosine of double the angle ZPO) will be $=\frac{cy-d}{sx}$; from the resolution of which equation the value of y will be found $=0.3094$, answering to Jan. 28, and Nov. 12, nearly.



Mr. *R. Butler*'s process is different, but his final equation and conclusion is exactly the same as the above.

This question was likewise answered by Mr. *Tho. Allen*, *Birchoverensis*, Mr. *Wm. Chapman*, Mr. *Rich. Holden*, Mr. *J. Randles*, Mr. *W. Spicer*, and several others.

XIII. QUESTION 487 *answered by Mr.* W. Kingston.

Let $d=$ tang. 21° 54′ (the sun's declination May 30.); $a=$ tang. 20° 11′ (the sun's declination Jan. 20); m and n equal the sine and cosine of 10°; s and c the sine and cosine of 9°; and let the sine and cosine of the ascensional difference in the latitude of the place A, on May 30, and Jan. 20, be denoted by x and y, and z and v respectively; then will $nx+my$, and $cz+sv$, express the sines of the ascensional dif-

difference in the latitude of the place *B*, on the said two days respectively. Then, per spherics, $\frac{z}{a} = \frac{x}{d} =$ the tangent of the latitude of the place *A*, and $\frac{nx+my}{d} = \frac{cz+sv}{a}$ = the tangent of the latitude of the place *B*. From the first of these equations z is found $= \frac{ax}{d}$, and consequently v $(\sqrt{1-zz}) = \frac{\sqrt{dd-aaxx}}{d}$; which values substitute for z and v in the second equation, and it becomes $\frac{nx+my}{d} = \frac{acx+s\sqrt{dd-aaxx}}{da}$; whence $a\times\overline{n-c}\times x+amy = s\sqrt{dd-aaxx}$; and consequently (by involution, &c.) $\frac{nn-cn}{m}\times xx+\frac{aamm-ddss}{2aam} = \overline{c-n}\times xy$, or $bxx+p = qxy$ (putting $\frac{nn-cn}{m}=b$, $\frac{aamm-ddss}{2aam}=p$, and $c-n = q$); and this involved again, in order to exterminate y, becomes $bbx^4+2bpxx+pp = qqxxyy = qqxx-qqx^4$, or $\overline{bb+qq}\times x^4+\overline{2bp-qq}\times xx = -pp$; whence $x^4-2rx^2 = -t$ (putting $\frac{2bp-qq}{bb+qq} = -2r$ and $\frac{pp}{bb+qq} = t$), and consequently $2xx = 2r \mp 2\sqrt{rr-t}$ = the versed sine of double the ascens. difference at the place *A* on May 30; from whence the latitudes of the places *A* and *B* are readily found equal to 45° 32′ and 54° 26′, respectively.

Answers to this question have likewise been received from *Birchoverensis*, Mr. *R. Butler*, Mr. *C. Green*, Mr. *T. Harris*, Mr. *R. Holden*, Mr. *Ed. Kimpton*, Mr. *Rich. Mallock*, Mr. *Paul Sharp*, Mr. *W. Spicer*, Mr. *Wm. Stokes*, Mrs. *Eleanor Sugget*, Mr. *Tho. Walker*, and many others.

XIV. QUESTION 488 *answered by Mr.* W. Spencer, *of* Stannington, *near* Sheffield, *in* Yorkshire.

Let x = the abscissa, and y = the ordinate of the required curve; then will $\frac{y\dot{x}}{\dot{y}} = x+y$, (per the quest.) and consequently $\frac{y\dot{x}-x\dot{y}}{yy} = \frac{\dot{y}}{y}$: The fluent of which gives $\frac{x}{y}$ = hyp.

log. y; whence $x =$ hyp. log. y^y; the equation of the curve required.

But, to find the area, we have $y\dot{x}$ (the general expression for the fluxion of the area) $= y\dot{y} + x\dot{y}$ (from above); whence fluent of $y\dot{x} = \frac{yy}{2} + xy$ — fluent of $y\dot{x}$, or twice the fluent of $y\dot{x}$ (= twice the required area) $= \frac{yy}{2} + xy$, and consequently the area itself $= \frac{yy}{4} + \frac{xy}{2} - \frac{1}{4} = \frac{3yy}{4} - \frac{1}{4}$ * (when $x = y$).

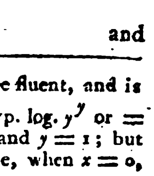
P. M. of *Durham*, after having found the equation of the curve and area the very same as above exhibited, adds, moreover, that when $x = y$ ($\frac{x}{y}$ being then $= 1 =$ h. log. y), y will be $= 1 + 1 + \frac{1}{2} + \frac{1}{2.3}$, &c. $= 2{\cdot}71828$, &c. and consequently the area required (in that case) $= 5{\cdot}36831$.

This question is also answered by *Birchoverensis*, Mr. *R. Butler*, Mr. *W. Chapman*, Mr. *W. Embleton*, Mr. *C. Green*, Mr. *T. Harris*, and others.

The PRIZE QUESTION *answered.*

Suppose *AP* to represent part of the required orbit of the planet commencing motion at *A*, at the given distance *AC* from the center of force *C*, and put

The radius *AC* (*CD*) of the circle *ADK* $= 1$,
The radius vector *CB* —— —— $= x$,
The arch *AD*, measuring the angle *ACD*, described about the center of force *C* $= z$,
The measure of the celerity with which the area *ACB* increases $= u$,
The measure of the centripetal force tending to the center *C* $= Q$;



and

* The quantity $\frac{1}{4}$ is the proper correction of the fluent, and is thus found: In the equation of the curve ($x =$ hyp. log. y^y or $= y \times$ hyp. log. y) when $x = 0$, then log. $y = 0$, and $y = 1$; but when $x = 0$, the area ought to be $= 0$; and since, when $x = 0$, and $y = 1$, the area $\frac{yy}{4} + \frac{xy}{2}$ is $= \frac{1}{4}$, $\therefore \frac{1}{4}$ is the correction to be subtracted.

and then the centrifugal force at B and A will be found to to be $\frac{4u^2}{x^3}$ and $4u^2$ respectively: And $\frac{\ddot{w}}{\dot{z}\dot{z}} = 1 - w - \frac{Q}{4u^2 \times \overline{1-w}|^2}$, per p. 139 and 140 of Simpson's Miscellan. Tracts (w being put $= -\frac{1}{x} + 1$, and no force being supposed to act here, besides that tending to the center of force C, and consequently u a constant quantity).

If now the value of the centripetal force at A be to the centrifugal force *there* in any given ratio of $\overline{1-e}$ to 1, then will the centripetal force at A be $= 4uu \times \overline{1-e}$, and consequently that at $B = \overline{1-e} \times 4uu \times \overline{1-w}|^2$ (the centripetal force being, in this case, as the square of the distance inversely); whence we shall have $\overline{1-e} \times 4u^2 \times \overline{1-w}|^2 \times \frac{\text{cos. } z}{n} =$ the perturbating force acting on the planet at B (n being the given multiple of the radius) and consequently $\overline{1-e} \times \frac{n - \text{cos. } z}{n} \times 4u^2 \times \overline{1-w}|^2 =$ the whole force urging it towards the center of force C in the direction of the radius vector CB, or the whole force whereby it tends to the center C, in that direction; and this value substituted for Q in the above equation, it becomes $\frac{\ddot{w}}{\dot{z}\dot{z}} = -w + e + \frac{\overline{1-e} \times \text{cos. } z}{n}$: From the resolution of which equation, w will be found $= e + \frac{\overline{1-e} \times \overline{B + \frac{z}{2}} \times \text{sin. } z}{n}$ (B being any constant quantity at pleasure); from whence the orbit AP, and every thing else required, may be readily determined, &c.

All our contributors who have solved this question agree so nearly in the above method of solution, that we cannot attribute it to any one in particular; but the 1st prize of 12 diaries fell to the lot of Mr. *Morland*, and the 2d of 8 to Αυτόματικος.

The Eclipses calculated for 1762.

In the revolution of this year will happen four eclipses, two of the sun, and as many of the moon; three of which are visible to these nations.

The first is an invisible solar eclipse, April 24th, between five and six in the morning.

The second is a partial eclipse of the moon, May 8th, in the morning, of which some calculations are as follow.

Calculated by	Beg. h. m.	Mid h. m.	End h. m.	Dur. h. m.	Dig. ° ′
Mr. W. Chapman, for Leicester	2 24	3 56	5 29	3 5	9 50
Mr. C. Green, for London	2 28	4 0	5 33	3 5	9 55
Mr. Metcalfe, for London	2 25	3 58	5 32	3 7	9 49
Mr. Metcalfe, for Wentworth	2 19	3 53	5 26	3 7	9 49
Mr. T. Harris, for London	2 28	4 0	5 33	3 5	9 54
Mr. Allen, for Spalding, Lincoln	2 26	3 58	5 31	3 5	9 54
Mr. E. Greensted, for London	2 27	4 0	5 33	3 6	9 58

Mr. *Metcalfe* observes that the end of this eclipse cannot be seen here, as the moon sets that morning above an hour before.

The third is a visible solar eclipse, October 17th, in the morning, of which we have received the following calculations.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° ′
Mr. W. Chapman, from Manuscript Tables, for Leicester	6 55	7 46	8 44	1 49	6 12
The same Gent. for Petersburgh	9 51	10 56	12 9	2 18	11 47
Mr J. Walker, of Colnbridge, in Yorkshire, for London	6 59	7 50	8 48	1 49	5 20
Mr J. Walker, of Colnbridge, in Yorkshire, for Colnbridge	6 53	7 45	8 41	1 42	5 13
Mr. C. Green, for London	6 57	7 49	8 47	1 50	5 59
Mr. J. Metcalfe, for London	6 52	7 42	8 36	1 44	4 56
Mr. J. Metcalfe, for Wentworth	6 45	7 36	8 32	1 47	5 43
Mr. T. Harris, for London	6 57	7 49	8 47	1 50	5 53
Mr. T. Allen, for Spalding	6 55	7 47	8 4[illegible]	1 5[illegible]	5 5[illegible]
Mr. E. Greensted, for London	7 0	7 53	8 4[illegible]	1 4[illegible]	6 [illegible]

The

The general Phænomena of the Solar Eclipse, Oct. 17th, by Mr. T. Harris.

	Lat. N. °	′	Longitude °	′	⊙ Limb
The eclipse begins at sun-rise	56	18	1	10 W.	Upper
Rises centrally eclipsed —	75	10	4	18 E.	Total
Centrally eclipsed at noon	52	2	50	19 E.	Total
Centrally ecliysed in the 90°	38	12	63	30 E.	Total
Sets centrally eclipsed —	23	39	109	2 E.	Total
The end at sun-set — —	2	50	95	23 E.	Upper
The contact of the two limbs	9	48	50	19 E.	Upper
Eclipsed in the meridian and the horizon near 10 digits	80	40	50	19 E.	Lower

The last eclipse is of the moon, Nov. 1st, in the evening, of which our correspondents exhibit the following account.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° ′
Mr. W. Chapman, for Leicester	7 18	8 41	10 4	2 46	6 48
Mr. C. Green, for London	7 22	8 45	10 8	2 46	6 46
Mr. J. Metcalfe, of Wentworth, for London	7 10	8 34	9 59	2 49	7 25
Mr. J. Metcalfe, of Wentworth, for Wentworth	7 5	8 29	9 54	2 49	7 25
Mr. T. Harris, for London	7 22	8 45	10 8	2 46	6 46
Mr. T. Allen, for Spalding	7 20	8 43	10 6	2 46	6 45
Mr. E. Greensted, for London	7 20	8 42	10 4	2 44	6 43

New Questions.

I. QUESTION 489, *by Mr.* T. Baker; *in which Quest.* 3. *in the last Year's Diary is answered.*

By a quadratic it appears right plain,
Five hundred y——four hundred x explain:
Now, sir, if in return you'll tell to me,
The age and fortune of my charming she;
Which from the giv'n equation * will appear:
I'll strive to do as much for you next year.

$$* \left.\begin{aligned} xy &= 59700 + 12y \\ x - y &= 2375 \end{aligned}\right\} \quad \begin{aligned} x &= \text{her fortune.} \\ y &= \text{her age.} \end{aligned}$$

II. QUESTION 490, *by Mr.* Tho. Atkinson.

Given the sum of the natural sines of the acute angles in a right-angled plane triangle $= 1\frac{2}{7}$ (radius being equal unity) and

and the base or longer leg of the triangle is $=40$; required the hypothenuse and perpendicular of the said triangle.

III. QUESTION 491, *by Mr.* John Hampson.

Required to find two such non-quadrate numbers, whose product shall be a square, and the said product added to the square of either number shall be a square also.

IV. QUESTION 492, *by Mr.* Edw. Kimpton.

Observed three stars *A*, *B*, and *C*, all in the arc of a great circle; the distance of *A* and *B* was found to be $=10°$, and that of *B* and $C=20°$: The difference of the azimuths of *A* and *C* was 90°; and the middlemost *B* was the nearest distance possible to the zenith; it is required, from this data, the altitudes of the three stars.

V. QUESTION 493, *by* Nosnihctuh.

To draw a right line through a given point betwixt two right lines given by position, so that the rectangle under the parts thereof, intercepted by that point and those lines, may be a minimum.

VI. QUESTION 494, *by Mr.* Joseph Fisher.

The gravity of salt water being to that of fresh, as 138 is to 135; in what latitude will a gallon of fresh water be to a gallon of salt water under the equinoctial, as 136 is to 138?

VII. QUESTION 495, *by Mr.* W. Spicer.

The legs of a plane triangle being given equal to 30 and 40 respectively; and if a line be drawn from the vertical angle to the middle of the opposite side, the rectangle of the said line and base is a maximum; required the bisecting line and base.

VIII. QUESTION 496, *by* P. M. *of* Durham.

The greatest plane triangle, having a given angle and any other limit, is that whose sides about the given angle are equal; required the demonstration.

IX. QUESTION 497, *by Mr.* Stephen Ogle.

Required to draw a right line through the focus of any given

given Apollonian parabola, so as to divide the area thereof into two such parts as shall obtain a given ratio.

X. QUESTION 498, *by Mr.* Tho. Barker.

Given $BE = 45$, $ED = 21$, and the time of descent of a heavy body through BA = time of descent through BC; required the lengths of the inclined planes AB, BC, and also their position with regard to the vertical line, or plane, BD, so that the rectangle under AE, CD (perpendiculars upon BD) may be the least possible. [*See the fig. to the solution.*]

XI. QUESTION 499, *by Mr.* Cha. Green.

Observed the sun's true altitude, at Greenwich observatory, to be 45° 41′, and one hour afterwards found it increased by just 8° 4′; required the day and hour when these observations were made.

XII. QUESTION 500, *by Mr.* Tho. Harris.

'Tis just one-half o'th' time till * noon,
When rad'ant Sol due East does shine
Upon the twenty-first of June;
Say † where——and Phillis shall be thine.

* From sun-rise. † In what north lalitude.

XIII. QUESTION 501, *by Mr.* T. Moss.

From the given point P, to draw, geometrically, a right line, cutting the sides of the given triangle ABC, so that the segments AE, BF, shall obtain a given proportion. [*See the fig. to the solution.*]

XIV. QUESTION 502, *by Mr.* T. Allen, *of* Spalding.

In the equation $\frac{\dot{y}}{y} - \frac{\dot{x}}{x} = \frac{x^m \dot{x}}{ay^n}$; it is required to determine the relation of the fluents x and y, supposing that when $x = a$, y is also $= a$.

The PRIZE QUESTION, *by Mr.* G. Witchell.

It is known, that the sine of the moon's parallax, at any apparent altitude, is in proportion to the sine of the horizontal parallax, as the cosine of the said altitude is to the

radius

radius: But it is frequent, in such operations, to use the true altitude instead of the apparent. Now I should be glad to know the quantity of the greatest error that can arise from hence, and what the true altitude will at that time be, on supposition that the horizontal parallax is just 1 degree.

1763.

Questions answered.

I. QUESTION 489 *answered by Mr.* Tho. Sadler.

THE age of your most charming she,
Twenty-five years appears to be:
Her fortune likewise I have found
Two thousand and four hundred pound.

For multiplying the latter of the two given equations by y, and subtracting it from the former, there results $yy + 2363y = 59700$; from whence y will be found $= \frac{\sqrt{4 \cdot 59700 + 2363^2} - 2363}{2} = 25$ years, the lady's age; and thence x will come out $= 2400$ l. her fortune.

Mr. *R. Gibbons* thinks that a great age for such a fortune to lie upon hand; and Mr. *Malachy Hitchins* seems partly of the same opinion, when he says,

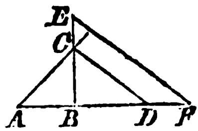
Twenty-five years her age I've found,
Her worth just twice twelve hundred pound:
Dear *Baker*, pray don't tarry longer;
I'd rather have one eight years younger.

Much after the same manner it is answered by Messrs. *J. Aistrope, Tho. Baker (the proposer), J. Barker, Tho. Barker, E. Batten, Tho. Bosworth, T. Bromhall, T. Bramley, J. Buddle, R. Butler, W. Chapman, T. Davidson, R. Dening, J. Doleman, J. Eadon, J. Giles, John Hampson, W. Hardy, E. Hare,* jun. *Sam. Harlock, W. Harvey, J. Hitchcock, S. Hodges, T. Hopkinson, J. Hudson, S. Kemp, E. Kimpton, W. Kingston, J. Launders, W. Mathewson, T. Orme, J. Potter, J. Probert, W. Rawle, T. Robinson, Alex. Rowe, T. Sanderson, P. Sharp, C. Shooter, R. Spencer, W. Spicer, J. Stokes, Elean. Suggett, R. T. Tho. Walker, W. Walton, Matt. Ward, W. Wells,* and many others.

II. QUES-

II. QUESTION 490 *answered by Mr.* Wm. Embleton.

CONSTRUCTION. Through A, the extremity of the right line AD ($=1\frac{4}{5}$) draw AC, making an angle therewith $=45°$, and about the other extreme D, as a center, with the radius (1), describe an arc of a circle cutting AC in C; draw CB perpendicular upon AD, and produce BD till it becomes $= BF$ (40); then draw FE parallel to DC, meeting BC produced in E, and EBF will be the triangle required.



DEMONSTRATION. The angle BAC being $=45°$, and B a right angle (by construction) BC will be $=AB$, and consequently the sum of BC and BD (the sines of the angles F and E, to radius $DC=1$) will be $=AD=1\frac{4}{5}$, and BF being $=40$ (per construction) the whole is manifest, &c.

CALCULATION. As rad. $(CD) : s\angle A\ (45°) :: AD$ (the given sum of the signs) : $s\angle ACD$, the excess of which above ACB (45°) is $=BEF$, the greater of the two acute angles required.

Mr. *R. Butler* and Mr. *Da. Kinnebrook* constructed it in the same manner exactly.

An Algebraic Solution to the same by Mr. John Hudson, *Land-Surveyor.*

Let s $(=\frac{7}{10})=$ half the given sum of the signs, and x $=$ half their difference; then will $s+x=$ sine of the greater, and $s-x=$ that of the lesser of the angles E, F; whence, per 47 Euc. 1, $\overline{s+x}|^2+\overline{s-x}|^2$, or $2s^2+2x^2=1$; and consequently $x=\sqrt{\frac{1-2s^2}{2}}=\frac{1}{10}$; whence the sines of the said two angles appear to be $\frac{6}{10}$ and $\frac{8}{10}$, and the leg EB and hypothenuse EF, 30 and 50 respectively.

According to this method, nearly, it is answered by Messrs. *J. Barber*, *Tho. Barker*, *E. Batten*, *T. Bosworth*, *T. Bromhall* (of Mr. *Allen*'s school) *T. Bramley*, *J. Buddle*, *W. Chapman*, *R. Dening*, *J. Giles*, *R. Gibbons*, *J. Hampson*, *W. Hardy*, *E. Hare*, jun. *S. Harlock*, *Hitchins*, *S. Hodges*, *T. Hopkinson*, *S. Kemp*, *E. Kimpton*, *W. Kingston*, *J. Launders*, *R. Mallock*, *J. Potter*, *T. Robinson*, *Alex. Rowe*,

Rowe, T. Sanderson, P. Sharp, W. Spicer, J. Stokes, Elean. Sugget, T. Walker, W. Walton, M. Ward, W. Wells, and others.

III. QUESTION 491 *answered by Mr.* R. Mallock.

Constitute a right angle ACD [*See fig. last question*] with two lines AC, CD, respectively measuring the sides of any two known squares at pleasure, and let the points A, D be joined; draw CB perpendicular upon AD, and AB and DB will be the measures of the corresponding numbers required. For $BD \times AD$ or $BD^2 + BD \times AB = DC^2$, and $AB \times AD$ or $AB^2 + AB \times BD = AC^2$, by a well-known property of right angled triangles, and $AB \times BD$ $(= BC^2$, by Euc. 6. 8.$) = \frac{AC^2 \times CD^2}{AD^2}$, which is manifestly a square number, as AC^2 and CD^2 are such, by supposition, &c.

EXAMPLE. Suppose $AC = 3$, and $DC = 4$; then will $AD = 5$ (per 47. Euc. 1.) and thence will be found AB and $DB = \frac{9}{5}$ and $\frac{16}{5}$ respectively; the first equimultiples of which numbers, in integers, are 9 and 16, but, being square numbers, they are not for the purpose: But the next pair of integers, 18 and 32, (found by multiplying each of the preceding ones by 2) will answer the conditions of the question; and, by proceeding in this manner, innumerable other answers may be discovered.

The same answered otherwise by Mr. John Hampson, *the Proposer.*

Let bba and a represent the two numbers required, the product whereof $aabb$ will evidently be a square number; but, per nature of the question, $b^4aa + bbaa$ and $aa + bbaa$ must each be a square number, and consequently $bb + 1$ = a square number (as $b^4aa + bbaa$ and $aa + bbaa$ are evidently $= bbaa \times \overline{bb+1}$ and $aa \times \overline{bb+1}$ respectively) which suppose $= \overline{b-c}|^2$; whence b will be found $= \frac{cc-1}{2c}$; but as a may be taken at pleasure, suppose it $= 4ccd$, and then $bba = d \times \overline{cc-1}|^2$; from whence may be found any number of answers in whole numbers at pleasure, making c not less than 2, and d any non-quadrate number.

EXAMPLE. If c and d are each $= 2$, then will $bba = 18$, and $a = 32$, the first answers; but if $d = 3$, then $bba = 27$, and

and $a = 48$, the next anſwers: And if $d = 5$, then bba will be $= 45$, and $a = 80$, &c. which anſwers are all in the ratio of 9 to 16. Again, ſuppoſing $c = 3$, and expounding d by 2, 3, 5, 6, 7, &c. ſucceſſively, another ſet of anſwers, each in the ratio of 9 to 16, will be obtained; and thus we may proceed as far as we pleaſe.

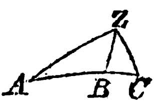
The ſame anſwered otherwiſe by Mr. T. Allen.

Let the required numbers be denoted by x and nx, and then nxx, $\overline{n+1} \times xx$, and $n \times \overline{n+1} \times xx$ being ſquare numbers, by the queſtion, n and $n+1$ muſt therefore be ſquare numbers likewiſe; whence, putting z and $z+a = \sqrt{n}$ and $\sqrt{\overline{n+1}}$ reſpectively, $2az + aa$ $(= \overline{z+a}|^2 - zz)$ will be $= 1$, and conſequently $z = \frac{1-aa}{2a}$, where aa may be any ſquare number leſs than unity, and x any number at pleaſure, &c.

Ingenious ſolutions to this queſtion have likewiſe been received from Mr. *J. Barber*, Mr. *Tho. Baker*, Mr. *J. Buddle*, Mr. *R. Butler*, Mr. *J. Chapman*, Mr. *Steph. Hodges*, Mr. *Da. Kinnebrook*, Mr. *Tho. Sanderſon*, Mrs. *Eleanor Suggett*, Mr. *W. Wells*, and others.

IV. QUESTION 492 *anſwered by Mr.* Rich. Gibbons.

Let A, B, C repreſent the three ſtars, and Z the zenith of the place. Then, the arc ZB being perpendicular upon AC, per the nature of the queſtion, we ſhall, per ſpherics, (prop. 35 Emer. Trig.) have ſine of $AB + BC$ (30°) : ſine of $AZB + BZC$ (90°) :: ſine of $BC - AB$ (10°) : ſine of $BZC - AZB$ (20° 19′ 20″). Therefore $AZB = 34° \; 15' \; 20''$, and $BZG = 55° \; 9' \; 40''$; the altitude of the ſtar $A = 72° \; 18' \; 14''$, of $B = 75° \; 19' \; 32''$, and that of $C = 65° \; 22' \; 23''$.

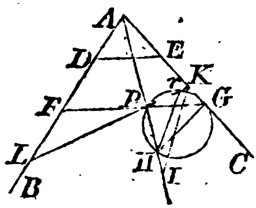


Mr. *Tho. Allen* and Mr. *E. Hare*, jun. anſwer it in this manner exactly.

We have alſo been favoured with elegant and conciſe algebraic ſolutions to this queſtion from Mr. *Tho. Boſworth*, Mr. *J. Buddle*, Mr. *Rob. Butler*, Mr. *W. Chapman*, Mr. *J. Hampſon*, Mr. *W. Hardy*, Mr. *S. Harlock*, Mr. *Malachy Hitchins*, Mr. *Steph. Hodges*, Mr. *Tho. Hopkinſon*, Mr. *J. Hudſon*, Mr. *W. Kingſton*, Mr. *Da. Kinnebrook*, Mr. *Ja. Launders*, Mr. *Tho Sanderſon*, and others.

V. QUESTION 493 *answered by Mr.* Da. Kinnebrook, *and Mr.* T. Mols.

CONSTRUCTION. Let P be the given point, and AB and and AC the two lines given in position, in which take $AD = AE$, and let the points D, E be joined; then through P draw FG parallel to DE, and the thing is done.



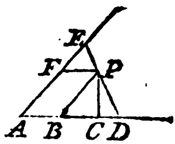
DEMONSTRATION. Join the points A, P, and draw the right line GH (meeting AP produced in H) so as to make the $\angle PGH = \angle FAP$, and through the points H, P, G, conceive the periphery of a circle to be described; then the $\angle PGH$ being $= \angle FAP$ (by hypothesis), and the $\angle APF = \angle HPG$, the $\angle AFP$ will be $= \angle PHG$ (32 Euc. 1); likewise the $\angle AFP = \angle AGP$ (by construct.) and consequently the $\angle AGP = \angle PHG$; wherefore the circle touches the right line AC in the point G (32 Euc. 3). But the triangles APF, HPG, are similar; and therefore $AP : PF :: PG : PH$; whence $PF \times PG = AP \times PH$, and consequently, when the rectangle $PF \times PG$ is a minimum, its equal $AP \times PH$ or PH (as AP is a constant quantity) will be a minimum also, which will be in the case above-mentioned, viz. when the line FG is drawn so as to cut off the $\triangle FAG$ isosceles; for let LK be any other right line drawn through the said given point P, and from the point r, where that line and the periphery of the circle intersect; draw rH, and parallel thereto draw KI; then, because AC is a tangent to the circle, PK will evidently be greater than Pr. And therefore, as KI is parallel to Hr, PI will be greater than PH, and consequently $LP \times PK$ ($= AP \times PI$) will every where be greater than $FP \times PG$ or its equal $AP \times PH$. *Q. E. D.*

Very ingenious and neat geometrical constructions to this question have likewise been received from Mr. *Rob. Butler*, Mr. *Rich. Gibbons*, *P. M.* of *Durham*, and several others.

The

The same solved Algebraically by Messrs. T. Allen, T. Bromhall, *and* E. Hare, *jun.* &c.

Let P be the given point, and draw BP, FP, parallel to AE and AD respectively, and PC perpendicular upon AD. Put $FP = a$, $BC = b$, $PC = c$, and $CD = x$. Then will $\sqrt{cc+xx} = PD$, and per similar triangles, $b+x\ (BD) : \sqrt{cc+xx}\ (PD) :: a\ (FP) : \frac{a\sqrt{cc+xx}}{b+x} = EP$; whence $PE \times PD$ is $= a \times \frac{cc+xx}{b+x}$, which is to be a minimum, per question; and therefore, in fluxions, $2bx\dot{x} + xx\dot{x} - cc\dot{x} = 0$; from whence x is found equal $\sqrt{cc+bb} - b$; and from thence the position of the line ED may be determined.



The same answered otherwise by Mr. Wm. Embleton.

Put $PB = a$; $PF = b$; $BC = c$, and $BD = x$; and then will PD be found $= \sqrt{aa+xx-2cx}$, by 9. 2. of Simpson's Elem. of Plane Geom. 1st edition: whence, per similar triangles, BDP, FPE, $x\ (BD) : \sqrt{aa+xx-2cx}\ (DP) :: b\ (PF) : \frac{b}{x}\sqrt{aa+xx-2cx} = PE$; and consequently $PE \times PD$ will be $= \frac{b}{x} \times \overline{aa+xx-2cx}$, which, according to the question, is to be a minimum: And consequently, in fluxions, $-\frac{aa\dot{x}}{xx} + \dot{x} = 0$; whence x is found $= a$, or $AE = AD$; from which the position of the required line becomes known.

True and concise answers to this question have also been received from Mr. *Tho. Barker*, Mr. *E. Batten*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *J. Hampson*, Mr. *Wm. Hardy*, Mr. *Malachy Hitchins*, Mr. *Steph. Hodges*, Mr. *Tho. Hopkinson*, Mr. *J. Hudson*, Mr. *S. Kemp*, Mr. *Wm. Kingston*, Mr. *J. Launders*, Mr. *Tho. Robinson*, Mr. *Tho. Sanderson*, Mr. *P. Sharp*, Mr. *W. Spicer*, and others.

VI. QUESTION 494 *answered.*

The numerical data of this question being misprinted, no solution to it has been received. Some, however, observe, that it is no more than to find x the sign of the latitude of the place where a body, weighing w at the equator, shall equiponderate with another given homogeneous weight W, which is easily found $=\sqrt{\frac{66240}{519}\times\frac{W-w}{w}}$ * (the ratio of the diameters of the earth being as 230 to 231, and the gravity and centrifugal force, at the equator, 1 and $\frac{1}{289}$ respectively.

VII. QUESTION 495 *answered by Mr.* W. Embleton.

Let ABC represent the triangle required, and BD the line bisecting its base AC. Now, when $BD\times 2AD$ $(=BD\times AC)$ is a maximum, $AD^2-2BD\times AD+BD^2$ or $\overline{AD \backsim BD}^2$ $(=AD^2+BD^2$, a constant quantity by theo. 11. book 2. Simpson's Elem. of Plane Geom. 2d

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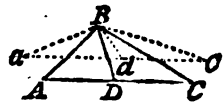
* This expression may be thus found: as 288 : 289 :: w (the weight or grav. at the equator diminished by the centrif. force) : $\frac{298\,w}{298}$ the weight *there* if the centrif. force were taken off, or if the earth did not revolve on its axis; also as 230 : 231 :: $\frac{289\,w}{288}$: $\frac{231\cdot 289\,w}{230\cdot 288}$ the weight at the pole; hence $\frac{231\cdot 289\,w}{230\cdot 288}-w$ $=\frac{519\,w}{66240}$ is the whole weight gained at the pole: Then, by art. 401 Simpson's Flux. 1 : xx :: $\frac{519\,w}{66240}$: $\frac{519\,wxx}{66240}$ the weight gained in the lat. whose sine is x, which must be $=W-w$; hence $x=\sqrt{\frac{W-w}{519\,w}\cdot 66240}$, as above.

Now the limit of the encrease being $\frac{519}{66240}=\frac{1}{128}$ nearly, and the increase proposed in the quest. being from 135 to 136, or $\frac{1}{135}$ part only, it is evident that it is proposed within the limit; and, by writing 135 and 136 instead of w and W in the above value of x, it becomes $x=\cdot 9723207=$ the sine of 76° 29′ 17″ nearly, the latitude required.

2d edition, diminished by the greatest value possible of the variable quantity $2\ BD \times AD$) will manifestly be a minimum, and consequently $AD \backsim BD$ a minimum, which, it is evident, can only be when it is $=0$; whence it clearly follows that the required triangle will be right-angled at B, because (since AD is now proved $= BD = DC$) it may be circumscribed by the periphery of a semi-circle of that radius described about the center D;——hence by 47 Euc. 1. AC will be found $=50$; and consequently $BD = 25$.



COROLLARY. If Ba, Bc, be supposed any other position of the two given sides BA, BC, different from that here determined, then will $AD + BD$ be always greater than $ad + Bd$; for $AD^2 + BD^2$ being $= ad^2 + dB^2$, and $2AD \times DB \sqsubset 2ad \times dB$ (by what has just now been demonstrated), $AD^2 + 2AD \times DB + DB^2$ will therefore be greater than $ad^2 + 2ad \times dB + dB^2$, and consequently $AD + DB \sqsubset ad + dB$.

According to this method of solution the question is also answered by Mr. *Rob. Butler*, Mr. *Rich. Gibbons*, Mr. *Da. Kinnebrook*, *P. M. of Durham*, Mr. *W. Spicer*, and some others.

An Algebraical Solution to the same by Mr. T. Bromhall.

Put $AB = 40 = a$, $BC = 30 = b$, $AD = x$, and $BD = y$; then, per 12. 2. Simp. Elem. Plane Geom. 1st edit. $2xx + 2yy = aa + bb$; whence $y = \sqrt{\frac{aa + bb - 2xx}{2}}$, and consequently $AC \times BD$ is $= 2x\sqrt{\frac{aa + bb - 2xx}{2}}$, which being to be a maximum per quest. $4xx \times \frac{aa + bb - 2xx}{2}$ will therefore be a maximum, and consequently, in fluxions, $2aax\dot{x} + 2bbx\dot{x} - 8x^3\dot{x} = 0$; whence $aa + bb - 4xx = 0$, and consequently $2x = \sqrt{aa + bb} = 50 = AC$; and hence it appears that $BD = \frac{1}{2}AC$, and also that ABC is a right angle.

This question was likewise truly answered by Mr. *Tho. Barker*, Mr. *E. Batten*, Mr. *Tho. Bosworth*, Mr. *Tho. Bramley*, Mr. *John Buddle*, Mr. *Wm. Chapman*, Mr. *J. Hampson*,

Mr. *Wm. Hardy*, Mr. *Edw. Hare*, jun. Mr. *Sam. Harlock*, Mr. *Malacty Hitchins*, Mr. *Steph. Hodges*, Mr. *Tho. Hopkinson*, Mr. *J. Hudson*, Mr. *Sam. Kemp*, Mr. *Wm. Kingston*, Mr. *Ja. Launders*, Mr. *R. Mallock*, Mr. *Tho. Robinson*, Mr. *Tho. Saunderson*, Mr. *P. Sharp*, Mr. *Jo. Stokes*, Mrs. *Eleanor Suggett*, Mr. *Wm. Walton*, and others.

VIII. QUESTION 496 *answered by Mr.* Rich. Mallock.

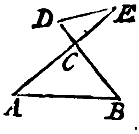
The other given limit, here meant, must necessarily be such as the base, the perimeter, the radius of the circumscribing circle, or the like, which would confine the increase of the required triangle within certain bounds or limits, and affect each of the including sides exactly alike, and then their sum may be always denoted by a certain right line determinable by means of the said limit, and consequently (the area of every plain triangle being = the rectangle under any two of its sides drawn into half the sine of their included angle, the triangle sought will be a maximum, when the said right line is so divided, that the rectangle of its two parts may be a maximum (the sine of the included angle being a constant quantity) *i. e.* when they are equal, which is too well known to need a particular demonstration here.

Mr. *J. Hudson* puts b for the base, a for the sine of the given angle, s and c for the sine and cosine of half the sum, and x and y for the sine and cosine of half the difference of the other two angles, and then the area of the required triangle is readily found $= bb \times \frac{ss - xx}{2a}$, which, it is manifest, will continually increase, as b increases, and x, at the same time, decreases, and consequently that it will (under any assigned possible value of b) be greatest, when x is least, that is, when x vanishes, and the required triangle becomes isosceles, whatever the other given limit may be, unless it restrains the including sides from becoming equal at the same time that, per question, they are supposed every way capable of being so, which would be absurd indeed, and incompatible with the true nature of the question!——If either of the including sides, or their difference, or the difference of their squares, or the sum or difference arising from either of them, being added to or subtracted from n times the other (except when n equals unity), or the like, were to be given, then, they affecting the including sides not alike, the question, instead of being limited thereby, would become either absurd or infinite.

The

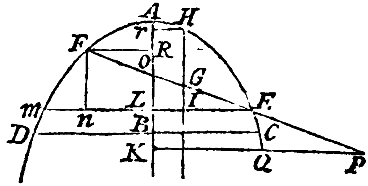
The same answered by P. M. *of* Durham (*the Proposer*). *Taken from the Diary for the Year* 1764.

Assume the points D, E, in the sides BC, AC, comprehending the given angle, produced, and let the right line DE be drawn, and then the $\triangle DCE$, as also each of its sides, will be given. But the triangles DCE, ACB, are the halves of parallelograms standing about the common angle at C, and parallelograms, standing about a common angle, are directly as the rectangles of their sides (23 Euc. 6), therefore also their halves will be in the same ratio, *i. e.* $\triangle DCE : \square DCE :: \triangle ACB : \square ACB$. Whence the $\triangle ACB$ having a given $\angle$ and any other limit whatever, is in a constant ratio to the rectangle under the sides about the said given $\angle$, and consequently when the $\triangle$ is greatest, the rectangle will be the greatest also. But the sides AC, BC, have a constant analogy to the sines of the opposite angles at the base, and may mutually be represented by each other; therefore, when the rectangle under the said sides is a maximum, it is manifest that the rectangle under the sines of the angles at the base must be a maximum also: But the sum of these angles being a constant quantity, the rectangle under their sines will be the greatest when the said angles are equal, *i. e.* when the sides subtending them are of consequence equal (6 Euc. 1). *Q. E. D.*



IX. QUESTION 497 *answered by Mr.* Rob. Butler.

Let DAC represent the given parabola, and EF the given dividing line passing through the focus O, and let the ordinates FR, EL, be drawn perpendicular upon the axis AL, putting $AO = a$, $FR = x$, and $EL = y$; then, per conics, $\frac{xx}{4a} = AR$, and $\frac{yy}{4a} =$



AL ($4a$ being $=$ the parameter); whence RO ($= a - \frac{xx}{4a}$) $= \frac{4aa - xx}{4a}$, and $OL = \frac{yy - 4aa}{4a}$; and therefore $\frac{1}{2}x \times$

4aa

$$\frac{4aa-xx}{4a}+\frac{2}{3}x\times\frac{xx}{4a}+\frac{2}{3}y\times\frac{yy}{4a}-\frac{1}{2}y\times\frac{yy-4aa}{4a}=$$

$\frac{4aax-x^3+4aay-y^3}{8a}+\frac{x^3+y^3}{6a}=$ the area of the figure $FAE=$ a given quantity A, by the nature of the question (because the area of the whole, DAC, and the ratio of the parts $DFEC$, FAE being given, per the question, the parts themselves will from thence likewise be given); but, by similar triangles, $x\ (FR):\frac{4aa-xx}{4a}\ (RO)::y\ (EL):\frac{yy-4aa}{4a}\ (OL)$; whence $4aa=\frac{xyy+yxx}{x+y}=xy$, which value substituted in the preceding equation of the area, &c. it becomes $\frac{x^3+y^3}{6a}+\frac{yxx+xyy-x^3-y^3}{8a}=A$, or $y^3+3y^2x+3yx^2+x^3=24aA$: And hence, by extraction, $y+x$ comes out $=\overline{24aA}|^{\frac{1}{3}}$; from the square of which subtracting the equation $4yx=16aa$ (found from above), and there results $yy-2xy+xx=\overline{24aA}|^{\frac{2}{3}}-16aa$, and consequently $y-x=\sqrt{\overline{24aA}|^{\frac{2}{3}}-16aa}$; and hence y is found $=\frac{1}{2}\sqrt{\overline{24aA}|^{\frac{2}{3}}-16aa}+\frac{1}{2}\times\overline{24aA}|^{\frac{1}{3}}$, and $x=\frac{1}{2}\times\overline{24aA}|^{\frac{1}{3}}-\frac{1}{2}\sqrt{\overline{24aA}|^{\frac{2}{3}}-16aa}$; from either of which equations the position of the line EOF is determined.

Mr. *Da. Kinnebrook* answered it according to this method of solution exactly.

The same answered otherwise by Mr. Wm. Embleton.

Draw Hr perpendicular upon AK, and put $x=Ar$; $p=$ the parameter of the principal diameter or axis AK, and suppose the right line FE (passing through the focus O) to be drawn as is required; and let the area of the space $FAHE$ (which is given by the nature of the question) be denoted by a; then, per conics, HG will be found $=\frac{4x+p}{4}$, $FE=4x+p$, $Hr=\sqrt{px}$, &c. Whence a $(=HG\times\frac{2}{3}FE\times s.\angle HGF)$ will appear $=\frac{\sqrt{p}}{6}\times\overline{4x+p}|^{\frac{3}{2}}$: And conse-

quently

quently $x = \frac{1}{4} \times \overline{\frac{36aa}{p}}\Big|^{\frac{1}{3}} - \frac{p}{4}$; from whence the position of the line FE may be readily determined.

Mr. *John Hudson* solved it according to this method very near.

The same answered more generally by P. M. *of* Durham.

This truly ingenious gentleman supposes the point P, through which the dividing line is required to pass, to be given any where in the plane of the given parabola DAC (either within it or without), and likewise that the right line PEF is drawn as required, cutting the axis AB in O, and dividing the given parabolic area DAC in the given proportion of m to n.

Then, bisecting EF in G, and drawing the diameter HG through G, and the ordinate Em through E, meeting the axis AB in L, and the diameter HG, produced, in I, the area DAC : area $EHF :: m+n : n :: BC^3 : EI^3$ (cor. 2. 52 Sim. 5); whence EI is given. Draw PK perpendicular to the axis in K, and meeting the curve in Q, as also Fn perpendicular to the ordinate Em. Then (by a property of the parabola) $AO \times p = \square ELn$; also (by another property) $Fn \times p = \square Enm$: But $AK \times p = QK^2$; therefore $OK \times p = QK^2 - \square ELn$: Whence $QK^2 - \square ELn : \square Enm :: (OK : Fn ::) PK : En$, i. e. $QK^2 - \square ELn = nm \times PK$. Call $PK = a$, $QK = b$, $EI = c$, and $IL = x$, then will $EL = c + x$, $Ln = c - x$, and $nm = 2x$: Wherefore $bb - cc + xx = 2ax$, and consequently $\overline{a-x}|^2 = aa - bb + cc$.

COROLLARY 1. If the point P be in the curve, then $aa - bb = 0$, and $a - x = c$.

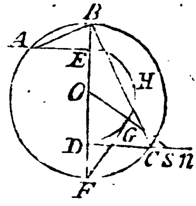
COROLLARY 2. But if the point P be in the axis, then $a = 0$, and $xx = cc - bb$.

This question is likewise ingeniously answered by Mr. *J. Hampson*, Mr. *P. Sharp*, and others.

X. QUESTION 498 *answered.*

If BD be supposed to be produced out to F till the time of perpendicular descent along it shall be equal to the time of descent along BA or BC, and the circumference of a

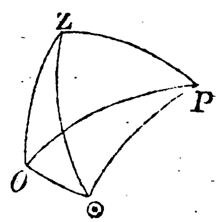
circle



circle be described about BF as a diameter, it will, it is very well known, pass through the points A and C (see art. 204 p. 230 and 131 of Simpson's Fluxions, 2d edition); whence, DC^2 being $= BD \times DF$ and $AE^2 = BE \times \overline{ED + DF}$, by the property of the circle, we shall have $AE^2 \times CD^2 = BE \times BD \times \overline{ED \times DF + DF^2}$, which it is evident, by inspection, can admit of neither max. nor minimum; but if it were required to be equal to a given quantity (m^2), then, $ED \times DF + DF^2$ ($= DF \times \overline{ED + DF}$) being $= \frac{m^2}{BE \times BD}$ (which suppose $= nn$) we shall, by describing a semi-circle EHD upon ED, as a diameter, and drawing a tangent to it such, that the distance intercepted between the point of contact G and its intersection F with ED produced may be $= n$, find the value of DF required.——Thus, in Dn, perpendicular to ED, take $Ds = n$; join the points O, s, with a right line, and from the point G, where it intersects the said semi-circle, draw GF perpendicular to Os, meeting OD produced in F, and the thing is done; for GF is manifestly $= Ds$ ($= n$, by construction).

XI. QUESTION 499 *answered by Mr.* R. Butler.

Put a and b equal to the cosines of $\odot Z$ and OZ, the co-altitudes at first and second observations, respectively; m and n equal to the sine and cosine of ZP (the co-latitude) and x and y equal to the sine and cosine of $\odot P$ or OP (the sun's distance from the north pole); then, per spherics, $\frac{a - ny}{mx} =$ cosine of the $\angle \odot IZ$; the sine of which is $\frac{\sqrt{mmxx - \overline{a - ny}|^2}}{mx}$: Whence,



putting p and q equal to the sine and cosine of $\angle OP\odot$ ($15°$), the cosine of $\angle ZPO$ will be found $= \frac{qa - nqy}{mx} + \frac{p\sqrt{mmxx - \overline{a - ny}|^2}}{mx}$; and consequently $qa - nqy + p\sqrt{mmxx - \overline{a - ny}|^2} + ny = b$ (by Anderson's theorem); from

from whence y is found = sine of $23\frac{1}{2}$ degrees, answering to the 21st of June; in the forenoon of which, at the hours of 9 and 10, the observations appear to have been made.

The same answered otherwise by Mr. T. Allen.

For the cosines of the given coaltiudes $\odot Z$ and OZ put a and b respectively and c and d = the sine and cosine of ZP, the colatitude; m and n = the sine and cosine of the given interval of time ($OP\odot = 15°$) between the two observations, and x and y = the since and cosine of $\odot P$ (OP) the codeclination (radius 1); then will $\frac{b - dy}{cx}$ and $\frac{a - dy}{cx}$ be equal to the cosines of ZPO and $ZP\odot$ respectively. Whence, per spherics, and the nature of the question, $n \times \frac{a - dy}{cx} + m \sqrt{1 - \frac{\overline{a - dy}|^2}{ccxx}} = \frac{b - dy}{cx}$; and hence, by substituting t for $b - na$, and s for $d - nd$, we get $m \times \sqrt{1 - \frac{\overline{a - dy}|^2}{ccxx}} = \frac{t - sy}{cx}$, and consequently (by involution, &c. and putting $f = ss + mmdd + mmcc$, $g = mmda + ts$, and $-k = mmcc - mmaa - tt$) $fyy - 2gy = -k$; from the resolution of which y is found $= \frac{g \pm \sqrt{gg - fk}}{f}$, expressing the sine of the declination required; from whence every thing else wanted, may be readily determined.

Mr. *Charles Green* (the proposer) answers it by a process not materially different from that above exhibited; and from thence finds $x = .917r796$ = cosine of 23° 29′ nearly. Whence the hours of observation appear to be 9 and 10 on the 21st of June; which agree very near with observation: —— but theory and calculation will differ a few seconds, nay even minutes, sometimes, from the nicest observations.

Mr. *Rich. Gibbons*, Mr. *Da. Kinnebrook*, and *P. M.* of *Durham*, by very curious and short processes, have likewise brought out the same conclusion.

Ingenious answers to this question have likewise been received from Mr. *T. Barker*, Mr. *J. Buddle*, Mr. *T. Hopkinson*, Mr. *J. Hudson*, Mr. *W. Kingston*, Mr. *J. Potter*, Mr. *Tho. Robinson*, Mrs. *Eleanor Suggett*, Mr. *W. Sutton*, and others.

XII. QUES-

XII. QUESTION 500 *answered by Mr.* John Potter; *to Mr.* Tho. Harris, *the Proposer.*

SIR,

The latitude below is shewn *,
Yet Phillis still may be your own.

* In 64° 35′ 48″ north latitude.

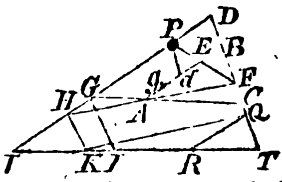
Another Answer to the same.

Let s and c = the sine and cosine of $P\odot$ [*See last fig.*] or PO (= 66° 31′) the sun's distance from the north pole P; y and x = the sine and cosine of ZP (the colatitude) and z and u = those of the $\angle ZPO$ (= $\odot PO$ per nature of the question; then will $2uu - 1$ = cosine $\angle ZP\odot$, and per spherics, $2uuys - sy + cx$ (= the cosine of $Z\odot$) will be = 0 (per question); whence $2uus = s - \frac{cx}{y}$: And in the right-angled spherical triangle OZP, $u : 1 :: \frac{c}{s} : \frac{c}{us} = \frac{x}{y}$ = cotangent of ZP; from which is derived $2uus = \frac{2ccyy}{sxx}$: Whence, by equality, $s - \frac{cx}{y} = \frac{2ccyy}{sxx}$; and consequently $\frac{x}{y} = 2{\cdot}1057018$ = tang. of 64° 35′ 48″, the latitude required.

According to this method, very nearly, the answer is given by Mr. *T. Allen*, Mr. *Tho. Barker*, Mr. *R. Butler*, Mr. *W. Chapman*, Mr. *J. Hampson*, Mr. *Da. Kinnebrook*, and Mr. *Paul Sharp*——This question is likewise ingeniously answered by Mr. *Tho. Bosworth*, Mr. *John Buddle*, Mr. *Rich. Dening*, Mr. *Rich. Gibbons*, Mr. *Sam. Harlock*, Mr. *Malachy Hitchins*, Mr. *Steph. Hodges*, Mr. *Tho. Hopkinson*, Mr. *John Hudson*, Mr. *W. Kingston*, *P. M.* of *Durham*, Mr. *Wm. Spirer*, Mr. *Ja. Stokes*, Mrs. *Eleanor Suggett*, Mr. *W. Sutton*, Mr. *Matt. Ward*, and others.

XIII. QUESTION 501 *answered by Mr.* D. Kinnebrook.

CONSTRUCTION. Through the given point P draw PI parallel to the side AB, of the given plane triangle ABC, meeting the other side thereof, BC, produced in D, and take



take DI to DT in the given ratio, and let the points I, T, be joined: Take DQ to PI as DT to DI, and draw QR parallel to AB meeting IT in R; through A draw AG parallel to IT meeting ID in G, and from thence let GL be parallel to DT meeting IT in L: Then from R towards I take RK so, that $RK \times LK$ may be $= RT \times AG$, and join the points K, Q, and through the point A, and parallel to KQ, let the right line HAF be drawn meeting ID and DT in H and F: Lastly, join the points P, F, and that will be the line required to be drawn, it cutting AB in E so, that $AE : BF :: ID : DT$, the ratio given, by construction.

DEMONSTRATION. By construction, $LK : RT :: AG : RK ::$ (by sim. triangles) $HG : RQ$: whence, by alternation and equality, $LK : HG :: RT : RQ ::$ (by sim. triangles) $IT : ID :: IL : IG$; and hence it follows, that the right line HK (when drawn) will be parallel to GL or DT (by corol. theor. 12. 4. of Simp. Elem. of Plane Geom. 2d edit.), and consequently that, HF being parallel to KQ (by construction), $HKQF$ will be a parallelogram; whence (per sim. triangles) $HK (FQ) : IH :: DT : DI :: DQ : PI$ (by construction), and consequently $DF (DQ - FQ) : PH (PI - IH) :: DQ : PI$; but $DF : PH :: BF : AE$ (by sim. triangles): Whence, by equality, $BF : AE :: DQ : PI$, that is, in the given ratio.

Mr. *R. Butler* has constructed this question in the same manner, very near.

The same answered otherwise by P. M. *of* Durham, *and* *Mr.* T. Moss (*the Proposer*).

Suppose the thing done, and PEF to be the position of the right line required; draw Pd parallel to BC, and take dg to Pd in the given ratio of AE to BF, and then $AE : gd :: (BF : Pd) :: BE : dE$ (by sim. triangles), and, compoundly, $AE : AB :: gd : gE$; wherefore AB and gd being both given in length and position, it is now only required to produce the given right line Ag to E so, that the rectangle under AE and gE may be equal to the given rectangle under AB and gd, which is elegantly done in Simpson's Select Exercises, p. 94.

Mr. *Wm. Embleton* also gave a curious construction, which is unluckily omitted.

XIV. QUESTION 502 *answered by* Mr. W. Spencer.

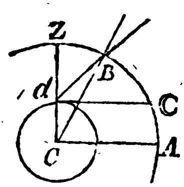
Multiply each side of the given fluxion. equation ($\frac{\dot{y}}{y} - \frac{\dot{x}}{x} = \frac{x^m \dot{x}}{a y^n}$) by $\frac{y^n x^n}{x^{2n}}$, and it becomes $\frac{x^n y^{n-1}\dot{y} - y^n x^{n-1}\dot{x}}{x^{2n}} = \frac{x^{m-n}\dot{x}}{a}$; the equation of the fluents of which is evidently $\frac{y^n}{nx^n} = \frac{x^{m-n+1}}{a \times \overline{m-n+1}}$ or $\frac{1}{n} = \frac{a^{m-n}}{m-n+1}$ when x and y are each $= a$; whence the equation of the fluents truly corrected, according to the conditions of the question, will be $\frac{y^n}{nx^n} - \frac{1}{n} = \frac{x^{m-n+1}}{a \times \overline{m-n+1}} - \frac{a^{m-n}}{m-n+1}$; and consequently $y^n = \frac{nx^{m+1} + \overline{m-n+1}.ax^n - na^{m-n+1}x^n}{a.\overline{m-n+1}}$ will be the equation, exhibiting the relation of the fluents x and y required.

COROLLARY. If $n = 1$, then will $\frac{y}{x} = \frac{x^m - a^m + ma}{ma}$, the equation of the fluents in this case, &c.

True answers to this question have been received from Mr. *T. Allen* (the proposer), Mr. *R. Butler*, Mr. *Wm. Embleton*, Mr. *Da. Kinnebrook*, *P. M.* of *Durham*, and others.

The PRIZE QUESTION *answered by* Mr. T. Allen, *of* Spalding, *in* Lincolnshire.

Let C represent the earth's center, and the arc AB any altitude of the moon, considered as variable. Put the sine of $\angle d$ ☾ C (the horizontal parallax) $= \cdot 0174524 = a$ (radius 1.) and the sine of the $\angle ZdB$ (the moon's visible zenith distance) $= x$, and then the sine and cosine of $\angle dBC$, the true parallax in altitude, will readily be found $= ax$ and $\sqrt{1 - aaxx}$ respectively: Whence $x\sqrt{1-aaxx} - ax\sqrt{1-xx} =$ sine of $\angle ZCB$, the true zenith distance; and



and consequently, $1 : x\sqrt{1-aaxx} - ax\sqrt{1-xx} :: a : ax\sqrt{1-aaxx} - aax\sqrt{1-xx}$, the sine of the erroneous parallax. Therefore $ax - ax\sqrt{1-aaxx} + aax\sqrt{1-xx}$, or, which is the same thing, $x - \sqrt{xx-aax^4} + a\sqrt{x^2-x^4}$ is required to be a maximum: In fluxions, $\dot{x} + \frac{2a^2x^2\dot{x} - \dot{x}}{\sqrt{1-aaxx}} + a \times \frac{\dot{x} - 2x^2\dot{x}}{\sqrt{1-xx}} = 0$; whence $\frac{1-2aaxx}{\sqrt{1-aaxx}} + a \times \frac{2xx-1}{\sqrt{1-xx}} = 1$: From the resolution of which equation the value of x is found $= \cdot 71039 =$ the sine of $45^\circ\ 16'$ very near. Whence the moon's true zenith distance (ZCB) is readily found $= 44^\circ\ 33'\ 23''$, and the error of parallax $= 31''\ 40'''$; which was required.

This question is likewise answered by Mr. *R. Butler*, Mr. *Jos. Fisher*, Mr. *Cha. Green*, Mr. *D. Kinnebrook*, and others; but the above answer, and Mr. *G. Witchell's*, being the only true ones that came to hand within the limited time, the authors thereof are therefore entitled to the prizes, the former of them to the lot of 12, and the latter to that of 8 Diaries.

The Eclipses calculated for 1763.

There will be only two eclipses this year, both of the sun, and both invisible to the inhabitants of this part of the earth. They happen in the following order.

The first is on April 13th, a little after ten in the morning, and will be very conspicuous to the Ethiopians, Abyssinians, &c.

The second, October 7th, about one in the morning, and will be seen by those who are that time sailing on the Pacific Ocean, near the equinoctial.

Mr. *Stephen Hodges*, at the Right Hon. Lord Spencer's, at Althorp, near Northampton, observes the same as above.

Mr. *J. Metcalfe* has obliged us with with a general calculation of these eclipses, but as they are invisible here, and we are straitened for room, we must omit them.

New Questions.

I. QUESTION 503, *by Mr.* Tho. Sadler.

At Marbury a maid doth dwell,
Whose wit and beauty most excel;
She seems to rival ev'ry fair;
They're few that can with her compare:
Her height, age, fortune, you will know
From th' equations propos'd below.

$$\left.\begin{array}{r} x+2y+z=281{\cdot}5294. \\ xyz=170988{\cdot}22. \\ xx=4yy+zz. \end{array}\right\} \begin{array}{l} \text{Where } x = \text{her fortune.} \\ \quad y = \text{her height in inches.} \\ \quad z = \text{her age—to be solved} \\ \qquad \text{by a quadratic.} \end{array}$$

II. QUEST. 504, *by the Rev. Mr.* Wyvell Blennerhassett.

A ship, at a certain port *A*, observes two islands *B* and *C*; *B* bears from her N. N. E. and *C*, E. S. E. From thence sailing E. by N. 5 miles, finds *B* and *C* equally distant from her, and continuing the same course 5 miles farther, she had the said islands in a right line; required the port's distance from the two islands *B*, *C*, and also the ship's distance from them at the second and third observations.

III. QUESTION 505, *by Mr.* J. Hudson, *Land Surveyor.*

There are two places in the same parallel of north latitude, differing in longitude 20°, and their distance on the parallel is known to exceed their distance on the arc of a great circle by 2·335 miles; required the latitude and nearest distance of the said two places, supposing the earth spherical, and 60 miles = a degree.

IV. QUESTION 506, *by* Geometricus.

In two similar right-angled triangles, there is given the base of the one, and the perpendicular of the other, in one sum; and it is proposed to determine the triangles such, that their hypothenuses shall form the legs of another triangle similar to them; and so, that the sum of their three areas may be of a given magnitude.

V. QUES-

V. QUESTION 507, *by Mr.* W. Spicer.

Given the area of a plane triangle = 235, and its vertical angle = 73° 34′, and the side of its inscribed square = 9.6 to describe the triangle.

VI. QUESTION 508, *by Mr.* T. Barker, *of* Westhall.

Supposing *A* and *B* to represent two right lines, whereof the sum is given = *s*, and $\frac{m}{n}$ to denote any given ratio. Required to determine, by a geometrical construction, what *A* and *B* themselves must be, so that $A^2 + \frac{m}{n} \times B^2$ may be $= cc$, a given square.

VII. QUESTION 509, *by Mr.* T. Moss.

A lays *B* one hundred guineas to 1, that in 25 throws with 5 half-pence, he does not throw, precisely, 4 heads the first 5 throws, 6 heads the next 5 throws, 8 the 3d, 10 the 4th, and 12 the last 5 throws: Required the respective values of these two gamesters expectations.

VIII. QUESTION 510, *by Mr.* Rob. Butler.

Suppose in latitude 52° north, a semi-circular plane, whose radius is 20 feet, to be placed in the plane of the meridian, with its base downwards; at what time in the afternoon, on 21st June, will its shadow cover just one acre on an horizontal plane?

IX. QUESTION 511, *by Mr.* Malachy Hitchins.

In latitude 50° north, there is a cylinder, whose internal diameter is = 6 feet, and length = 12 feet, lying on the horizontal plane, and making an angle with the meridian = 20° east; at what time of the day, on May 10, 1763, will the greatest part of its internal superficies be enlightened by the sun; and what ratio will that enlightened space bear to the whole internal superficies?

X. QUESTIION 512, *by Mr.* Joseph Walker.

Two right lines, drawn from the same point, being given both in length and position; 'tis required so to apply a right line, of a given length, between them, and subtending the

given angle, that the part of the one of them, next the said angle, may be to the alternate part of the other, in the given ratio of p to q.

XI. QUESTION 513, *by Mr.* T. Moss.

Let the ratio of the head and bung diameters be what it will, within certain limits, a spheroidal cask may be so formed, that the diagonal line, such as is now graduated upon gauging rods, will exhibit the true content of the cask: 'Tis proposed to find, by a general method, what those limits are, and how near the head and bung diameters can approach to the ratio of equality, before the above circumstance fails.

XII. QUESTION 514, *by Mr.* William Embleton.

Supposing a triangular prism, whereof the dimensions of the sides are 8, 10, and 12 inches, to be laid in still water; I would know in what position, or positions, it will rest, and what part of its surface will be immersed, supposing its specific gravity to be in proportion to that of the water, as 2 to 3.

XIII. QUESTION 515, *by Mr.* Abr. Botham.

Suppose a perfectly elastic ball, of 2 inches diameter, to be let fall upon the surface of an hemisphere whose diameter is 30 inches, from the height of 5 feet above the plane of the horizon or base of the hemisphere; to find at what distance from the axis the ball must descend, so that, after being reflected at the surface, it shall impinge upon the horizon at the greatest distance possible from the center of the hemisphere.

XIV. QUESTION 516, *by Miss* Ann Nicholls.

The latitude pray shew; the hour o'th' night * too tell,
When the three stars below † are in one parallel. ‡

* On Jan. 1, 1763. † Sirius, Cor Leonis, and Aliah. ‡ Of Altitude.

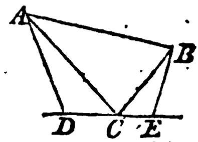
XV. QUESTION 517, *by Mr.* William Spencer.

Given $4\dot{x}^2 + 8y^2\dot{x}^2 + \dot{x}^2\dot{y}^2 + 4y^4\dot{x}^2 - x^3\dot{y}^2 = 0$, to find the value of x when $y = 2$, supposing that $x = 2$, when y vanishes or becomes $= 0$.

The

The PRIZE QUESTION, *by Mr.* T. Moss.

Through the angular point C, of a given plane triangle ABC, to draw, geometrically, a right line so, that two others AD, BE, being drawn, from the given points A, B to meet the same, and make given angles with the right line AB, whose sum is less than two right angles, their rectangle ($AD \times BE$) may be of a given magnitude.



1764.

Questions answered.

I. QUESTION 503 *answered by* Rusticus.

TWENTY your charmer's age appears to be—
Height sixty-five*, as near as I can see: * *Inches.*
Her fortune too will easily be found,
Nearly a hundred, thirty, and one pound.

The same answered by Mr. Edw. Griffiths, *Writing-master, in* Ellesmere, Shropshire.

Put $281 \cdot 5294 = a$; $170988 \cdot 22 = b$: and then, by transposition, involution, &c. will be found $aa - 2ax + xx = 4yy + zz + 4yz = xx + 4yz$ (per equality, &c.): Whence $aa - 2ax = 4yz$, and consequently $aax - 2axx = 4xyz = 4b$ (per quest.); from the resolution of which equation, x comes out $= 131 \cdot 5294$l. and from thence her age is readily found $= 20$ years, and her height $= 65$ inches.——In the same manner the answer is given by Mr. *J. Hampson*, who thinks it a very great pity that so tall a beauty should have no greater fortune.

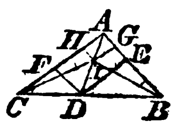
This question is likewise truly and concisely answered by Mess. *Tho. Baker*, *Tho. Barker*, *Geo. Bland*, *T. Bromhall* (both of Mr. Allen's school), *J. Eadon*, *Rob. Forster*, *Rich. Gibbons*, *Steph. Hodges*, *J. Potter*, *T. Sadler*, (the proposer),

Tho.

Tho. Sanderson, Wm. Sewell, Paul Sharp, Wm. Spicer, J. Stokes, Eleanor Suggett, J. Swan, Isaac Tarratt, Francis Taylor, J. Walker, Tho. Walker. J. Wood, Ja. Young, and others.

II. QUESTION 504 *answered by Mr.* T. Bromhall, *at Mr.* Allen's, *at* Spalding.

Let A represent the port; B, C, the two islands; AD the ship's course; and I, D, her places at the second and third observations: and let the points A, C; A, B; B, C; C, I, and B, I, be joined, and draw DF and $IH \parallel BA$, and DE and $IG \parallel CA$ respectively; and then, by trigonometry will be readily found $DF\ (2IH) = 5{\cdot}55 = b$, and $DE\ (= 2IG) = 8{\cdot}316 = c$.



Put AD $(= 10$ miles$) = a$, and $CF = x$; and then, per similar triangles, $x : b :: c : \frac{bc}{x} = EB$: Whence $CH = x + \frac{1}{2}c$, $BG = \frac{bc}{x} + \frac{1}{2}b$, and consequently (47 Euc. 1) $xx + cx + \frac{1}{4}cc + \frac{1}{4}bb = \frac{bbcc}{xx} + \frac{bbc}{x} + \frac{1}{4}bb + \frac{1}{4}cc$ $(= CI^2 = BI^2$, per quest.) i. e. $xx + cx = \frac{bbcc}{xx} + \frac{bbc}{x}$: Whence, $\overline{x^3 - bbc} \times \overline{c + x}$ $(= x^4 + cx^3 - bbcx - bbcc) = 0$, and consequently $x^3 - bbc = 0$, or $x^3 = bbc$, and $x = \sqrt[3]{bbc} = 6{\cdot}35488$; and hence will be found $AC = 14{\cdot}6695$, $AB = 12{\cdot}8247$, $IC\ (= IB) = 10{\cdot}8730$, $DC = 8{\cdot}2409$, and $DB = 11{\cdot}0441$ miles respectively.

In this manner nearly, the answer is given by Messrs. *Blennerhassett* (the proposer), *Tho. Bramley, John Cross, J. Eadon, Rich. Gibbons, J. Hampson, Steph. Hodges, Steph. Ogle, Tho. Sanderson, P. Sharp, Wm. Spicer, Eleanor Suggett, Tho. Walker, Ja. Young*, and others.

Mr. *Wm. Embleton* constructs and solves it by means of the semi-cubical parabola only, to which it evidently appertains; and *Sangrado*, jun. constructs and solves it generally, viz. when AI and ID are in any given ratio whatever, by means of the intersection of a circle with a hyperbola described, to the asymptotes AC, AB, through the given point D, &c.

From

From the information received from ſeveral correſpondents, it appears that this queſtion, after the copy of the laſt year's Ladies' Diary was delivered, has been propoſed and anſwered in ſeveral of the magazines; an affair the more ſurprizing, as we cannot prevail on ourſelves to think that the ingenious propoſer would act ſo diſhonourably, as to ſend any thing he had devoted to the ſervice of the Ladies' Diary to another place.

III. QUESTION 505 *anſwered by Mr.* Wm. Spicer.

Put $a = 600$ ($= 10° \times 60 = \frac{1}{2}$ the given difference of longitude in geographical miles), $b =$ nat. ſine of $10°$, $2c = 2{\cdot}335$ (the given exceſs of the diſtance on the parallel above that on the arc of a great circle), and $x =$ the coſine of the latitude required (rad. $= 1$); and then ax will $= \frac{1}{2}$ the diſtance of the two places on the parallel, and $bx =$ the ſine of $\frac{1}{2}$ their diſtance on the arc of a great circle: Whence, putting $v =$ the degrees in the ſaid arc of a great circle, we ſhall have $60v = ax - c$, per queſt. and conſequently $x = \frac{60v + c}{a} =$ ſine of $32° \ 43' \ 25''$; and hence the latitude appears to be about $57° \ 16' \ 35''$, and the neareſt diſtance of the two places $646{\cdot}3635$ geographical miles.

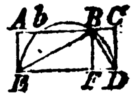
Mr. *J. Hampſon* and Mr. *C. Hutton* put $r = 3437{\cdot}64$ ($=$ number of geographical miles in the earth's radius); $y =$ ſine of $\frac{1}{2}$ the required diſtance of the two places on the arc of a great circle (rad. 1); $\frac{a}{b} - r = d$; all the reſt as above; and then, by a very eaſy and ſhort proceſs, find $dy - \frac{ry^3}{2.3} - \frac{3.3ry^5}{2.3.4.5} - \frac{3.3.5.5ry^7}{2.3.4.5.6.7} -$ &c. $= c$; from which, by reverting the ſeries, y comes out $= {\cdot}09255$; and from thence they find every thing elſe nearly the ſame as above.

Mr. *Rich. Gibbons*, Mr. *Paul Sharp*, Mr. *Joſ. Webſter*, jun. and ſome others, have alſo anſwered this queſtion.

IV. QUES-

IV. Question 506 *answered by Mr.* Rich. Gibbons.

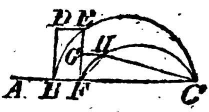
Upon ED, the given sum of the base and perpendicular, constitute the rectangle $EC =$ the sum of the three given areas, and also about the same, as a diameter, conceive a semicircle to be described, cutting AC in B; join the points E, B, and D, B; and EAB, BCD, and DBE, will be the three triangles required; which is too evident to need a demonstration.



In this manner exactly it is constructed by Mr. *Rob. Butler*, Mr. *Wm. Embleton*, Mr. *Tho. Sanderson*, *Sangrado*, jun. and others; and Mess. *T. Barker*, *Geo. Bland*, *T. Bromball*, *J. Eaden*, *J. Hampson*, *B. Robinson*, *Paul Sharp*, *W. Spicer*, *Eleanor Suggett*, *Tho. Walker*, *W. C. Mathematicus*, &c. have sent algebraic solutions to it.

V. Question 507 *answered by Mr.* C. Hutton.

Construction. Produce AB, the side of the given inscribed square, to C, so that $AB \times BC$ may be $=$ twice the given area of the triangle, and take BD, $\perp AC$ at B, a mean proportional between AB and BC, and on BC conceive a semicircle BEC to be described, and draw $DE \parallel AC$ meeting the same in E, from whence let EF be drawn $\parallel DB$ meeting AC in F: Then, if on FC a segment FHC of a circle, to contain the given vertical angle, be described, and FG be taken $= FB$, and GH drawn $\parallel FC$, meeting the arc FHC in H, and the points H, F, and H, C, be joined, FHC will be the triangle required.



Demonstration. The $\angle FHC =$ the given angle, per construct. and twice the area of the $\triangle FHC = FC \times FG =$ (by construct.) $FC \times FB =$ (per prop. of the cir.) $FE^2 = BD^2 = AB \times BC =$ twice the given area, by construct. And (by prob. 3d part 2d Simp. Select Exercises) $\frac{AB \times BC}{FC + BF} = \frac{AB \times BC}{BC} = AB$, the side of the given inscribed square, by construction. Q.E.D.

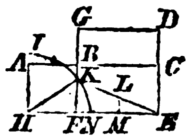
Note, When GH neither cuts nor touches the segment FHC, this quest. is, manifestly, impossible.

Mr.

Mr. *Wm. Embleton*, Mr. *Steph. Ogle*, and *Sangrado*, jun. construct it nearly in the same manner.——Mess. *T. Barker*, *T. Bromhall*, *Geo. Bland*, *J. Hampson*, *Paul Sharp*, &c. have however pointed out ingenious methods whereby it may be solved algebraically.

VI. QUESTION 508 *answered by Mr.* Wm. Embleton.

CONSTRUCTION. Upon $AC = s$, the given sum of the two right lines, constitute the rectangle $AE = cc$, and take $AB = AH$, and then AB, BC will be the measures of A and B respectively, and in the given ratio of m to n For, drawing $BF \parallel AH$, and producing it and EC till they become each $= FE$ (BC), and joining their extremes G, D, then, supposing $AB : BC :: m : n$, it will be $m : n :: AB \times BC (= BF \times BC = BE) : BC^2$: Whence $BE = \frac{m}{n} \times BC^2$; to which equal magnitudes, the equal magnitudes AF, AB^2, being respectively added, it will become $AB^2 + \frac{m}{n} \times BC^2 = AF + BE = AE = cc$, per construction, &c.



NOTE, This question will be found useful in the solution of some difficult problems relating to triangles.

The same answered by Mr. Da. Kinnebrook.

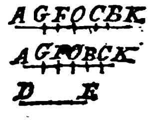
CONSTRUCTION. Take $HE = s$, the given sum of the two right lines, and from the center H, with radius c, suppose the circular arc IN, cutting HE in N, to be described, and draw LM perpendicular any where upon NE, and so as that LM^2 be to $ME^2 :: m : n$, and let the points E, L, be joined by a right line produced, till it meets the said arc in K; from whence draw $KF \perp HE$, and HF, FE will be the measures of A and B required.——For, joining the points H, K, $n : m :: FE^2 : FK^2 = \frac{m}{n} \times FE^2$ (by construction and sim. triangles, EML, EFK), and (by 47 E. 1) $HF^2 + \frac{m}{n} \times FE^2 (= FK^2) = cc (= HK^2$, per construction).

Mr. *Rob. Butler* and Mr. *Tho. Sanderson* also construct it exactly according to this method.

A more

A more general Answer to the same, by P. M. *of* Durham.

Suppose in a right line the two points A, B were given, and it was required to determine another point C therein, such that $m \times AC^2 + n \times BC^2$ might be $= p \times DE^2$ (DE being another given right line.



Bisect AB in O, and divide it in F in the given ratio of m to n; take OK, so that $m+n$ may be to p in the duplicate ratio of DE to OK, and, making FG (towards opposite parts of F) $= FO$, take GC, so that the rectangle under it and CO may be equal to the given rectangle under AK and KB (Simp. Geom. Pr. 6), and C shall be the point required.

DEMONSTRATION. For, since AB is bisected in O, $\square AKB + AO^2 = OK^2$ (6 Euc. 2) and $\square GCO = \square GOC + OC^2$ (3 Euc. 2) $= 2\square FOC + OC^2$, because $GO = 2FO$ (hyp.). But $\square GCO = AKB$ (hyp.); therefore $2\square FOC + OC^2 + AO^2 = OK^2$, and, multiplying both sides by $\overline{m+n}$, $\overline{m+n} \times 2\square FOC + \overline{m+n} \times \overline{OC^2 + AO^2} = \overline{m+n} \times OK^2 = p \times DE^2$ (hyp.). Again, because AB is divided in F in the ratio of m to n, and bisected in O, therefore $m+n : m-n :: (AO : FO ::) \square AOC : \square FOC$; and consequently $\overline{m+n} \times \square FOC = \overline{m-n} \times \square AOC$: Whence ($\overline{m-n} \times 2\square AOC + \overline{m+n} \times \overline{AO^2 + OC^2} = m \times \overline{AO+OC}^2 + n \times \overline{AO-OC}^2$ $=$) $m \times AC^2 + n \times BC^2 = p \times DE^2$.

COROLLARY 1. As the point C falls within or without the terms (or points) A, B; the right line AB will be the sum or difference of the right lines sought.

COROLLARY 2. The numerical solution is had from this equation $\overline{m-n} \times 2\square AOC + \overline{m+n} \times \overline{AO^2 + OC^2} = p \times DE^2$, where p is universal, but in the question is limited to the value n.

Neat geometrical constructions to this question have also been received from Mr. *T. Barker* (the proposer), *Sangrado*, jun. Mr. *W. Spicer*, and others.

VII. QUES-

VII. QUESTION 509 *answered by Mr.* W. Spencer.

The throwing of m heads, precisely, in n throws with p halfpence, being evidently the same as throwing m heads precisely in $n \times p$ throws with one half-penny only, it follows, from corol. to prob. 5 on page 12, &c. of Simp. Nat. and Laws of Chance, that, if the 5th, 7th, 9th, 11th, and 13th terms of the binomial $1+1$ raised to the 25th power, be each divided by $\overline{1+1}|^{25}$, they will be the respective probabilities required, *i. e.* putting $\frac{25}{1} \times \frac{24}{2} \times \frac{23}{3} \times \frac{22}{4} \times \frac{1}{\overline{1+1}|^{25}} = p$; then p, $p \times \frac{21}{5} \times \frac{20}{6}$, $p \times \frac{21}{5} \times \frac{20}{6} \times \frac{19}{7} \times \frac{18}{8}$, $p \times \frac{21}{5} \times \frac{20}{6} \times \frac{19}{7} \times \frac{18}{8} \times \frac{17}{9} \times \frac{16}{10}$, and $p \times \frac{21}{5} \times \frac{20}{6} \times \frac{19}{7} \times \frac{18}{8} \times \frac{17}{9} \times \frac{16}{10} \times \frac{15}{11} \times \frac{14}{12}$ will be the respective probabilities of throwing the number of heads as specified in the question, considering them as independent one of another: and therefore the probability of their all happening together, according to some order or other, will, by corol. to prob. 1st of Simp. Laws of Chance aforesaid, be expressed by $p^5 \times \frac{21^4}{5^4} \times \frac{20^4}{6^4} \times \frac{19^3}{7^3} \times \frac{18^3}{8^3} \times \frac{17^2}{9^2} \times \frac{16^2}{10^2} \times \frac{15}{11} \times \frac{14}{12}$; but there being $1 \times 2 \times 3 \times 4 \times 5$ ($=120$) permutations, or different orders or ways in which 5 such events may all happen together, the probability of their happening in the precise order required in the question will therefore be $\frac{p^5 \times 21^4 . 20^4 . 19^3 . 18^3 . 17^2 . 16^2 . 15 . 14}{120 \times 5^4 . 6^4 . 7^3 . 8^3 . 9^2 . 10^2 . 11 . 12}$. From whence the values of A and B's expectations may readily be found, &c.

Much after the same manner the answer is given by Mr. *Wm. Embleton*, Mr. *C. Hutton*, Mr. *Moss* (the proposer) *Sangrado*, jun. and some others.

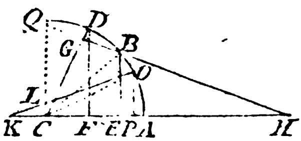
VIII. QUESTION 510 *answered by* P. M. *of* Durham.

It may be easily shewn, in the projection of any regular figure, whether right-lined or a conic section, that the area of the plane of the figure is to the area of its projection, as the height of the plane above the horizon multiplied by

 radius,

radius, is to the shadow of the said height multiplied by the sine of the angle, which the said shadow makes with the meridian; but this angle is the sun's azimuth; and the area of the plane, its projection, and also its height, are given: And therefore, calling its height $=a$, its shadow $=y$, the given ratio of the plane's surface to its projection as m to n, and, likewise, denoting the cosines of the given latitude and declination by c and q, the sine and versed sine of the hour angle sought by x and v, and the cosine of the difference between the lat. and declin. by d, we shall have $m : n :: ar$ (r being rad.) : $y \times$ s. azim. but s. alt. : cos. alt. :: $a : y$, and cos. alt. : $x :: q$: s. azim. Whence s. alt. : $x :: qa : y \times$ s. azim. and consequently $rr \times$ s. alt. : $qrx ::$ ($ar : y \times$ s. azim. ::) $m : n$; but (by spherics) $rr \times$ s. alt. $= rrd - cqv$, and therefore $rrd - cqv : qrx :: m : n$, i. e. $\frac{rrd}{cq} - v : x :: \frac{mr}{c} : n$; whence this easy

GEOM. CONSTRUC. Let AB, AD, represent the complements of the latitude and declination to the rad. AC ($=r$), and then $BE = c$, $DF = q$), and $CG = d$; produce GB till it cuts the rad. CA produced in H, and take AK a fourth proportional to EH, CH, and AC (12 Euc. 6).



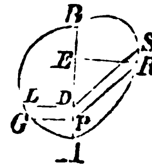
Assume KC, CL in the given ratio of $\frac{mr}{c} : n$, and draw $CL \perp KC$ at the point C, and through the points K, L draw a right line, cutting the circle in O, and AO will be the measure of the hour angle sought.

For, producing HG, CL, to meet in Q, and drawing OP, the sine of the arc AO; then, by similar triangles, $DF : DC :: CG : CQ$, and $DF \times BE : \square DCG ::$ ($BE : CQ :: EH : CH ::$) $AC : AK = \frac{rrd}{cq}$; but KP ($AK - AP$) : $OP ::$ ($KC : CL ::$) $\frac{mr}{c} : n$, i. e. because $AK = \frac{rrd}{cq}$, and AP is the versed sine of the arc, of which OP is the sine; therefore $OP = x$, and $AP = v$.

The

The same answered otherwise by Mr. Robert Butler, *(the Proposer).*

Suppose $ARSB$ represents the required shadow; $s =$ area of the given semicircle AGB; a and $b =$ sine and cosine of the lat. x and $y =$ those of the sun's alt. z and $u =$ those of his azim. from the north; $d =$ sine of his decl. (rad. $= 1$) and $A = 43560$, the feet in an acre: Then, GP being perpendicular to the horizontal plane $ARBA$, $\frac{y}{x} \times PG = PR$ will be the length of its shadow; and, drawing DL, DS, indefinitely near, and $\parallel PG$, PR, and meeting the curves in L and S, respectively, and $RE \perp AB$, then will $PG \times PD$, and $PD \times ER$, be respectively $=$ the areas of the parallelograms PL and PS: but z being the sine of EPR, RE is $= z \times PR = \frac{zy}{x} \times PG$, and therefore $PD \times ER = \frac{zy}{x} \times PG \times PD$, i. e. the area of the parallelogram PL drawn into $\frac{zy}{x} =$ area of the parallelogram PS, and consequently $s \times \frac{zy}{x} = A$, let the curve ARB be what it will; but, by sperics, &c. z is found $= \frac{\sqrt{bbyy - \overline{d - ax}^2}}{by}$, which substituted above, gives $\frac{s}{bx} \times \sqrt{bbyy - \overline{d - ax}^2} = A$: From whence x is found $=$ ·0111641, and consequently the sun's azim. from the west $= 32^\circ\ 19'$, and the time required $= 9'\ 16''$ past 8 o'clock.—— No regard is here had to the sun's semidiameter and refraction; but the answer in such case will be had, by making $d =$ sine of $19'$ more than what it here is.



IX. QUESTION 511 *answered.*

Φιλομαθεμαχἱκος, and some others, (answering this question) make the time sought to be about 12′ past 11 in the morning, and find the illuminated part $= \frac{1}{9}$ the whole internal superficies nearly; but *Philotechnus* finds the illuminated internal surface to be always $=$ a rectangle under

the fine and fecant of the angle comprehended between the folar ray, iffuing from the center of the fun, and the diameter of the given cylinder, paffing through that point of the circumference of its end, whereon the faid ray impinges, to radius = the diameter of the faid cylinder; from which (as it depends wholly on the fun's altitude and azimuth conjunctly) being fluxed, put = 0, and reduced, the time required, &c. may be determined.*

X. QUES-

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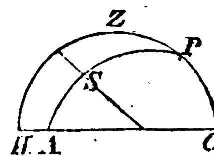
* IX. QUESTION 511.

As the anfwer to this queftion is partly falfe and partly incomplete, I fhall fupply the whole folution as below.

I. To find the Time of the greateft Illumination.

CONSTRUCTION. On the plane of the meridian of the place defcribe the primative circle, and therein take Z the zenith and P the pole; draw the horizon HO, on which take $HA =$ 20° the given horizontal angle made by the meridian and the axe of the cylinder; and through A draw the meridian ASP. So fhall APZ be the hour angle from noon required when the moft furface is illuminated.

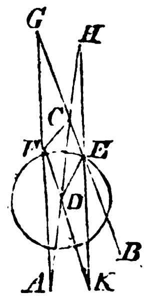


DEMONSTR. It is evident that the curve bounding the illuminated part of the internal furface, is always the fhadow of that half or femicircumference of the end of the cylinder which is turned towards the fun, that is, that femicircumference whofe diam. is perpendicular to the plane drawn through the fun and the axe of the cylinder. It is alfo evident that the moft furface will be illuminated when the folar ray penetrates to the fartheft diftance poffible within the cylinder; and this, it is farther evident, will happen when the ray makes the leaft poffible angle with the axe of the cylinder; and this angle being always meafured by the arc of a great circle paffing through the fun and that point A of the horizon to which the axe of the cylinder is directed, this arc will therefore be a min. that is, the arc intercepted by A and the parallel of declination for the given day, is a minimum; in which cafe it is evident that the arc muft be perpendicular to the parallel of declination, and will therefore be a meridian paffing through A, as by the Conftruction.

CALCULATION. In the right-angled triangle AHP, it is as the s. of HP (130°) : radius : : tang. HA (20°) : tang. hour $\angle P$ =

X. QUESTION 512 *answered by Mr.* T. Moss.

CONSTRUCTION. Suppose $DH =$ one of the given lines comprehending the given angle, and from the center D with radius DE ($=$ the given line to be applied) conceive the circumference of a circle to be described, and make the $\angle HDK =$ the supplement of the given angle to 180°, taking $DK : DH :: p : q$, viz. the given ratio, and joining the points H, K, by a right line cutting the circumference of the said circle in E: From whence drawing $EC \parallel DK$, meeting DH in C, and producing DH till $DA = CH$, and CE, till CB becomes $=$ the other given comprehending line, and then, joining the points D, E, the thing is done.——For, by sim. triangles, $DK : DH :: CE : CH$; and, per construction, $DK : DH :: p : q$; whence, per equality, $CE : DA (= CH$, per construction$) :: p : q$; and the $\angle BCA = \angle ADK$ (the given angle), and the right lines CA, CB, and DE are $=$ the given lines, by construction.



A

$= \frac{\text{tan. } HA}{\text{sin. } HP} = \frac{\text{t. } HA}{\text{s. lat.}} =$ tang. of 25° 21′ 47″, which answers to 1 h. 41 m. 39 s. and which being taken from 12 hours, there remain 10 h. 18 m. 21 s. or nearly 18 minutes past 10 in the morning is the time of the greatest illumination. And as radius : cos. HA :: cos. HP : cos. $PA = 127°\ 50'\ 30''$; from which take PS ($= 72°\ 23'$) the sun's polar distance, and the remainder $55°\ 27'\ 30''$ is $=$ AS the measure of the least angle formed by the solar ray and the axe of the cylinder.

SCHOLIUM. Hence we may observe that the time is the same whatever the declination is; and that the time given in the original answer is wrong.——It may also be remarked, that if the reasoning in the above, or in what follows, should not be clear to any person, he may assist his imagination by rolling up a rectangular piece of paper into the form of a cylinder, and exposing it to the solar rays, or to the light of a candle.

A CONSTRUCTION *to the same by Mr.* Da. Kinnebrook, *and* P. M. *of* Durham.

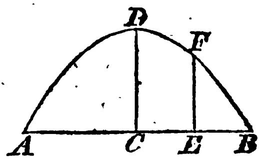
Produce one of the given, comprehending lines BC till CG is to the other of them (AC) in the given ratio of p to q, and let the points A, G, be joined; and about the center C with radius CF (= the given line to be applied) conceive a circular arc to be described cutting AG in F; from whence draw $FD \parallel CG$ meeting AC in D, and, joining the points F, C, draw $DE \parallel$ thereto, and that will be the position of the line required: For, by sim. triangles, FD (= CE, per construction) : DA : : (CG : CA : :) p : q, per constr. and, DC being a parallelogram (per constr.), DE will be = CF = the given line to be applied (per construction).

Mess. *Rob. Butler*, *Wm. Embleton*, *Sangrado*, jun. and some others, have also constructed it in an elegant and concise manner; and Mess. *T. Bromhall*, *J. Hampson*, *Tho. Sanderson*, *P. Sharp*, &c. have given curious algebraic solutions to it.

XI. QUES-

II. *To find the Quantity of surface illuminated.*

Conceive the cylinder to be cut thro' on one side in the direction of its axe, and to be spread flat out on a plane; then will the illuminated part appear as in the annexed figure, where AB is the semicircumference as stretched out in a right line; CB is a quadrant; CD, perpendicular to AB, is the projection of the diameter of the end; or it might be considered as the shadow of the diameter; and, in like manner, every other perpendicular EF is the projection or shadow of double the sine of its corresponding arc BE.



Put now t for the cotang. of the angle formed by CD or by the axe of the cylinder and the solar ray, or the cotang. of the arc AS ($55° \ 27' \ 30''$) in the first figure to the radius 1; z = the arc CE, and x = its sine to the radius r (3) of the cylinder. Then, by plane trigonometry, $2tr = CD$, $\sqrt{rr - xx}$ = s. BE (or cos. CE), and $2t\sqrt{rr - xx} = EF$. But the fluxion of $CDFE$ is $= \dot{z} \times EF$, and $\dot{z}$ is $= \frac{r\dot{x}}{\sqrt{rr - xx}}$; hence $\dot{z} \times EF$ is $= \frac{r\dot{x}}{\sqrt{rr - xx}}$

$\times 2t$

XI. QUESTION 513 *answered by* Mr. T. Moss, (*the Proposer*).

Let $d =$ the given diagonal of the spheroidal cask required, $x =$ the bung diameter, the ratio of the head and bung as n to 1, and $p = \cdot 7854$; and then, by the well-known theorem, its content will be $\frac{2p}{3}\sqrt{dd - \overline{\frac{1+n}{2}}\big|^2 xx} \times x \times \overline{2xx + nnxx}$: Which, put into fluxions (supposing n constant) and made $= 0$, and reduced, gives $x = \frac{2d\sqrt{2}}{1+n\sqrt{3}} =$ the bung diameter, in this circumstance, when the cask, under the given diagonal d, is the greatest possible; and this value, substituted for x in the above expression, gives $\frac{16pd^3}{9\sqrt{3}} \times \frac{2+nn}{\overline{1+n}|^2} =$ the content of the greatest spheroidal

cask

$\times 2t\sqrt{rr - xx} = 2tr\dot{x}$; the fluent of which is $2trx =$ the area $CDFE$. And when E arrives at B, then x is $= r$, and the area CDB is $2trr$, the double of which is $ADB = 4.rr = tdd$, putting d for the diameter of the cylinder; which, since d is $= 6$, will be $36t = 24\cdot 78068$, the illuminated surface required.

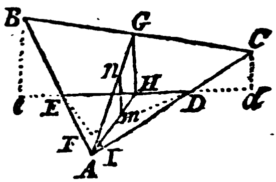
Now, the length of the cylinder being double its diameter, its whole internal surface will be $2pdd$, putting $p = 3\cdot 1416$; and therefore the whole is to the illuminated part, as $2p$ to t, that is, as $6\cdot 2832$ to $\cdot 68835$, or as $9\cdot 12788$ to 1.

COROLLARY. From the above it appears that the projection or shadow ($2\,CDFE = 4trx$) of a middle zone of the circular end of the cylinder, *upon its internal surface*, is always equal to the shadow or projection, *on the plane of the horizon*, of the rectangle under the diameter and distance between the two rectilinear ends of the zone; and that the projection of the whole circular end, on the internal surface, (that is, the whole illuminated part) is equal to the projection, on the horizontal plane, of a square whose side is equal to the diameter of the cylinder ——Or, it farther appears, that the projection of the middle zone, is to the rectangle under the sine and secant of the complement of the angle made by the solar ray and the axe of the cylinder, to a radius equal to the diameter of the cylinder, as the sine of the arc CE is to the radius. And, also, that the whole illuminated part is equal to a rectangle under the sine and secant of the same angle, to the same radius equal to the diameter of the cylinder: which last is also remarked by *Philotechnus* in the original solution.

cask having the given diagonal d, and corresponding to any assignable possible value of n: And when it is less than $d^3 \times \cdot 6283$, the content, as found by the diagonal line, the problem is then manifestly impossible; suppose it therefore equal thereto, in order to determine the limit of n (required), *i. e.* suppose $\frac{16pd^3}{9\sqrt{3}} \times \frac{2+nn}{1+n^2} = d^3 \times \cdot 6283$, or $\frac{2+nn}{1+n^2} = \cdot 7796$; whence n is readily found $= \cdot 898$, &c.——And hence it appears that the present constructed diagonal line, will not exhibit the true content of a spheroidal cask, when the ratio of the head and bung diameters approaches nearer to an equality than that of ·898 to 1 (or 9 to 10, nearly), in any circumstance whatever.

XII. QUESTION 514 *answered by Mr.* Wm. Embleton.*

Suppose ABC represents a section of the given prism parallel to its ends, AED the immersed part thereof, AG, AH, right lines drawn from the angular point A, to the middle points of BC and ED, respectively: Then, if the centers of gravity n, m of the triangles ABC, AED (which are known to be $\frac{1}{3}$ of AG and AH from G and H, respectively) be joined by the right line nm, the prism will, by the laws of hydrostatics (see prop. 87 sect. 9 of Emer. Mechan.) be quiescent, whenever that line becomes perpendicular to the hori-



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* XII. QUESTION 514.

In order to bring this problem to a final solution; On DE, produced both ways, demit the perpendiculars Cd, Be, which will therefore be parallel to GH; also, on AB and AC, the perpendiculars DF and EI.

Since then GB is $= GC$, and Be, GH, Cd are parallels, He will be $= Hd$; but $HE = HD$, $\because$ $Ee = Dd$. Again, since the vertical angles eEB and FED are equal, as also dDC and IDE, the two triangles BeE and DFE are similar, as also the two CdD and EID; hence $\left\{\begin{array}{l} EB : Ee :: ED : EF \\ DC : Dd :: DE : DI \end{array}\right\}$, and as the two means are the same in both, therefore $EB : DC :: DI : EF$.

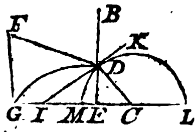
Put

horizontal line ED: But GH will then also be perpendicular to the same (by cor. to theor. 12 B. 4 Simp. Geom. 2d edit.) and so the question be reduced to that of dividing the given triangle ABC into two given parts by the right line ED, so that GH, connecting the middle points of BC, and ED, may be perpendicular to the latter of them; which is a problem purely geometrical; and from whence an answer in numbers may be readily derived.——This problem may be of use in determining the structure of a ship, so as to be least subject to roll, &c.

Mr. *J. Moreland*'s answer is the same as the above, very near.

XIII. QUESTION 515 *answered by Mr.* Da. Kinnebrook.

Suppose the ball to fall from B, on the point D, of the given hemisphere MDL, and, after reflection, to describe the curve DG, meeting the horizon in the point G; and draw DF, a tangent to the same at D: Suppose, also, FG, DE, to be perpendicular to the horizon, and KDI to be a tangent to the hemisphere at D,



and put DC ($= 1\frac{1}{4}$ foot) $= r$, $16\frac{1}{12}$ feet $= s$, $DE = x$; and then the direction of the ball DF, after reflection, being such as that the $\angle BDK = \angle FDI$ (by the nature of reflection), and its velocity therein $= 2\sqrt{4rs - sx}$ (as both ball and hemisphere are perfectly elastic), we shall, by the laws of falling bodies, trigonometry, &c. find $GC =$ $\frac{8x \times \overline{4r - x} \times \overline{2xx - rr}}{r^4} + 1 \times \sqrt{rr - xx}$, a max. (per quest.)

Put, now, $b = AB$, $c = AC$, m and $n =$ sine and cos. $\angle A$, $x = AE$, and $y = AD$. Then $nx = AI$, and $ny = AF$; hence $EF = x - ny$, and $DI = y - nx$; also $BE = b - x$, and $CD = c - y$; therefore the last proportion becomes $b - x : c - y :: y - nx : x - ny$, which produces the equation $x^2 - bx + bny = y^2 - cy + cnx$. But the $\triangle AED = \frac{1}{2}mxy$ is $= a$, a given magnitude, by the quest. and hydrostatics; therefore y is $= \frac{2a}{mx}$; which being substituted in the equation, it becomes at last $x^4 - \overline{b + cn} . x^3 + \overline{c + bn} . \frac{2ax}{m^2} - \frac{4a^2}{m} = 0$. From whence the value of x may easily be found; and thence that of y.

queſt.) from which, thrown into fluxions, and reduced, &c. x comes out $= \cdot 4646$ &c. and hence every thing elſe required may be readily found.

Much after the ſame manner the anſwer is given by Mr. *C. Hutton*, Mr. *J. Moreland*, and ſome others.

XIV. QUESTION 516 *anſwered by Mr.* T. Allen, *of* Spalding.

Put x and y for the ſine and coſine of the co-latitude required, v and z for thoſe of the arc of time that Sirius is then ſhort of the meridian. Alſo, let a and b, c and d, e and f, be the reſpective ſines and coſines of the given polar diſtances of the three given ſtars, Sirius, Cor Leonis, and Aliah; and m and n, p and $q =$ the ſines and coſines of the difference in right aſcenſion betwixt Sirius and the other two of the ſaid ſtars, reſpectively, rad. $= 1$; and then, by ſpherics, the coſines of their zenith diſtances will be found equal to $axz - by$, $cx \times \overline{zn - vm} + dy$, and $fy - eqxz + epxv$, reſpectively; which are all equal, by the queſt. From the firſt and ſecond, y is found $= x \times \frac{az + cmv - cnz}{b + d}$, which, ſubſtituted in the third, and ordered, gives $\frac{z}{v} =$ $\frac{cm . \overline{b + f} + ep . \overline{b + d}}{a . \overline{d - f} + cn . \overline{b + f} + eq . \overline{b + d}} =$ tang. of the arc Sirius is ſhort of the meridian at the time required; from whence the hour of the night, and latitude of the place, will be eaſily determined.

XV. QUESTION 517 *anſwered by Mr.* T. Allen, *and* P. M. *of* Durham.

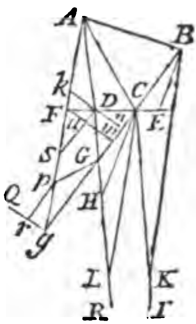
From the given equation is readily derived $\frac{\dot{x}}{x\sqrt{x - 1}} =$ $\frac{1}{2} \times \frac{\dot{y}}{1 + yy}$; whence, putting $\left\{ \begin{matrix} a \\ b \end{matrix} \right\} =$ circular arc, whoſe radius is unity and $\left\{ \begin{matrix} \text{ſec. } \sqrt{x} \\ \text{tang. } y \end{matrix} \right\}$, and $c =$ the arc of $45°$, whoſe ſec. $= \sqrt{2}$, the equation exhibiting the relation of the fluents will be found $2a = \frac{1}{2}b$: Which, corrected, according to the conditions ſpecified in the problem, becomes $2a = \frac{1}{2}b + 2c$; and hence, by a table of natural or logarithmic ſines, &c. the value of a will readily be found $= 1 \cdot 06218$, when $y = 2$; and thence $x = 4 \cdot 21683$, &c. which was required.

In

In this manner, nearly, the anſwer is alſo given by Meſſ. *R. Butler, Wm. Embleton, C. Hutton, Da. Kinnebrook, Wm. Spencer* (the propoſer), and ſome others.

The PRIZE QUESTION *anſwered by Mr.* T. Moſs, (*the Propoſer*).

Imagine DCE to be the poſition of the line required, and produce BC (ſee fig. to queſt.) to meet $Ag \parallel BE$ in g, and then, $AD \times BE$ being a given quantity (per queſt.), it is evident, from ſim. triangles BCE, gCF, that $AD \times Fg$ is a known quantity alſo, it being to the given $\square\, AD \times BE :: Fg : BE$. Moreover, Dk being $\perp Ag$, it is very evident that $Fg \times AD : Fg \times Dk$ in the known ratio of $AD : Dk$; whence it follows that $Fg \times Dk$, or twice the meaſure of the $\triangle\, DFg$, becomes known. Now, ſuppoſe Fm and $Dn \perp Cg$, and produce AD to meet BC produced in G, and then it is manifeſt that $Cg \times \overline{Fm - Dn} =$ twice the meaſure of the $\triangle\, DFg (= Fg \times Dk)$; but Cg is known, and conſequently $Fm - Dn$ is known alſo: Hence the problem is reduced to that of drawing a right line from the given point C to cut the right lines Ag, AG, given in poſition, ſo that the difference of the perpendiculars Fm, Dn, falling from the points of interſection F, D, upon Bg, may be $=$ a given right line, *i.e.* equal to twice the given meaſure of the $\triangle\, DFG$ divided by the given line Cg; of which the following is the



CONSTRUC. In $gQ \perp gB$, take $gr =$ the given difference of the perpendiculars Fm, Dn, and draw $rp \parallel gB$ meeting Ag in p; join p, G, and draw (by prob. 37 p. 242 of Simp. Geom. 2d edit.), from the given point C, a right line cutting Ag, AG, ſo that the ſegments Fp, DG, cut off thereby, may be to each other in the given ratio of Ag to AG, and the thing is done.

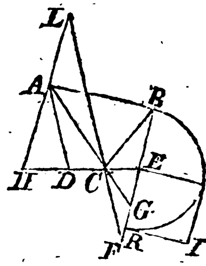
DEMONSTRA. Draw $Ds \parallel Cg$ cutting Fm in u; then (by conſt.) $Ag : AG :: Fp : DG$, and, by ſim. triangles, and diviſion of ratios, $Ag : AG :: sg : DG$; whence, by equality, $Fp = sg$, and, taking sp from each, $gp = Fs$, and conſequently $Fu = gr =$ the given difference, by conſtruc. ——This method contains the ſolution of another problem of equal, if not ſuperior, difficulty.

Mr

Mr. *Da. Kinnebrook* draws CH and $CI \parallel BE$ and AD respectively, and produces AD to meet CH in H, and takes CK in CI, so that $CH \times CK$ may be $=$ the given $\square AD \times BE$, and, joining B, K, draws $CL \parallel$ thereto, meeting AH produced in L, and then, continuing BC till it meets AL in G, divides AL in D, so that $AD \times DL$ may be $= AH \times GL$; from which point D, through C, drawing the right line DCE, and the thing is done.——The demonstration whereof is extremely natural and easy.

The same answered by Mr. Wm. Embleton, *and* *Mr.* C. Hutton.

CONSTRUC. Through the given point C, draw $CL \parallel DA$, meeting BE produced in F; and $AL \parallel BE$, in L; produce AC to meet BF in G, and take GR so, as that $CL \times GR$ may be $=$ the given $\square AD \times BE$, and upon BR describe the semicircle BKR: erect $RI \perp BR$, and $=$ to a mean proportional between BF and GR; draw $IK \parallel BR$, meeting the circumference in K, and $KE \parallel RI$ meeting BF in E, the point through which the required line ECD must pass.



DEMONSTRA. Produce LA to meet ECD, produced, in H; and then, RE (by prop. of the circle) being $= \frac{EK^2}{BE} = \frac{RI^2}{BE} = \frac{BF \times RG}{BE}$ (by construc.), and $GE = RE - RG = \frac{BF \times RG}{BE} - RG = \frac{BF - BE}{BE} \times RG = \frac{FE \times RG}{BE}$ (per the fig.), we shall (by theor. 20 B. 4 of Simp. Geom. 2d edit.) have $CL : DA$ $(:: HL : HA) :: FE : (GE =) \frac{FE \times RG}{BE}$ (the triangles HCL, ECF, being similar, and the right line ACG making equal angles with the homologous sides thereof, &c. whence, multiplying extremes and means, $AD \times BE = CL \times GR$ $=$ the given rectangle, by construction. *Q. E. D.*

NOTE, When IK neither cuts nor touches the semicircle BKR, this problem is impossible.

Mess. *Rob. Butler*, *J. Barnard*, *J. Moreland*, *Plus Minus*, *J. Walker*, *T. Walker*, and some others, have likewise given curious

curious and elegant constructions to this question; but t[illegible] prizes of 12 and 8 Diaries, for the solution thereof, ha[illegible] fallen to the lots of Mr. *Rob. Butler*, and Mr. *Wm. Emblet[illegible]* respectively.

The Eclipses calculated for 1764.

There will be four eclipses this year, two visible and tw[illegible] invisible, which will happen in the following order. App.

1. A visible lunar eclipse, March 17th, at night, of whic[illegible] our ingenious correspondents have obliged us with the fo[illegible]lowing calculations.

Calculated by	Beg. h. m.	Mid. h. m	End h. m.	Dur. h. m.	Dig h. m
Mr. T. Hopkinson, for London	10 48	12 10	13 34	2 46	8 2
Mr. Chapman, for Foxton, Leicest.	10 39	12 1	13 22	2 43	8 3
Mr. J. Metcalf, for London	10 50	12 13	13 36	2 46	8 3
Mr. J. Metcalf, for Wentworth	10 44	12 7	13 30	2 46	8 3
Mr. T. Harris, for London	10 41	12 6	13 29	2 48	8 3
Mr. T. Harris, for Brington	10 37	12 2	13 25	2 48	8 3
Mr. T. Bramley, for London	10 21	11 42	13 4	2 43	8 [illegible]
Mr. J. Webster, for Loughbor. by Dr. Halley's tables	10 39	12 1	13 22	2 43	8 1
Mr. Hampson, for Leigh, Lancash.	10 33	11 56	13 19	2 46	8 1

2. A great annular eclipse of the sun, April 1st, in th[illegible] morning; it will be visible in most parts of Europe, an[illegible] nearly annular at London.

Calculated by	Begin. h. m.	Mid. h. m.	End h. m	Dur. h. m.	Dig.
Mr. T. Hopkinson, for London	9 19	10 46	12 18	2 59	annul
Mr. Chapman, for Foxton, Leicest.	9 15	10 42	12 15	3 0	annul
Mr. J. Metcalf, for London	9 16	10 43	12 14	2 52	annul.
Mr. J. Metcalf, for Wentworth	9 12	10 39	12 9	2 57	annul.
Mr. J. Harris, for London	9 15	10 45	12 20	3 5	annul.
Mr. J. Harris, for Brington	9 9	10 39	12 14	3 5	annul.
Mr. J. Harris, for Rome	10 8	11 45	1 3	2 55	7 30
Mr. J. Harris, for Edinburgh	9 7	10 37	12 12	3 5	11 0
Mr. J. Harris, for Paris	9 21	10 48	12 23	3 2	11 6
Mr. T. Bramley, for London	9 1	10 27	11 49	2 48	9 51
Mr. J. Webster, for Loughboro'	9 18	10 23	11 18	2 0	11 9
Mr. Greensted, for Waterinbury	8 40	10 4	11 30	2 50	10 3
Mr. Hampson, for Leigh, Lancash.	9 10	10 36	12 4	2 54	10 40
Mr. E. Kimpton, for London	9 15	10 44	12 17	3 2	annul.
Mr. Smithurst, for London	9 2	10 27	11 58	2 56	11 5
Mr. Smithurst, for Manchester	8 53	10 18	11 49	2 56	10 50
Mr. C. Green, for Greenwich	9 2	10 30	12 2	3 0	11 0
Mr. G. Witchell, for London	9 3	10 28	11 59	2 56	10 59

Mr. *Smithurst* observes that the cental shade of this eclipse will enter Europe near Cape St. Vincent, and travel through Portugal, Spain, France, Holland, Denmark, and Sweden, till it quits the continent at the North Cape.

The 3d eclipse will be of the moon, September 10th, about 7 in the morning, and consequently invisible to us.

The last is a solar eclipse, September 25th, about 5 in the afternoon, but invisible to these northern parts of the world.

Besides the above, we have also been favoured with calculations of the eclipses from Mess. *Stephen Hodges*, *T. Quanbrough*, *Thomas Sanderson*, *John Stokes*, and others.

New Questions.

I. Question 518, *by Mr.* Tho. Sadler.

Old Simon's dead, and Margery is found
A bucksome widow with a thousand pound;
'Cause she hath gold, many a courting go,
Both old and young, the clown and fribbling beau—
She treats them kindly, ladies, you must know.
Her age and number of sweethearts you'll find
From the equations which are here subjoin'd,

$$\left.\begin{aligned}\frac{xy^{\frac{3}{2}}}{\sqrt{60x+5y}} &= 71\cdot5544.\\ xy-y &= 780.\end{aligned}\right\}\begin{aligned}x &= \text{her age,}\\ y &= \text{her number of sweethearts,}\end{aligned}$$

II. Question 519, *by Mr.* Rich. Gibbons.

Required the values of x and z, when $x^{z^{\frac{1}{2}}} = z^{x^{\frac{2}{3}}} = 100$?

III. Question 520, *by Mr.* Tho. Baker; *addressed to Mr.* Mal. Hitchins.

SIR,

Your kind advice I wou'd, with joy, obey'd,
But 'tis too late! I've lost the lovely maid)

That

That charming form, I thought wou'd make me bleſt,
Is in another's cold embraces preſt.
Ingenious algebraiſt (well known to fame)
Tell all the world my hateful rival's name.*

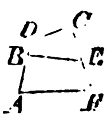
*From the given equations, viz. $\left\{\begin{array}{r} m+w+x+y+z=37, \\ w^2-x+y-z=35, \\ wz+w+z=53, \\ xy+wz=59, \\ \overline{my-x}\times wz=3000. \end{array}\right\}$
In which the values of m, w, x, y, and z, denote the places, in the alphabet, of the letters compoſing the required name.

IV. Question 521, *by Mr.* Joſeph Walker.

From the equation $\overline{x+1}\times\overline{x^2+1}\times\overline{x^3+1}=30x^3$, to find the value of x, by means of a quadratic only.

IV. Question 522, *by Mr.* Wm. Embleton.

To divide the given trapezium $ADCF$ into two parts AE, DE (geometrically) by the right line BE, ſo that $AB:EC::AE:DE::m:n$, i. e. in the given ratio of m to n.



VI. Question 523, *by Mr.* Wm. Spicer.

In a quadrangular piece of ground, whereof the ſides are 39, 47, 68, and 57 in a ſucceſſive order, and the diagonal joining the two ſhorteſt of them, 64, 'tis propoſed to form a ſquare fiſh-pond, having its angular points ſituated in the ſides thereof; required a method how to effect the ſame.

VII. Question 524, *by Mr.* Tho. Barker.

Four ſpires ſtanding directly north and ſouth, and at the reſpective diſtances of 2, 3, and 4 miles from each other, were obſerved by a traveller, on a road tending to the north-eaſt, the 1ſt and 2d, and alſo the 3d and 4th, appearing under equal angles, which they alſo did a 2d time, after travelling two miles farther on the ſame road; required his diſtance from them at each obſervation.

VIII. Question 525, *by Mr.* Paul Sharp.

Given the area of a curve $=198\cdot9333\times aa$ whoſe equation is $aaxx+y^4-aaxy=2axyy$; required the content of the ſolid generated by the rotation of the ſaid curve about its axis.

IX. QUESTION 526, *by* Geometricus.

Required the nature, area, and description, &c. of the curve, whose abscissa being the same with that of a given circle, its ordinate shall be every where equal to the difference between the corresponding ordinate and abscissa of the said circle.

X. QUESTION 527, *by Miss* Ann Nicholls.

On October the 28th, in the latitude 32° north, I observed the brightest of the Pleiades and Aldebaran, both in the same vertical circle; from whence is required the hour of the night, and greatest latitude possible, where such a phænomenon can be observed.

XI. QUESTION 528, *by Mr.* Wm. Toft.

At what time on the 10th of June, in the latitude of 12° 30′ north, will the sun's azimuth be the greatest?

XII. QUESTION 529, *by* Plus Minus.

In a certain piece of ground stand two trees, an oak and an ash, distant from each other 10 chains; the piece is surrounded by a ditch of so odd a cut, that if you draw lines from any point of it to the trees, and another from tree to tree, a triangle will be formed, such that the half complement of the angle at the ash to 90°, shall be = the angle at the oak; what is the area of the piece?

XIII. QUESTION 530, *by* Philotechnus.

A person, aged 35, is desirous of paying 20l. per annum into the insurance office, as long as he lives, on condition that his wife, whose age is 30, shall, after his decease, receive an annuity of 60l. per annum for the remainder of her life after his; required whether the insurer or the insured would have the advantage, interest of money being at 3 per cent.

XIV. QUEST. 531, *by Mr.* Wm. Spencer, *of* Stannington.

To determine the equation, the area, and the length of the curve, whose tangent and subtangent have always the same given difference (d).

XV. QUES-

XV. QUESTION 532, *by Mr.* Rob. Butler.

To find the sun's longitude, when his declination alters the fastest possible.

XVI. QUESTION 533, *by Mr.* T. Allen, *of* Spalding.

Let one end A of a straight inflexible rod AB, 60 feet long, be laid on an horizontal plane, and the other end B on a hemisphere (whose diameter = 30 feet) standing with its base on the said plane, and suppose the end A to advance, in a right line, along the plane, until it reaches and touches the said hemisphere; required the greatest distance the end B will have receded from this position of the rod, and the area of the curve space described thereby, and bounded by the said perpendicular position of the rod, and another right line drawn from the point B, at the commencement of the motion, perpendicular to the same.

XVII. QUESTION 534, *by* Curiosus.

Suppose one end of a thread AB, 100 feet long, is fastned to the circumference of a circle (or cylinder) whose diameter = 4 feet, and, being drawn tight, in the direction of the radius thereof, a ball, weighing 5 pounds, is fastned to its other end, and impelled, with a velocity of 20 feet per second, in a direction perpendicular to AB, by a force acting in the plane of the said circle; required the ball's position when it has been 45 seconds in motion, the tension of the string, &c.

The PRIZE QUESTION, *by* Philalethes Londinensis.

Supposing the sphere to be projected on the concave surface of an infinitely long cylindric tube of paper, &c. touching it every where on the equator, by the eye at the center, and afterwards opened, through any of the projected meridians, and spread or stretched upon a plane; required the figure of the rumbs in that projection.

NOTE, This is proposed with an intent to rectify an error in a mathematical tract wrote by the late Rev. Mr. West, of Exeter, and published anno 1762, which, if unnoticed, might prove of fatal consequence to the navigator, it being pretended to demonstrate there, that the rumbs, in this projection, will be right lines; but the contrary will be proved hereafter.

1765.

Questions answered.

I. QUESTION 518 *answered by Mr.* Malachy Hitchins.

MARGERY's age is forty years,
 Her sweethearts are just twenty;
How plainly avarice appears
 In bringing her such plenty.
Had she ten thousand charms in store,
 But wanted one in money,
I dare affirm, of all the score,
 Scarce one would be so funny.

The same answered by Mr. Isaac Tarratt.

From the 1st given equation, squared, &c. is derived $x^2y^3 = 60aax + 5aay$ (putting $71{\cdot}5544 = a$, and $780 = b$), and, from the second, $x = \frac{b+y}{y}$: Whence $\frac{bb+2by+yy}{yy} \times y^3 = 5aay + \frac{60aab+60aay}{y}$; solved, $x = 40 =$ her age. $y = 20 =$ her sweethearts.

A buxsome widow sure! of wond'rous parts,
Thus to attract, or wound so many hearts!
Widow! set up a school, instruct old maids:
If this thou can'st, 'twill be the best of trades.

Mess. *J. Ashmore*, *T. Baker*, *T. Barker*, *W. Barnes*, *T. Bramley*, *J. Hitchcock*, *Rd. Mallock*, *Jos. Mountfort*, *J. Probert*, *B. Rotherham*, *T. Sadler* (the proposer), *W. Sewell*, *Sopor*, *W. Spicer*, Mrs. *Suggett*, *T. Walker*, *J. Young*, and several others, have likewise answered this question.

II. QUESTION 519 *answered by Mr.* Tho. Walker.

Let X denote the log. of x, Z the log of z, and L that of 100, and then $z^{\frac{x}{z}} \times X = x^{\frac{z}{x}} \times Z = L$ (per the nature of

of logarithms and the quest.); whence $Z = \frac{L}{x^{\frac{2}{3}}}$. Assume $x = 47.5$, and then $x^{\frac{2}{3}} = 13.115$, and consequently $\frac{2.000000}{13.115} = Z = 0.152421$ the log. of $1.4215 = z$, according to this assumption; but $\overline{1.4215}|^{\frac{1}{2}} \times X$ is $= 1.9986 =$ log. of 99.69, which is too little by 0.31: therefore, now, suppose x to be $= 47.6$, and then z comes out $= 1.42$, and the 2d error $= 0.16$; whence $0.15 : 0.1 :: 0.16 : 0.106$, &c. which added to the last assumed value of x, gives 47.706, &c. for its true value, very near; and thence z is found $= 1.42$, &c.——And thus, by repeating the operation, we may arrive at any degree of accuracy required.

In this manner, nearly, the answer is given by Mess. *Tho. Baker*, *Tho. Barker*, *Wm. Barnes*, *S. Beeken*, *R. Gibbons* (the proposer), *J. Probert*, *Tho. Sadler*, *P. Sharp*, *Isaac Tarratt*, and others.

III. QUESTION 520 *answered by Mr.* Malachy Hitchins.

Could'st thou, dear Baker, sing of Hervey's DEATH,
And not perceive the fleeting state of breath,
Nor know delays to dang'rous issues tend,
Ere thou hadst lost thy lover and thy friend?
'Tis now in vain thy mournful tale is told!
Thy charmer's lost!—and her more charming gold!

Mr. *Paul Sharp*, by an easy process, derives the two following equations, viz.

$$\overline{43y - xyy - xy - yy - x} \times \overline{59 - xy} = 3000, \text{ and}$$

$$24xy + yxx + xxyy + 6y - 6x - xyy - \frac{\overline{88 + x - y}|^2}{xy - 5} = 233;$$

from which, by the method given by the late truly ingenious and profoundly great mathematician Mr. Thomas Simpson, on page 82 of his Essays, he finds $x = 1$ and $y = 19$; and from thence is readily found $m = 4$, $w = 5$, and $z = 8$; which point out the ingenious proposer's rival to be DEATH.

This question is also answered algebraically by Mess. *J. Ashmore*; *T. Baker*, *T. Barker*, *W. Barnes*, *S. Beeken*, *R. Gibbons*, *J. Hitchcock*, *J. Mountfort*, Miss *Ann Nicholls*, *J. Probert*, *T. Sadler*, *Sopor*, *W. Spicer*, *W. Sewell*, *J. Tarratt*, *W. Toft*, *T. Walker*, *J. Young*, and others.—But

as

as solutions this way turn out somewhat troublesome and uncouth, the following method, in point of ease and expedition, may perhaps claim the preference, viz.——The 3d given equation ($wz+w+z=53$) being evidently no more than to find two whole positive values of w and z such, that their sum added to their product may be $=53$, it appears, from very little consideration, that they must be 5 and 8, and consequently that their product (wz) is $=40$; whence it, with certainty, follows (from the 4th given equation) that xy is $=19$, and consequently that the number 1 must appertain to either x or y, and 19 to the other of them; and thence m is, certainly, found $=4$ (from the 1st given equation): whence it also, certainly, follows that $x=1$, because if it be supposed $=19$ (and either 1 or 19 it must be) then the last given equation will become impossible. In like manner it appears that w must be $=5$, because if it be taken $=8$ (and one of them it must be), then the 2d given equation becomes impossible. Hence the values of m, w, x, y, and z are found to be 4, 5, 1, 19, and 8, answering to the letters (of the alphabet) D, E, A, T, H, respectively.

IV. Question 521 *answered by Mr.* Thomas Barker.

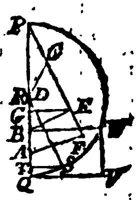
From the given equation, the following one, viz. $x^6+x^5+x^4-28x^3+x^2+x+1=0$, is readily derived; which, divided by x^3, becomes $x^3+x^2+x-28+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}$, or, putting $x+\frac{1}{x}=z$, $\overline{zz+4z+10}\times\overline{z-3}$ ($=z^3+z^2-2z-30$) $=0$: Whence $z-3=0$, and $z=3$; which is the only possible value of z, in the equation, and which, substituted for z in the assumed equation $x+\frac{1}{x}=z$, gives $x+\frac{1}{x}=3$; whence $xx-3x=-1$, and $x=\frac{3\pm\sqrt{5}}{2}=\left\{\begin{matrix}2{\cdot}618\\0{\cdot}382\end{matrix}\right\}$; both which numbers answer the conditions of the question.

Mess. *T. Baker*, *R. Gibbons*, *J. Probert*, *T. Sadler*, *W. Spencer*, *J. Tarratt*, *W. Toft*, &c. have likewise answered it according to this method, nearly.

V. Ques-

V. QUESTION 522 *answered by* Mr. C. HUTTON.

If the sides FC, AD, of the given trapezium $ADCF$, be produced to meet in P, the $\triangle PDC$ will be of a given magnitude; and since the said trapezium is to be divided in a given ratio, by the right line BE, each of the parts DE and AE will also be of a given magnitude; and consequently the $\triangle PBE$ will be of a given magnitude, which call aa. Then the question is reduced to this, viz. to draw the right line BE so, that AB may be to EC :: $m : n$, and the $\triangle BPE =$ the given square aa.



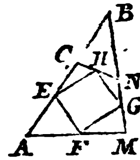
CONSTRUC. In PA produced, take AQ to PC in the given ratio of m to n, and on the diameter PQ describe the semicircle PWQ; and from the center R thereof draw the rad. $RS \parallel PF$ and $ST \perp PQ$. Then if QV be made $\perp PQ$, and $=$ a mean proportional between $\frac{m \times PQ}{n}$ and $\frac{aa}{ST}$, and VW and WB be drawn $\parallel PQ$ and QV respectively, we shall have B for one point through which the required line must pass: and if CE be taken to AB in the given ratio of n to m, E will be the other.

DEMONSTRA. Draw $EG \perp PA$: then, by the sim. triangles TSR, GEP, $SR : ST :: EP : \frac{EP \times ST}{SR} = EG$; whence (triangles being the halves of their circumscribing parallelograms, per Elements of Geom.) $\frac{1}{2} PB \times \frac{EP \times ST}{SR}$ or $\frac{PB \times EP \times ST}{PQ}$ ($2RS$ being $=$ the diameter PQ) will be the measure of the $\triangle BPE$. But (per construc.) $PB \times \frac{m \times EP}{n}$ ($= PB \times BQ$) is $= BW^2 = QV^2 = \frac{m \times PQ}{n} \times \frac{aa}{ST}$ (per construction); and consequently each of these equal magnitudes, or spaces, being multiplied by $\frac{n \times ST}{m \times PQ}$ (or repeated that number of times), aa will be found $= \frac{PB \times EP \times ST}{PQ} =$ the measure of the triangle BPE (the same as found above).

VI. QUES-

VI. QUESTION 523 *answered by Mr*. C. Hutton.

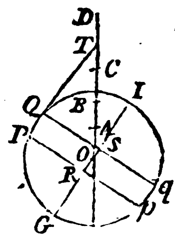
This quest. may be constructed from prob. 41 on page 246 of Simp. Geom. 2d edit. And if the sides *AC*, *MN*, of the given trapezium *AMNC*, be produced to meet in *B*, the construction here will be the same as in that prob. (only observing that here *C* and consequently *E* fall between *A* and *B*); for which reason it seems needless to repeat the construction.



Algebraic solutions to this question have been received from Mr. *T. Barker*, the Rev. Mr. *John Cross* (of *Pocklington* Grammar School, near *York*), Mr. *M. Hitchins*, Mr. *Tho. Sadler*, Mr. *W. Spicer*, Mrs. *Suggett*, Mr. *J. Tarratt*, and others: From which (by means of quadratics only) the required side of the inscribed fish-pond may be found, and appears to be = 39, nearly; but, for want of room, we are obliged to omit them.

VII. QUESTION 524 *answered by Mr*. C. Hutton.

From the construc. on page 238 of Simp. Geom. 2d edit. it appears, that the locus of concourse of lines drawn from the given points (or places of the spires) *A*, *B*, *C*, *D*, so that the angles subtended by *AB* and *CD* may be equal, will be the circumf. of a circle; which cir. *IPQ*, let be described by that prob.



Then, to find in which two points a right line, drawn in the given direction (north east), shall cut the circumf. of his circle so, that the part of it intercepted by those points may be of the given length (2 miles), through the center *O*, draw *GOI* in the given direction, and on it, from *O*, each way, set off *OR*, *OS*, each = (1 mile) half the given distance of the stations: Then through *R* and *S* draw *PR* and *QS* ⊥ *GI*, and the points *P*, *Q*, of their intersection with the circumf. of the said circle, will be the two stations (in the road *PT*) required; which is too evident to need any further demonstration.

Mr. *Tho. Barker* (the proposer) and Mr. *Malachy Hitchins* have also constructed this problem; and many contributors have sent algebraic solutions to it.

VIII. QUES-

VIII. QUESTION 525 *answered by Mr.* Rob. Butler.

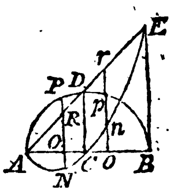
By transposing the given equation, and extracting the square root, it becomes $yy - ax = \pm a\sqrt{xy}$, or, putting $\frac{b^2x}{z^2} = y$, $\frac{b^4x^2}{z^4} - ax = \pm\frac{abx}{z}$; whence $x = \frac{az^4}{b^4} \pm \frac{az^3}{b^3}$, and $y = \frac{az^2}{b^2} \pm \frac{az}{b}$ (where b may be any quantity at pleasure, and so the curve may be constructed). Suppose $b = 1$; and then $\dot{x} = 4az^3\dot{z} \pm 3az^2\dot{z}$, and $y\dot{x}$ (the flux. of the area) $= 4a^2z^5\dot{z} \pm 7a^2z^4\dot{z} + 3a^2z^3\dot{z}$, and the correct fluent thereof $\overline{\frac{2z^6}{3} \pm \frac{7z^5}{5} + \frac{3z^4}{4} - d} \times aa = 198{\cdot}9333aa$ per quest. (d being $= \frac{2}{3} \pm \frac{7}{5} + \frac{3}{4}$); from whence, when a is expressed in numbers, z, and consequently x and y, may be found in numbers also.

In like manner (putting $p = 3{\cdot}1416$) the flux. of the solidity ($pyy\dot{x}$) is found $= p \times \overline{azz \pm az}|^2 \times 4az^3\dot{z} \pm 3az^2\dot{z}$, and consequently $pa^3 \times \frac{1}{2}z^8 \pm \frac{11}{7}z^7 + \frac{5}{3}z^6 \pm \frac{3}{5}z^5 - n$ = the solidity itself (n being = the sum of the coefficients of the powers of z, &c.).

Mess. *W. Embleton*, *C. Hutton*, *D. Kinnebrook*, and *P. Sharp* (the proposer) answer it in this manner nearly.

IX. QUESTION 526 *answered by Mr.* Da. Kinnebrook.

Let ADB represent the given semicircle, and ACE the required curve, which, it is easy to perceive, will pass through the center C, of the circle, and meet a tangent to it at B, in the point E, whose distance from B is = the given diameter AB.



Join the points A, E, and draw the ordinates PO, po, &c. cutting AE in R, r, &c. and AB in O, o, &c. respectively, and take ON, on, &c. thereon every where $= PR$, pr, &c. and N, n, &c. will be the points in the required curve.——For $AB = BE$ (per nature of the quest. and construc.) and consequently $AO = OR$, &c. (per

(per sim. triangles) whence $PO \backsim AO = PR = NO$, &c. (per construction). Q. E. D.

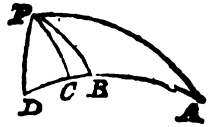
Put, now, AO (or Ao, &c.) $= x$, $AB = 2a$, NO (or no, &c.) $= y$, and then (per prop. of cir. and quest.) $\sqrt{2ax - xx} = PO$, and y (NO, &c.) $= \sqrt{2ax - xx} \backsim x$, and $\overline{x \pm y}|^2 = 2ax - xx = \frac{2a\sqrt{2} - x\sqrt{2}}{2aa} \times aax\sqrt{2}$: Whence $\overline{x \pm y}|^2 : x\sqrt{2} \times \overline{2a\sqrt{2} - x\sqrt{2}} :: aa : 2aa$, i.e. $NR^2 : AR \times RE :: CD^2 : AD^2$; and hence it appears, that the required curve ACE is an ellipse, whose center is D (the rad. DC being perpendicular upon the diameter AB), and AE a diameter thereof, the conjugate to that diameter being DC.——And if an infinite number of parallels to PN be conceived to be drawn, the circular and elliptic areas $APDR$, $ANCO$ will be divided into an infinite number of parallelograms, whose bases PR, NO, &c. and altitudes are equal; and therefore the circular area $APDR =$ the elliptic area $ANCO$.——In like manner the area DEB is proved to be $=$ the area CEB; and thence the semiellipse $ANCED$ appears to be $=$ the semicircle $ACBD$.

In this manner, nearly, the answer is given by Mr. *Rob. Butler*, and Mr. *C. Hutton*: But Mr. *T. Allen*, after proving the nature and description, &c. of the required curve to be the same as above, puts AO, or Ao, &c. $= x$, the corresponding ordinate ON, or on, &c. $= y$, the rad. $AC = a$, and finds the fluxions of the spaces ANO, $Cno = \dot{x}\sqrt{2ax - xx} - x\dot{x}$ and $x\dot{x} - \dot{x}\sqrt{2ax - xx}$ respectively; and the fluent of the former of them (when $x = a$) is $=$ the measure of the circ. quad. $ADC - \frac{1}{2}AC^2$, and the correct fluent of the latter $= \frac{xx - aa}{2} -$ the measure of the circ. seg. $CDpo =$ (when $x = 2a$) $\frac{3AC^2}{2} -$ the measure of the circ. quad. $CDB =$ the measure of the whole space CBE; to which if that of the space ANC ($=$ the measure of the circ. quad. $ADC - \frac{1}{2}AC^2$) be added, the area of the whole curve space $ANCEBC$ will be found $= AC^2$; *A thing somewhat remarkable!*

X. QUES-

X. QUESTION 527 *answered by Mr.* T. Allen.

Let A and B be the places of Aldebaran and the brightest of the Pleiades respectively, P the pole, PC the meridian, and CA the vertical circle passing through the two stars. In the spherical triangle ABP, are given AP the polar distance of Aldebaran $= 73° 58' 54''$, BP that of the brightest of the Pleiades $= 66° 58' 54''$, and the included angle, he diff. in right ascension of the two stars $= 12° 13' 30''$, whence the $\angle PBC$ is found $= 60° 7' 15''$; then in the spherical triangle PBC are given PB and the $\angle PBC$ as above, and PC the comp. of the given lat. $= 58°$, from whence the $\angle CPB$ is found $= 28° 55' 40''$, which is the arc that the brightest of the Pleiades is short of the meridian at the time required.

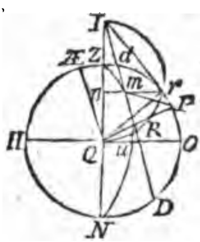


Lastly, the greatest lat. in which this phænomenon can be observed, is, when the $\angle PCB$ becomes $= PDB$ a right angle. Therefore, rad. : sine of PB :: sine of the angle PBD : cosine of $37° 14' 53''$, the greatest lat. required.

Mess. *R. Butler, W. Embleton, D. Kinnebrook, Tho. Sanderson, Jos. Webster*, and some others, answer it in this manner very near.

XI. QUESTION 528 *answered by Mr.* Wm. Toft, (*the Proposer*).

In the orthographic projection of the sphere, on the plane of the meridian, the horizon HO, the equator $Æ Q$, the prime vertical ZN, the parallel of declination Dd, &c. will, it is well known, be projected into right lines, and the azimuth circles into ellipses, each intersecting Dd in two points, except that (ZuN) which touches it; and which, therefore, is the azimuth required.



For the sun coming afterwards to the other points of intersection with Dd, will apparently recede back again, and be seen successively on the same points of the compass as before, and so the azimuth decrease again by the same steps it before increased; an appearance surprizing enough to those who are ignorant of the true cause thereof!

Therefore to determine the position (or projection) of this azimuth circle, (and consequently to solve the prob.) produce NZ and Dd till they meet in I, and conceive a semicircle to be described upon the diameter IQ, cutting the primitive in r, and let rn be drawn $\perp NZ$, and its intersection m with Dd will be the place of the sun at the time required, or the point through which the projected azimuth circle required must pass.——For, supposing the points m, I; r, I; and Q, r to be joined, and the semi-periphery of an ellipse, to the transverse diameter ZN, to be described through the given point m: then will mI be a tangent to it in m, by the property of the ellipse and its circumscribing circle (see theor. 49th of the ellipse in Steel's Conic Sections), because rI is a tangent to the primitive circle in r (QrI being a right angle, by construction).

The method of calculation is from hence also extremely natural and easy; for QR ($\perp Dd$) being the nat. sine of the given declin. to rad. of the primitive; QI, and consequently Qn ($=\frac{Qr^2}{QI}$, per prop. of the circle) are from thence readily found, the latter of them being the nat. sine of 33° 27′, the sun's alt. at the time required; and thence nI and nm become known: Whence, by another prop. of the ellipse and its circumscribing circle, $nr : nm :: QO : \frac{nm \times QO}{nr} = Qu$, the nat. cos. of 70° 24′, the sun's azimuth required. And hence the time from noon, when this phænomenon happens, is easily found = 3 h. 55 m. nearly.

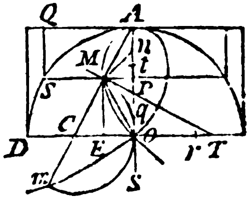
Mess. *Robert Butler*, *Da. Kinnebrook*, and *Lycidas*, by spherics (from the spherical triangle mZP, where ZP is the complement of the given lat. Pm the complement of the sun's declin. and mZ that of his alt. at the time required), find $\frac{\text{sine of } \angle ZmP \times \text{sine } Pm}{\text{sine } PZ}$ = the sine of the $\angle PZm$, the azimuth required; which, it is manifest, will be greatest when ZmP is a right angle, because the sines of Pm and PZ are constant quantities; from whence the time required is directly found to be either 5 m. past 8 in the forenoon, or 55 m. past 3 in the afternoon, very near.

And Mess. *T. Allen*, *T. Barker*, *W. Barnes*, *W. Embleton*, *R. Gibbons*, *T. Sadler*, *P. Sharp*, *W. Spicer*, *T. Walker*, *J. Young*, and some others, determine the time sought nearly the same as above.

XII. QUES-

XII. QUESTION 529 *anſwered by Mr.* R. Butler.

Let A repreſent the aſh, O the oak, and with any rad. Oq (not exceeding $\frac{1}{2}OA$) and center q, deſcribe the circumference of a circle cutting OA in n, and on the diam. qA (the complement of Oq) conceive the circumference of another circle to be deſcribed cutting the former in M, a point in the curve (or figure of the ditch) required; and after this manner any number of other points may be found.——



For MP being drawn $\perp OA$ and the points M, q; M, A; M, n; and M, O joined, the $\angle AMP = MqP =$ double the $\angle AOM$, as required per the queſtion.

The ditch $OMAO$ being thus conſtructed, to find the area thereof, put $a = OA$ (= 10 chains), $x =$ abſciſſa OP, $y =$ ordinate PM; then, per ſim. triangles, $a - x$ (AP) : y (PM) :: y (PM) : $\frac{yy}{a-x} = Pq$; and $\frac{yy}{x} = Pn$: whence

$$nq = \frac{yy}{a-x} + \frac{yy}{x} = Oq = x - \frac{yy}{a-x},$$

and conſequently

$$y = x\sqrt{\frac{a-x}{a+x}} = \frac{ax-xx}{\sqrt{aa-xx}},$$

and $y\dot{x}$ (the fluxion of the area OMP)

$$= \frac{ax\dot{x}-x^2\dot{x}}{\sqrt{aa-xx}} = \frac{ax\dot{x}}{\sqrt{aa-xx}} + \frac{1}{2}\times\frac{aax\dot{x}-2x^3\dot{x}}{\sqrt{aaxx-x^4}} - \frac{\frac{1}{2}aa\dot{x}}{\sqrt{aa-xx}},$$

and the correct fluent thereof (or area OMP)

$aa - a\sqrt{aa-xx} + \frac{1}{2}x\sqrt{aa-xx} - \frac{1}{2}a \times$ circ. arc whoſe rad. $= a$ and right ſine $= x$; which, when $x = a$, becomes

* $aa - \frac{1}{2}a \times \frac{3.1416a}{2} = 21.46$ ſquare chains = the area of the ſpace $OMAO$; and doubled is 42.92 ſquare chains = the area of the whole ſpace required. Mr.

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* This expreſſion reducing to $aa \times \overline{1 - .7854}$, it may be remarked that $OMAO$, the ſemi-area, is equal to the difference between a ſquare whoſe ſide is a, and its inſcribed circle. Which alſo agrees with the other ſolution by *Plus Minus*.

Mr. *Da. Kinnebrook*'s answer is the same, exceeding near. And much after the same manner the answer is also given by Mess. *J. Barnard*, *W. Embleton*, *C. Hutton*, *Lycidas*, *W. Spicer*, and some others.

The same answered otherwise, by Plus Minus, *the Proposer.*

Let A and O be the places of the ash and oak, and M a point in the required curve. 'Tis plain, from the data, that the excess of the angle at the fence above $90° =$ the angle at the oak. Draw $MT \perp AM$, and then $TMO = MOA$; whence we have the method of describing the required curve, viz. draw $OD \perp AO$, and AMC at pleasure: With the center C, and radius CO describe the circumference of a circle which will cut AC in two points M, m of the required curve. Call AO, a; AP, x; and PM $(\perp AO)$ y; then (per sim. triangles APM, AOC) CO, or $CM = \frac{ay}{x}$, and $CE = \frac{ay}{x} - y$; but $EC^2 + EM^2$ (or PO^2) $= CM^2$, i.e. $2ayy - xyy = x^3 - 2axx + aax$, which is an equation for the hyp^a. defectiva nodata diametrum habens, (see Newton's Enumeratio Lin. 3tii Ord. Species 41^a.) and y

$$= \frac{\sqrt{x \times \overline{a-x}|^2}}{\sqrt{2a-x}} = \frac{a}{x^0\sqrt{-1+2ax^{-1}}} - \frac{1}{x^{-1}\sqrt{-1+2ax^{-1}}},$$

which belongs to tab. 2, form 4 cas. 2 and 3, of Newton's Quad. Curv. or $y = \frac{ax - xx}{\sqrt{2ax - xx}}$, which belongs to cas. 2 and 3 of form 8; and by either of them, it appears, that if you describe a quad. AD on the center O, with AO for a radius, and produce PM meeting AD in S, and draw SQ, QA parallel to OA and OD, the area APM shall be equal to the area ASQ; and the whole area of the field $= 2AO^2$ — a semicircle whose rad. is the same AO. That this is the whole of the area is plain, if we look at the equation $2ayy - xyy = x^3 - 2axx + aax$: for when $y = 0$, $x = 0$ or a; so that the trees stand in the fence: And that no other part of the curve beyond O is to be considered as belonging to the field, is also plain; because, by the data, the angle at the fence must never be less than a right one. When y is infinite, $x = 2a$, which gives the asymptote.

If

If you take Or, Os each $= \frac{1}{2}a$, and ſet Ar from s to t, you will have the foot of the greateſt ordinate.*

XIII. QUESTION 530 *anſwered by Mr.* E. Nott.

This gentleman conſiders the inſurer as certain of receiving 20l. per annum during the huſband's life, though the wife was to die immediately after the agreement, conſequently that (by tab. 5th on pag. 260 of Simpſon's Select Exerciſes) the preſent value of his expectation is worth 14·1 years purchaſe, or 282 l.

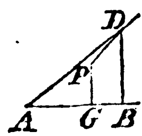
In like manner, the remainder of the wife's life after that of her huſband's being (by prob. 15 on pag. 285 of Simp. Select Exerciſes aforeſaid) worth 4·7 years purchaſe, the preſent value of her expectation will be found to be 60 × 4·7 = 282 l. which, being the ſame with the other, ſhews that neither ſide will have the advantage in this caſe.

And in this light it is conſidered by Meſſ. *R. Butler*, *W. Embleton*, *R. Gibbons*, *M. Hitchins*, *C. Hutton*, *D. Kennebrook*, *T. Sadler*, *J. Tarratt*, and ſeveral others.

But the propoſer, ſuppoſing the inſurer to be no longer intitled to receive than during the joint exiſtence of the man and his wife, makes the advantage about 22 l. on the ſide of the inſured: And if the intereſt the inſurer might make of the money he has a chance of receiving, from time to time, before he is liable to pay any, were to be taken into conſideration, the balance would then be found to turn ſomewhat in favour of him, &c.——His ſolution is from firſt principles, but our narrow limits oblige us to omit it.

XIV. QUESTION 531 *anſwered by Mr.* Rob. Butler.

Let PD repreſent the required curve, whoſe abſciſſa $GB = x$, and ordinate $BD = y$, and then $\frac{y\dot{x}}{\dot{y}}$ being = its ſubtangent AB and $\frac{y\sqrt{\dot{x}^2+\dot{y}^2}}{\dot{y}}$ = its tang. AD, $\frac{y\sqrt{\dot{x}^2+\dot{y}^2}-y\dot{x}}{\dot{y}} = d$,



per

* This will eaſily appear by putting the fluxion of ($y =$) $\frac{ax-xx}{\sqrt{2ax-xx}}$ equal to o.

per question. Whence $\dot{x} = \frac{y\dot{y}}{2d} - \frac{1}{2}d \times \frac{\dot{y}}{y}$; and y being $= d$ ($= GP$, the tangent at the vertex) when $x = 0$, per the nature of the question, the correct equation of the fluents will be $x = \frac{yy - dd}{4d}$ + hyp. log. of $\overline{\frac{d}{y}}\Big|^{\frac{d}{2}}$; the equation of the curve required.

Again, $y\dot{x} = \frac{y^2\dot{y}}{2d} - \frac{1}{2}d\dot{y}$, and the correct fluent, or area required, $= \frac{y^3}{6d} - \frac{1}{2}dy + \frac{1}{3}dd$.

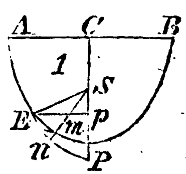
Lastly, $\dot{x}^2 + \dot{y}^2 = \dot{y}^2 \times \overline{\frac{yy - dd}{4ddyy}}\Big|^2 + \dot{y}\dot{y}$, and $\sqrt{\dot{x}^2 + \dot{y}^2} = \frac{y\dot{y}}{2d} + \frac{1}{2}d \times \frac{\dot{y}}{y}$; and the correct fluent $\frac{yy - dd}{4d}$ + h. log. of $\overline{\frac{y}{d}}\Big|^{\frac{d}{2}}$ is = the length of the required curve PD.

COROLLARY: It hence appears that the curve's length always exceeds the axis by the quantity $d \times$ hyp. log. $\frac{y}{d}$.—— It is also answered in this manner by Mr. *T. Allen*, and Mr. *Da. Kinnebrook*, exceeding near.

And Mess. *T. Barker*, *W. Barnes*, *W. Embleton*, *C. Hutton*, *P. Sharp*, *W. Spicer*, *T. Sanderson*, *T. Walker*, and some others, have likewise answered it.

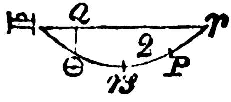
XV. QUESTION 532 *answered by Mr.* T. Allen.

Let $AEPB$ (fig. 1.) be one half of the ellipsis in which the earth revolves about the sun, in the focus S, P the place of the perihelion, and ESm an elliptic sector described in an indefinitely small particle of time $\dot{t}$. Put $CP = a$, $AC = CB = b$, $CS = c$, and let the cosine of ESP, the earth's distance from its perihelion $= x$, rad. $= 1$. Then, by the properties of the ellipsis, $ES = \frac{bb}{a + cx}$, and (by



trigonom.) $Ep = \frac{bb\sqrt{1 - xx}}{a + cx}$, and $Sp = \frac{bbx}{a + cx}$. Therefore

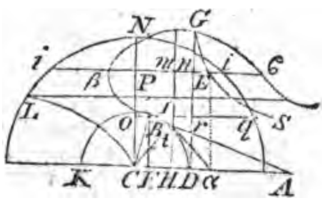
fore $\frac{bb\sqrt{1-xx}}{a+cx} : \frac{bb}{a+cx}$, or $\sqrt{1-xx} : 1 :: \frac{abb\dot{x}}{\overline{a+cx}|^2}$ $(=\text{flux. } Sp) : \frac{abb\dot{x}}{\sqrt{1-xx}\times\overline{a+cx}|^2}$, the fluxion or increase (En) of the true longitude in the circular arc EP, whose rad. is ES, in the time t'.



Now, let ♎♑♈ (fig. 2) represent the southern semicircle of the ecliptic, ♑ the Tropic of Capricorn, P the place of the perihelion, ⊖ the place of the earth in the ecliptic at the time proposed; then will ⊖Q = the sun's declination. Put m and n = the sine and cosine of ♑P, the distance of the perihelion from ♑, p and q = the sine and cosine of ⊖♎Q, and the cosine of P⊖ $= x$ (as above), then will $pnx + pm\sqrt{1-xx}$ = the sine of Q⊖ (see art. 478 of Simpson's Flux.) and $\frac{pn\dot{x}\sqrt{1-xx} - pmx\dot{x}}{\sqrt{1-\overline{pnx+pm\sqrt{1-xx}}|^2}\times\sqrt{1-xx}} =$ the fluxion or alteration of the declin. in the time t'. Consequently $\frac{\sqrt{1-\overline{pnx+pm\sqrt{1-xx}}|^2}}{n\sqrt{1-xx}-mx\times\overline{a+cx}|^2}$ by the quest. must be a min. By making the fluxion of which = o, the value of x may be found.

Mess. *R. Butler*, *D. Kinnebrook*, *J. Moreland*, and *A. Rowe* find, by very easy, short, and elegant processes, that at crossing the line the declination alters the fastest, supposing the earth's orbit to be a circle, and the apparent motion of the sun in the ecliptic uniform, &c.

XVI. QUESTION 533 *answered by* PLUS MINUS.



Let AB $(= 60 \text{ feet} = a)$ represent the first position of the given rod, and $\alpha\beta$ any other position of it, touching the given hemisphere DBK in t, and and draw the rad. Ct, and CP, $P\beta$ (perpendicular and parallel to CD); DG $(= AB)$

($= AB$) a tangent to the given circ. at D, αE ($\parallel DG$) meeting βP (produced) in E, and or, through B, perpendicular to CP and DG; and then, putting Ct ($= CD =$ 15 feet) $= r$, $CP = x$, and $P\beta = y$, x ($E\alpha$) will be to a ($\alpha\beta$) :: r (Ct) : $\frac{ar}{x} = C\alpha$, and $\beta E = y + \frac{ar}{x}$: but $\beta E^2 + \alpha E^2 = \alpha\beta^2$, and therefore $y = -\frac{ar}{x} \pm \sqrt{aa - xx}$; consequently, if about C, with the rad. AB, the circumference of a circle be described cutting CP, βE and Br, produced, in N, i, and q, and also an hyperbola through the point G to the asymptotes CP, CA, and meeting Bq, produced, in s, the area $G\beta\beta r$ will be $=$ the circular area Nqo $-$ the hyperbolic area $Grs = 1369{\cdot}58$; and the area of any portion $\beta\,\text{ɩ}\,\mathfrak{C}$ thereof, cut off by a right line $\beta\mathfrak{C} \parallel CA$ will be $=$ the circ. seg. iNi.—— The fluxion of y or $-\frac{ar}{x} \pm \sqrt{aa - xx}$, being equated to nothing, gives $x^3 = \pm ra\sqrt{aa - xx}$; therefore if the cubic parabola, whose equation is $x^3 = ray$, be described through C, it will cut the aforesaid circ. in L, the point through which the ordinate passes, which is a maximum ($= 38{\cdot}02$) on one hand, and gives the point of contrary flexure on the other.

In this manner, exceeding near, the answer is given by Mess. *T. Allen* (the proposer), *R. Butler*, *W. Embleton*, *C. Hutton*, *D. Kinnebrook*, *T. Sanderson*, *W. Spencer*, and some others.

XVII. QUESTION 534 *answered by Mr.* Da. Kinnebrook.

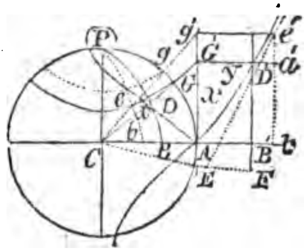
The ball, in the first part of its motion, moves through the quadrant of a circle, and in the latter part through the involute of the given circle or cylinder; in both which cases the thread is perpendicular to the curve, and therefore cannot affect the ball's motion, and consequently it moves uniformly through both curves. Now the length of the quadrant, to rad. 100 feet, is $= 157{\cdot}08$; whence $\frac{157{\cdot}08}{20} = 7{\cdot}854$ seconds, the time of its description, and $45'' - 7{\cdot}854'' = 37{\cdot}146''$ the time of the ball's moving in the involute; whence $37{\cdot}146 \times 20 = 742{\cdot}92$ the length of the involute. Put now $a = 100$, $b = 2$, $x =$ the rad. of involution, and $l =$ the length of the involute; then (per pag. 163 of Simp.

Simp. Flux. 2d edit.) $\frac{aa-xx}{2b}=l$; whence $x=\sqrt{aa-2bl}$ $=$ (when $l=742\cdot92$) $83\cdot836=$ the radius of involution, or the part of the thread or ſtring remaining unwound at the end of 45″: from whence the poſition of the ball may be found, &c.——The tenſion of the ſtring is $=\frac{5\times20^2}{32\frac{1}{6}\times83\cdot836}$ $=\frac{3}{4}$lb. nearly.

In this manner, very near, it is anſwered by Meſſ. *R. Butler*, *W. Embleton*, *C. Hutton*, *E. Nott*, &c.

The PRIZE QUESTION *anſwered by* Philalethes Londinenſis (*the Propoſer*).

Let $AGPDB$ be part of the ſphere, its center C, pole P, part of the equator AB, and PA, PB, Pb meridians, of which Pb is infinitely near PB. Let ADe be a loxodromic line that makes equal angles with all the meridians through which it paſſes, of which let t be the tangent to the rad. of the ſphere $AC=r$, and put AB $(=AB')=y$, $AG=l$, its tangent $AG'=x$, and then $\sqrt{rr+xx}\,(CG'):r\,(CA)$



$::\dot{y}\,(Bb):\frac{r\dot{y}}{\sqrt{rr+xx}}=Da$, and $r:t::\dot{l}\,(ea):\frac{r\dot{y}}{\sqrt{rr+xx}}$ (Da); whence $\dot{y}=\frac{t\dot{l}\sqrt{rr+xx}}{rr}$: but $\dot{l}=\frac{rr\dot{x}}{rr+xx}$, and therefore $\dot{y}=\frac{t\dot{x}}{\sqrt{rr+xx}}$, and (putting $2\cdot30258=M$) $y=$ $tM\times\log.\frac{x+\sqrt{rr+xx}}{r}$, the equation expreſſing the relation betwixt x and y on the globe, or, which is the ſame thing, between the abſciſſa and ordinate in the propoſed chart or projection. But $x+\sqrt{rr+xx}$ is $=$ tangent of $\frac{90^\circ+l}{2}$; therefore, laſtly, $y=tM\times$

log.

log. cotan. $\frac{90^\circ - l}{2}$ — log. r.——The curve may be very easily constructed by the table of logarithmic tangents; and the preceding figure is exact, when the angle of the loxodromic is 45°.

COROL. 1. Because $\dot{t}\dot{a} : \dot{a}D' :: \dot{x} : \dot{y} :: \sqrt{rr+xx} : t$, therefore if through any point D' of the curve a line be drawn perpendicular to the equator AB', and the point F be taken therein, so that $D'F = G'C$, and a line be drawn through $F \parallel AB'$, on which let the point E be taken such, that $FE =$ the tang. of the constant loxodromic angle, then a line drawn from E to D' will touch the curve in D'; from whence it follows that all loxodromics in this projection (except those whose direction are north and south or east and west) are mechanical or transcendent curves, having a point of contrary flexure when they cut the equator.

COROLLARY 2. Since $\dot{y} = \frac{t\dot{x}}{\sqrt{rr+xx}}$, the fluxion of the space $AB'D'$ $(= x\dot{y})$ will be $= \frac{t x\dot{x}}{\sqrt{rr+xx}}$, and that space itself $= t\sqrt{rr+xx} - tr = t \times GG'$.

COROLLARY 3. Since $\dot{y}$ is found $= \frac{t\dot{l}\sqrt{rr+xx}}{rr}$, if $r\dot{m}$ be put $= \dot{l}\sqrt{rr+xx}$ or rm for the sum of all the secants from 0 to l degrees, then will $\dot{y} = \frac{t\dot{m}}{r}$, or $ry = mt$, and therefore $r : t :: m : y$. But here m will be the meridional parts of the lat. l for the rad. r; whence the diff. of longitude found by Mercator's sailing corresponds exactly with what is derived from the globe itself.

The solutions by Mess. *T. Allen*, *R. Butler*, *W. Embleton*, *C. Hutton*, *Da. Kinnebrook*, *Plus Minus*, *W. Spencer*, and *G. W* agree with this so exactly, that, even if room would permit, it would be needless to repeat them here.

The prize of 12 *Diaries, for the solution of the prize question, was won by Mr.* David Kinnebrook, *and that of* 8 *by Mr.* Charles Hutton.

The

The Eclipses calculated for 1765.

There will happen six eclipses this year, four of the sun and two of the moon, according to the following order.

The first is an invisible eclipse of the sun, on Tuesday the 19th of February, 22 m. past 11 at night, apparent time. ——It will be seen only in the unknown parts of the great South Sea.

The second is a total and nearly central eclipse of the moon, on Thursday the 7th of March, 17 m. past one in the afternoon, and therefore invisible in Europe, but will be visible in the South Sea and Asia.

The third is another invisible eclipse of the sun, on Thursday the 21st of March, 2 m. after 1 in the afternoon.—— This will appear a small eclipse in North America, and to places near the north pole.

The fourth is also of the sun, on Friday the 16th of August, and will be visible from the beginning to the end, according to the following calculations.

Calculated by	Beg.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m.	h.	m.	h.	m.	°	′
Mr. William Baylis, for Warwick	3	32	4	12	4	49	1	17	1	50
Amicus —— ——	3	39	4	19	4	57	1	18	1	46
Mr. T. Quanbrough, for London	3	29	4	5	4	44	1	15	1	35½
Mr. Stephen Hodges, for Althorp, near Northampton	3	52¼	4	24	4	57	1	4¾	1	25⅙
Mr. T. Walker, for London	3	37¼	4	16	4	54¾	1	17½	2	3
Mr. J. Metcalfe, for London	3	37¼	4	16	4	54¼	1	17½	2	3
Mr. T. Allen, for Spalding, by Maskelyne's Tables and Construction	3	35	4	5	4	49	1	14	1	40

The fifth is another total eclipse of the moon, on Friday the 30th of August, 6½ m. past 2 in the afternoon, but invisible in Britain, by reason it will be over before the moon rises.

The sixth and last is of the sun, on Sunday the 15th of September, 35 m. after 4 in the morning, and consequently invisible here, and to all these parts of the world.

JOHN METCALFE.

New

New Questions.

I. QUESTION 535, by Mr. Tho. Sadler.

Declare ye wits, well known to fame,
A celebrated lady's name;
Whose equal's scarcely to be found
On this terraqueous globe around.
Mount-science she can scale with ease—
Ascend its summit, if she please.
She is the muses' fond delight;
Who're always pleas'd when she's in sight.
Artists, with ease, her name you'll find.
From th' equations here subjoin'd.

viz. $$\left.\begin{array}{l} w+x+y+z=31 \\ wz-x-y=58 \\ xy-w+z=22 \\ wy+xz=157 \end{array}\right\}$$ Where w denotes the 1st letter in the alphabet, x the 2d, y the 3d, and z the 4th, that compose this lady's name; and the 5th letter is the same with the 2d, and the 6th the same with the 3d.

II. QUESTION 536, by Mr. W. Spicer.

To find two such square numbers, that their sum may be a square, and their difference a cube number, and the sides of the said square and cube equal.

III. QUESTION 537, by Mr. W. Toft.

Given ($a =$) the sum of 3 numbers in harmonical porportion, and also ($b =$) their continual product; to find the numbers themselves.

IV. QUESTION 538, by Mr. Tho. Barker.

To determine a point within an equilateral plane triangle, whose perimeter is $= 195$, from which if right lines be drawn to the three angular points thereof, their sum and the sum of their squares shall be $= 113\frac{1}{2}$ and $4337\frac{1}{2}$ respectively.

V. QUESTION 539, by Mr. Paul Sharp.

In what latitude on the 21st of March at 9 o'clock, will the shadow of an object, standing perpendicular to the horizon, be = to the shadow of the same when standing in a position,

position to the horizon, so as to cast the longest shadow possible at 10 o'clock on the same forenoon.

VI. QUESTION 540, *by Mr.* Rob. Butler.

A gentleman has a triangular garden, whose sides are 50, 60, and 70 poles respectively, in which respectively grow three trees *A*, *B*, and *C*, so posited, that if right lines be drawn from tree to tree, a triangle will be so formed that the angles at *A*, *B*, and *C* will be = 50, 60, and 70 degrees respectively: Moreover the tree *A* (in the garden's lesser side) is just 30 poles from the garden's greatest corner. Hence is required the distance of the trees from each other (geometrically), and their situation in the sides of the garden.

VII. QUESTION 541, *by* Jack Hazard.

Three persons *A*, *B*, and *C* toss up, in their turns, and upon equality of skill, a die with two equal faces only, whereof the one is white and the other black, and he that brings up the white face first, is to be entitled to a deposit of 30 guineas: Required the value of each person's expectation.

VIII. QUESTION 542, *by* Curiosus.

An horizontal dial, brought from the coast of France in the late expedition (but without the gnomon), was put into my hands, and being desirous of knowing for what latitude it was made, I found, by means of a pair of compasses, that the angle included between the hour lines of 12 and 3 was exactly = the angle comprehended between those of 4 and 6; from which the latitude may be found, and is here required.

IX. QUESTION 543, *by Miss* Ann Nicholls.

One night I made observation of an eclipse of the moon, but clouds interposing, I could not see the beginning; however, some time after it cleared up, and at 9 h. 0' that evening, I observed her to be exactly 6 digits eclipsed on her lower limb: At 9 h. 36 m. 22 s. the digits eclipsed were 10° 18', and then clouds appearing again, prevented all further observation. By calculation I found the semidiameter of the moon = 16 m. her latitude at the middle = 30 m. north, and her horary motion from the sun = 33 m.——From this curious observation, the beginning, middle, and end of this eclipse are required.

X. QUESTION 544, *by Mr.* W. Toft.

A young man, at sea, in working his day's work, forgot to allow for the variation of the compass, and by that means made his difference of latitude too much by 22, and the departure too little by 34 miles; and the distance sailed that day was 100 miles in a direct course, in the north-west quarter: From whence is required the true difference of latitude and departure, and the nature and quantity of the variation.

XI. QUESTION 545, *by Mr.* T. Moss.

To investigate a general theorem (without finding the whole content) for determining the true ullage, in ale and wine gallons, of any standing spheroidical cask, whose bung, head diameters, length, and wet inches are given; and that by a rule fit for the practical gauger.

XII. QUESTION 546, *by* Lycidas.

In what point of the ecliptic between ♈ and ♋ does the sun's longitude exceed his right ascension by the greatest difference possible?

XIII. QUESTION 547, *by Mr.* Paul Sharp.

There is a conical tub whose bottom diameter, depth, and top diameter are in arithmetical proportion, whose common difference is 12 inches, and which, when full of water, will empty itself through a circular hole, of one inch diameter, in its bottom, in 15·261584 minutes: Quere its content in ale gallons?

XIV. QUESTION 548, *by Mr.* T. Moss.

Supposing there are two given concentric ellipses, whose given difference of transverse diameters is equal to that of their conjugates; 'tis required to demonstrate, in a general manner, whether the part of any common diameter, intercepted between the peripheries of those ellipses, is greater or less than half the said given difference of their transverse or conjugate diameters.

XV. QUESTION 549, *by Mr.* Rob. Butler.

A chain's length is 149 feet, and its variable thickness (or law of density) such, that if its ends are fastened to two pins

pins situated in the same horizontal line at the distance of 100 feet from each other, it will dispose itself into a circular form; from whence is required the law of density and the chain's weight, supposing one foot of either end to weigh one pound.

The PRIZE QUESTION, *by Mr.* Da. Kinnebrook.

Suppose a given right-angled plane triangle, whose hypothenuse is an uniform slender rod, and its base parallel to the horizon, to revolve about its perpendicular as an axis, whilst a ring slides freely along its hypothenuse: It is required to determine the time of the ring's descent along the said hypothenuse, its length being $= 50$ feet, the perpendicular $= 40$, and the time of one revolution round the axis $= 3$ seconds.

1766.

Questions answered.

I. QUESTION 535 *answered by Mr.* Rich. Gibbons.

FROM the addition of the two first equations is had the sum of the two quantities w, z added to their product $= 89$; which (as they must be whole positive numbers, each not greater than 24, per quest.), it is easy to perceive, will be 4 and 17, and consequently $x + y = 10$ and $xy = 9$, from the 1st and 3d given equations: whence x evidently appears to be 9 and $y = 1$: and these are also equal to the 5th and 6th values required, per quest. Consequently the required numbers are 4, 9, 1, 17, 9, 1, and answer to *D, I, A, R, I, A*; the lady's name.

In this manner, exceeding near, Mess. *J. Bliss*, *M. Gordon*, and *W. Rawle* have likewise answered it.

But if the four given equations are added together, &c. the following one will be derived, viz. $\overline{z+y} \times \overline{w+x} = 268 - 2z$: From which, and the first given equation ($\overline{z+y} + \overline{w+x} = 31$) will be found $\overline{z+y}^2 - 2 \times \overline{z+y} \times \overline{w+x} + \overline{w+x}^2 = 8z - 111$; and consequently $z + y = 15\frac{1}{2} + \frac{1}{2}\sqrt{8z - 111}$.

U 2 Then,

Then, as w, x, y, and z are to be whole positive numbers, each not exceeding 24 (per quest.), z must be such a one as will bring out $\sqrt{8z-111}$ not only a whole, but also an odd number; which, with very little trouble, will be found to be 17: Whence $y=1$, and the 1st and 3d given equations become $x+w=13$ and $x-w=5$; consequently $x=9$ and $w=4$, and the celebrated lady's name required is DIARIA.

It is also answered by Mess. *J. Ashmore, T. Atkinson, T. Barker, W. Barnes, T. Bosworth, W. Cole, J. Coulthred, Is. Dalby, R. Dening, H. Fry, W. Gough, R. Hale, R. Peckham, J. Potter, J. Probert, Tho. Robinson, Alex. Rowe, W. Sewell, P. Sharp, Edw. Smith, W. Spicer,* Mrs. *Eleanor Suggett, J. Swift, J. Tarratt, J. Thorne, T. Wilkin, J. Woolley, J. Young*, and some others.

II. QUESTION 536 *answered by Mr.* William Spicer (*the Proposer*).

Assume $\frac{1}{2}x^2+\frac{1}{2}x^3$ and $\frac{1}{2}x^2-\frac{1}{2}x^3$ for the two required square numbers, whose sum and difference are manifestly a square and cube number, having each the same root x; then, finding two other square numbers $4nn\times\overline{1+nn}|^{-2}$ and $\overline{1-nn}|^2\times\overline{1+nn}|^{-2}$, such that their sum may be $=1$, $\frac{1}{2}x^2+\frac{1}{2}x^3$ will be $=4nnxx\times\overline{1+nn}|^{-2}$ and $\frac{1}{2}x^2-\frac{1}{2}x^3=\overline{1-nn}|^2\times x^2\times\overline{1+nn}|^{-2}$, and consequently, the equal root, $x=\overline{6n^2-n^4-1}\times\overline{1+nn}|^{-2}=$ (when n is $\frac{1}{2}$) $\frac{7}{25}$; which value substituted for x in the above assumed expressions, they become $\frac{784}{15625}$ and $\frac{441}{15625}$, two numbers answering the conditions of the question.——Mr. *Wales* denotes them by $9xx$ and $16xx$ (it being evident, from 47 Euc. 1, that they will be in the ratio of 9 to 16); whence (their sum and diff. being $=25xx$ and $7xx$ respectively) $5x$ will be $=\sqrt[3]{7xx}$ (per quest.), and consequently $x=\frac{7}{125}$, and the required numbers the same as before, &c.——Mr. *R. Gibbons*' answer is not essentially different from this last; and Mr. *C. Hutton* answers it, very ingeniously, by a different method.

III. QUES-

III. QUESTION 537 *answered by Mr.* S. Woodbey.

Let x, y, and z denote the required numbers, and then, $x+y+z$ being $= a$, $xyz = b$ (per quest.), and $x : z :: x-y : y-z$ (per the nat. of harmonic proportion), $y \times \overline{x+z}$ will be $= 2xz$, or $yy \times \overline{a-y} = 2b$; from the resolution of which, the value of y, &c. may in any case be found.——If $a = 191$ and $b = 254016$, then will y, x, and z come out $=$ 72, 63, and 56 respectively.

And in this manner, nearly, the answer is given by Mess. *T. Barker*, *William Barnes*, *Isaac Dalby*, *R. Gibbons*, *J. Probert*, *T. Robinson*, *P. Sharp*, *W. Spicer*, *T. Wilkin*, and others.

But Mr. *W. Toft* (the proposer) conceives the three numbers sought to be represented by $z \times \overline{z+v}$, $\overline{z-v} \times \overline{z+v}$ and $z \times \overline{z-v}$ (viz. the numerators of three fractions) $\frac{1}{z-v}$, $\frac{1}{z}$, and $\frac{1}{z+v}$, having their denominators in arithmetic progression, reduced to a common denominator, it being easy to prove them to be in harmonic proportion), and then, $zz \times \overline{zz-vv}|^2$ being $= b$, and $3zz - vv = a$ (per quest.), $2z^3 - az + \sqrt{b}$ will be $= 0$; from whence the value of z, when b is a square number, may be easily found, by the method of divisors, and then v $(= \pm \sqrt{3zz-a})$, and the required numbers, will become known.——But when $\sqrt{b}$ is irrational, let it be divided by the greatest square number m^2 it will admit of, and the irrational quotient call n, and then the foregoing equation (supposing $x\sqrt{n} = z$) will become $2nx^3 - ax + m = 0$; from which x may be found, and consequently the values of z and v, and the numbers required.

EXAMPLE 1. Let $a = 26$ and $b = 576$; then the 1st of the preceding equations becomes $z^3 - 13z + 12 = 0$; whence $z = 3$ and $v = 1$, and consequently 12, 8, and 6 are the required numbers.

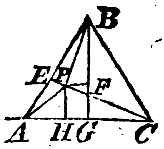
EXAMPLLE 2. Where $a = 13$ and $b = 72$. Then $\sqrt{b} = 6\sqrt{2}$, $m = 6$, and $n = 2$, and the equation in the 2d case becomes $4x^3 - 13x + 6 = 0$: From which $x = \frac{3}{2}$, $z = \frac{3}{\sqrt{2}}$, $v = \frac{1}{\sqrt{2}}$, and 6, 4, 3 = the three numbers required.

EXAMPLE 3. Where $a = 39$ and $b = 1944$.——Here $\sqrt{b} = 18\sqrt{6}$, $m = 18$, and $n = 6$, and the aforesaid equation becomes $4x^3 - 13x + 6 = 0$, as in the last example, and, of consequence, $x = \frac{3}{2}$ also: But $z = 3\sqrt{\frac{3}{2}}$; whence $v = \sqrt{\frac{3}{2}}$, and the required numbers are 18, 12, and 9.

SCHOL. The two foregoing general equations being of three dimensions, z and x will each have three values therein, and v as many corresponding ones; as for instance, in example 1, where the values of z are 1, 3 and -4, and those of v, corresponding, $\sqrt{-23}$, 1 and $\sqrt{22}$; from whence three sets of terms, answering the conditions of the prob. may be found, if those arising from the first and last (viz. from 1 and $\sqrt{-23}$, or from -4 and $\sqrt{22}$) of these corresponding values, can be called real quantities; and, what's somewhat odd, they, as well as the others, answer the conditions of the quest. notwithstanding they are absolutely imaginary: But perhaps neither of them can so properly be called answers, seeing there is one in whole positive numbers, which the prob. in some sort tacitly requires.

IV. QUESTION 538 *answered by Mr.* Isaac Tarratt.

Let P represent the required point within the given equilateral plane $\triangle ABC$, and let the perpendiculars PE, PH, BG, upon AB and AC, be drawn; also let PF be $\perp BG$, and put $a = 32{\cdot}5 = AG$, $b = 56{\cdot}29 = BG$, $c = 113{\cdot}5$, $d = 4337{\cdot}25$, $h = -4112\frac{3}{4}$, $x = BF$, and $y = PF = HG$.



Then (by 47 Euc. 1) $\sqrt{xx + yy} + \sqrt{\overline{a-y}|^2 + \overline{b-x}|^2} + \sqrt{\overline{a+y}|^2 + \overline{b-x}|^2} = c$, and $3xx + 3yy - 4bx = h$; whence $y = \sqrt{\frac{4bx - 3xx + h}{3}}$; and this value substituted for y, an equation, containing one unknown quantity only, will emerge, from which x comes out $= 40$; and thence y is found $= 5{\cdot}6$ (fere), and the distances AP, BP, $CP = 31{\cdot}44$, $40{\cdot}16$, and $41{\cdot}43$, very near.

In this manner, exceedingly near, it is answered by *Amicus*, Mess. *T. Barker* (the proposer), *R. Hale*, *J. Probert*, *Tho. Robinson*, *Alex. Rowe*, *Edw. Smith*, *W. Spicer*, *E. Suggett*, and *Ja. Young*.

V. QUES-

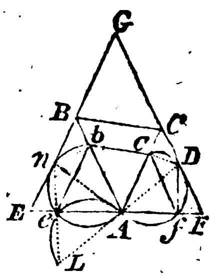
V. QUESTION 539 *answered by Mr.* Tho. Bosworth.

Put m, n = sine and cosine of the declin. p, q = cosines of 45° and 30° (the sun's distance from the merid. at 9 and 10 o'clock), x, y = sine and cosine of the required latitude; then, per spherics, $\overline{npy + mx} \times \overline{1 - \overline{npy + mx}^2}^{-\frac{1}{2}}$ and $nqy + mx$ will be the tangent and sine of the sun's alt. at 9 and 10 o'clock respectively; which, whenever an appearance happens, at those times, like what is mentioned in this question, will (as is easy to demonstrate) be equal to each other, or (when the sun is in the line, and $m = 0$, which is the case, very near, at the time specified in the quest.) $py \times \overline{1 - ppyy}^{-\frac{1}{2}} = qy$: From whence $y = \sqrt{\frac{1}{pp} - \frac{1}{qq}}$ = 0·816496, the cosine of 35° 15′ 52″ the lat. of the place required; which is the same as found by Mr. *Paul Sharp* (the proposer) by a different method.

The solution by Mr. *W. Wales* is nearly the same as this; and Mess. *W. Barnes*, Rev. Mr. *J. Cross*, *J. Dalby*, *R. Hale*, *S. Hodges*, *C. Hutton*, *W. Rawle*, *Edw. Smith*, *W. Spicer*, *J. Tarratt*, *Jos. Webster*, and *Ja. Young*, have likewise answered it.

VI. QUESTION 540 *answered by Mr.* Rob. Butler, (*the Proposer*).

On a right line Ab (of any assumed length, at pleasure) construct a plane triangle having its ∠s A, b, c = 50°, 60°, and 70° respectively, and upon Ab and Ac describe two segments of circles to contain 78° 28′ and 57° 7′ (the garden's greatest angles), and let the diameter AD be drawn, producing it till AD : AL in the given ratio of 2 to 3, and upon AL describe a semicircle cutting the segment Ab in e; through which point and A a right line being drawn, meeting the circumf. AD in f, produce it both ways till AF = 20 and AE = 30 poles; join the points



e, b and f, c, and parallel to those lines let EG and FG (meeting each other in G) be respectively drawn, producing Ab, Ac till they meet the same in B and C; then will EGF be

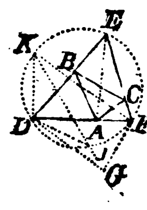
be the given triangle, and A, B, and C the places of the trees in the sides thereof.

DEMONST. Join the points e, L; D, f; and B, C: Then (per sim. triangles, &c.) $Af : Ae$ (:: $AD : AL$ :: 2 : 3) :: $AF : AE$ (per construction); also $\frac{Ab \times AE}{Ae} = AB$, and $\frac{Ac \times AE}{Ae} = AC$: Whence $AB : AC$ (:: $\frac{Ab \times AE}{Ae}$: $\frac{Ac \times AE}{Ae}$) :: $Ab : Ac$, and consequently BC is ∥ Bc,* and the $\triangle ABC$ sim. Abc; and the $\angle$s E, F being respectively = the $\angle$s e, f (the given angles per construction), and the side EF = the given side (per construction), EGF will be the given triangle, and A, B, C the true places of the trees in the sides thereof.

CALCUL. Draw the diam. An, and let the points e, n; b, n; and c, D be joined; then (by the common cases of plane trigonometry) AC will be found = 29·92, AB = 32·46, BC = 26·48, BE = 29·89, and CF = 32·46 poles.

Answer to the same, by Mr. W. Wales.

CONSTRUC. Conceive the given $\triangle DEF$ to be circumscribed by a circle $DKEF$, and upon its least side DF, constitute a $\triangle DGF$ similar to the required one. From the vertex E, to the given point A, draw the right line EA, and make the $\angle ADI = AEF$: Then, the intersection I, of the right line DI with the circumference and the point G being joined, make the $\angle$s EAC, EAB respectively = the $\angle$s IGF, IGD, and, joining C, B (the points where the right lines AC, AB meet the given lines FE, DE); CAB will be the triangle, and A, B, C the places of the trees required.



DEMONST. Produce GI to meet the circumference in K, and let the points D, K and F, K be joined: then (per cor. to

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* The parallelism of BC, Bc, may perhaps be proved more geometrically thus: Since, as is proved above, $Af : Ae :: AF : AE$, or $Af : AF :: Ae : AE$; but, by sim. triangles, $Af : AF :: Ac : AC$, and $Ae : AE :: Ab : AB$, ∴ $Ac : AC :: Ab : AB$, and therefore BC ∥ bc.

to 9. 3. of Simp. Geom. 1st edit.) the $\angle$s DKF, DEF are equal; and, since the $\angle$s FDI, AEF are equal (per construction), and FDI, FKI stand on the same arc FI, the $\angle$s FKI, AEF are equal. Whence the $\angle$s DKI, DEA are also equal; and, as EAC and IGF, EAB and IGD are equal (per construction), the $\triangle$s ABE and GDK, ACE and GFK are similar, and consequently $AC : FG :: EA : KG :: AB : GD$; whence the $\triangle$s ABC, GDF, having their respective sides proportional, must have their angles equal and be similar.——The given point A being the point of contact of the side DF, with a circle inscribed in the given $\triangle$ DEF, a very simple construction, in that particular case, may from thence be derived.

Mess. *R. Hale* and *C. Hutton* very easily reduce it to prob. 39 on pa. 244 of Simp. Geom. 2d edit.——And it may also be constructed from prob. 23 on p. 169 of his Geom. 1st edit.

VII. QUESTION 541 *answered by Mr.* Ja. Hazard.

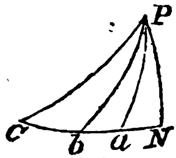
Put the given deposit (30 guineas) $= d$; then, the probability of bringing up a white face at any one throw being constantly $\frac{1}{2}$, the expectation of A on the 1st throw will be $\frac{1}{2} \times d$, which would also be the expectation of B on the 2d throw, were it certain that A would miss the 1st; but the probability of that being also $\frac{1}{2}$, his expectation can only be $\frac{1}{2}$ of $\frac{1}{2}d$, or $\frac{d}{4}$: In like manner, the value of the expectation of C, on the 3d throw, will be found to be $\frac{d}{8}$; for, it would be $\frac{d}{2}$, if it was certain that A would miss the 1st and B the 2d throw, but the probability of that being $\frac{1}{2} \times \frac{1}{2}$, his expectation thereon can only be $\frac{1}{2} \times \frac{1}{2}$ of $\frac{d}{2}$, or $\frac{d}{8}$: And by reasoning in this manner, their expectations on the succeeding throws, and consequently the values of their whole expectations (forming three infinite progressions) may be found, that of A's being $\frac{d}{2} \times : 1 + \frac{1}{8} + \frac{1}{8^2} + \&c. = \frac{d}{2} \times \frac{1}{1 - \frac{1}{8}} =$ 18l. that of B's $= \frac{d}{4} \times : 1 + \frac{1}{8} + \frac{1}{8^2} + \&c. = \frac{d}{4} \times \frac{1}{1 - \frac{1}{8}} =$ 9l. and that of C's $= \frac{d}{8} \times : 1 + \frac{1}{8} + \frac{1}{8^2} + \&c. = \frac{d}{8} \times \frac{1}{1 - \frac{1}{8}} =$ 4l. 10s. Or, the same conclusion may be derived without sum-

ſummation, by conſidering that, as their correſponding terms are every where in the conſtant ratio of 1, $\frac{1}{2}$, and $\frac{1}{4}$, their ſums will alſo be in the ſame invariable ratio.

The anſwers by Meſſ. *T. Barker*, *T. Boſworth*, *C. Hutton*, *P. Sharp*, *W. Spicer*, and *W. Wales*, are not materially different.

VIII. QUESTION 542 *anſwered by Mr.* W. Wales.

In the ſpherical triangles PNa, PNb, and PNc, all right-angled at N, are given their reſpective angles at the pole $P = 45°$, $60°$, and $90°$, and the arc $cb = aN$, to find PN, the lat. of the place, the ſine of which ſuppoſe $= x$: Then (per ſpherics) 1 (rad.) : x :: 1 (tang. $\angle NPa = 45°$) : x $=$ tang. arc Na, and, putting $t =$ tang. $\angle NPb = 60°$, $1 : x :: t : tx =$ tang. of arc Nb; whence its cotang. (or the tang. of the arc bc) is $\frac{1}{tx}$, which, per queſt. is $= x$, and conſequently $2xx = \frac{2}{t} = 1{\cdot}15475$, &c. the verſed ſine of 98° 52′, double the lat. required.



Mr. *Paul Sharp*'s anſwer is nearly the ſame as this.

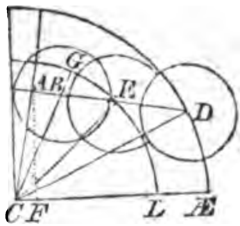
Meſſ. *T. Allen*, *T. Barker*, *W. Barnes*, *T. Boſworth*, *J. Chipchaſe*, *J. Dalby*, *R. Hale*, *C. Hutton*, *J. Probert*, *W. Rawle*, *T. Robinſon*, *Alex. Rowe*, *W. Spicer*, *J. Tarratt*, *Joſ. Webſter*, *Tho. Wilkin*, and *J. Young*, by eaſy ſhort proceſſes, find $x = t^{-\frac{1}{2}} = {\cdot}7598357$ the ſine of 49° 26′ 59″, the lat. required; and Mr. *John Potter*, by analogy, without algebra, very ingeniouſly derives the ſame concluſion.

It may alſo be anſwered another way, independent of algebra, whereby the ſine of the arc ab comes out = ſine of 15° ÷ ſine of 105°; from whence the lat. the dial was made for may be readily found.

IX. QUESTION 543 *anſwered by Mr.* T. Allen.

Let $ABED$ and $CFÆ$ repreſent the portions of the lunar orb and the ecliptic, E, B the places of the ☽'s cent. at the 1ſt and 2d obſervations, and AF ($= 30'$) her lat. at the middle of the eclipſe. Then, the $\angle CAF$ being found =

5° 15′



5° 15′ (from Aſtron. Tables), CA will be $= 30{\cdot}1517'$, which put $= a$, $AB = x$, BE $(= 20')$ $= c$, and $CE - CB$ $(= 11{\cdot}46') = n$; then will $\sqrt{aa + xx} = CB$, $\sqrt{aa + \overline{c + x}|^2} = CE = n + \sqrt{aa + xx}$, and (putting $cc - nn = 2m$), $x = \sqrt{\frac{cc}{4} + \frac{nnaa}{2m}} - \frac{m}{2} - \frac{c}{2} = 11{\cdot}84'$: Whence $AE = 31{\cdot}84'$, CE (the ſemidiameter of the ⊖'s ſhad.) $= 43{\cdot}85'$, and the meaſure of the ſemiduration $AD = 51{\cdot}7'$; and thence are eaſily deduced 8h. 23m. 54s. 9h. 57m. 54s. and 11h. 31m. 54s. for the beginning, middle, and end of the eclipſe, and 11° 8′ 12″ = the greateſt quantity eclipſed.

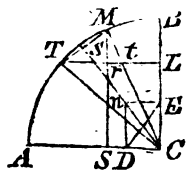
The ſame anſwered by Mr. Wm. Wales.

As 60′ : 33′ (the given hor. mot) : : 36′ 22″ (the time between the obſerv.) : 20′ $= EB = a$; alſo 6 dig. : 16′ (the ☽'s ſemidiameter) : : 4 dig. 18′ : 11·466′ &c. $= BG = b$; whence, putting $x = GE$ (the ſemidiam. of the earth's ſhad.), $a : 2x - b :: b : \frac{2bx - bb}{a}$, half of which = the diſt. of CA ($\perp DA$ and $= 30'$, the given lat. of the ☽ at A, the middle of the eclipſe) from the middle of the baſe BE of the triangle BCE; whence (per 47 Euc. 1) $xx - \frac{\overline{aa + 2bx - bb}|^2}{4aa} = 900$ (CA^2), and conſequently $x = 43{\cdot}6965'$, and $EA = 31{\cdot}7708'$: Then 33′ (horary motion) : 60′ : : 31·7708′ : 57′ 46″, which added to the time of the 1ſt obſervation, gives 9h. 57m. 46s. for the middle. Alſo (per 47 Euc. 1) $DA =$ $= 51{\cdot}6107'$ = (in time) 1h. 33m. 50s. which ſubtracted from and added to the middle, gives 8h. 23m. 56s. and 11h. 31m. 36s. for the beginning and end reſpectively.

Meſſ. *W. Barnes*, *T. Boſworth*, *R. Hale*, *C. Hutton*, *Rob. Peckham*, *W. Spicer*, and *Joſ. Webſter* have alſo anſwered it very ingeniouſly, and bring out nearly the ſame concluſion.

X. QUESTION 544 *anſwered by Mr.* Cha. Hutton.

CONSTRUC. About the center C with rad. CA $(= CB \perp CA) = 100$ (the diſt. failed) deſcribe the quad. AB, and take



take CD, CE thereon, respectively $= 22$ and 34 (the given err. in lat. and depart.), and let the points D, E be joined; thro' the center C and the middle of DE draw the right line Cs, and parallel thereto, and at the dist of $\frac{1}{2} DE$ therefrom, conceive two other right lines to be drawn intersecting the arc AB in T and M; join the points C, T and C, M, and draw $TL \parallel AC$, meeting CB in L; then is the $\angle TCB$ the measure of the *true* and MCB the *erroneous* course, and the $\angle TCM$ the variation, which is westerly; CL is the *true* diff. of lat. and TL the departure.——For, joining the points T, M, and drawing $MS \parallel BC$, cutting TL in r, and compleating the $\square\, DCEn$, the $\triangle$s TMr and DCE or DCn are equal and similar, because of the equal and perpendicular hypothenuses TM, Cn, and the parallelism of the legs, *i. e.* $Tr = CE = 34$, and $Mr = DC = 22$, as by the quest. they ought.——The calculation from this construction is extremely easy; whereby the $\angle TCM$ ($=$ variat.) and CL, TL ($=$ *true* diff. of lat. and depart.) come out $= 23^{\circ}\ 22'$, $71{\cdot}217898$ and $70{\cdot}199816$ miles respectively.

Mess. *T. Bosworth*, *W. Toft* (the proposer), and *W. Wales* likewise construct it in this manner, very near.

Mess. *T. Allen*, *T. Barker*, *W. Barnes*, *W. Cole*, the Rev. *J. Cross*, *J. Dalby*, *R. Gibbons*, *R. Hale*, *J. Probert*, *T. Robinson*, *Alex. Rowe*, *P. Sharp*, *W. Spicer*, Mrs. *Eleanor Suggett*, *Tho. Wilkin*, and *Ja. Young have likewise sent neat and concise algebraic solutions.*

XI. QUESTION 545 *answered by Mr.* Joseph Walker, *Supervisor in the Excise.*

Let $h =$ the head diameter, $b =$ the bung diameter, $l =$ the length, and $w =$ the wet inches of the proposed cask, (all which are given); then will $lb \div \sqrt{bb - hh} =$ the transverse of the generating semiellipse, and $\frac{1}{2} lb \div \sqrt{bb - hh} - \frac{1}{2} l =$ the part thereof intercepted between the vertex of the ellipse and the middle of the head diameter of the cask; and consequently, putting $a = {\cdot}7854$ and $x =$ any variable depth, estimated from the center of the bottom up the axis, the fluxion of the content corresponding will be $= a\dot{x} \times \overline{hh + \frac{bb - hh}{\frac{1}{4} ll} \times \overline{l - x} \times \overline{x - \frac{1}{4} ll}}$; and the fluent, viz.

aw

$aw \times bb - \frac{bb - hh}{\frac{1}{4}ll} \times \frac{1}{4}ll - \frac{1}{2}l - \frac{1}{3}w \times w$, when $x = w$, divided by 282 for ale, or 231 for wine, gives the ullage, in ale or wine gallons, required; and, expressed in words, furnishes the following

GENERAL RULE.

Multiply the difference between the semi-length and $\frac{1}{3}$ the wet inches by the wet inches, and subtract the product from the square of the semi-length, and multiply the remainder by the rectangle under the sum and difference of the bung and head diameters divided by the square of the semi-length; then, subtracting this last product from the square of the bung diameter, and multiplying the remainder by the wet inches divided by 359·65 for ale or 294·11 for wine, the result will be the ullage, in ale or wine gallons, required.

Ingenious answers to this question (with practical rules deduced therefrom) have also been received from Mess. *T. Allen*, *R. Gibbons*, *C. Hutton*, and *Barth. Sikes*; but our scanty room will not permit us to insert them.

The answer by Mr. *T. Moss* (the proposer) may now be seen in his excellent Treatise of Gauging, lately published.

XII. QUESTION 546 *answered by Mr.* T. Allen.

Put s = cosine of 23° 29′, the obliquity of the ecliptic, x = tang. of the ☉'s right ascension to rad. 1; then, per spherics, $\frac{x}{s}$ = tang. of the longitude, and $\frac{x - sx}{s + xx}$ (per pa. 55 of Simp. Trigo.) = tang. of their diff. which, per quest. is to be a max. and therefore its flux. made = 0, and reduced, gives $x = \sqrt{s}$ = ·957693 the tang. of 43° 45′ 43″: Whence their greatest diff. = 2° 28′ 35″, and the ☉'s place will be ♉ 16° 14′ 16″.

Mess. *T. Barker*, *T. Bosworth*, *W. Barnes*, *J. Dalby*, *R. Hale*, *Step. Hodges*, *J. Potter*, *J. Probert*, *T. Robinson*, *A. Rowe*, *P. Sharp*, *W. Spicer*, *J. Tarratt*, *W. Wales*, *T. Wilkin*, and *J. Young* answer it in this manner, and bring out the same conclusion, exceedingly near.

Astronomicus, by trigonometric reasoning only, finds the sine of the diff. of the ☉'s long. and his right ascension = the right angle under the sine of their sum and the versed

sine of the obliquity of the ecliptic directly, and the versed sine of the supplement of the obliquity of the ecliptic inversely, which, when a max. will evidently become = the rectangle under rad. and the versed sine of the obliquity of the ecliptic directly, and the versed sine of the supplement thereof inversely, = 0·043201, the sine of 2° 28¼′ nearly; from which the same conclusion as above exhibited may be found, very near.

XIII. QUESTION 547 *answered by Mr.* P. Sharp (*the Proposer*).

Put $a = 16\frac{1}{12}$ feet, $b = 0\cdot005454$ (the area of the circular hole), $c = 3\cdot8197$, and $x =$ the depth of the tub, in feet; then will $x + 1 =$ top, and $x - 1 =$ bottom diameter, $\frac{3x^3 + x}{c} =$ its content, and $\frac{3x^3 + x}{bc\sqrt{2ax}} =$ the time of evacuation with the 1st velocity (per pa. 173 of Emer. Mechan. 1st dit.): Whence $\frac{3x^3 + x}{bc\sqrt{2ax}} \times \frac{30xx - 20x + 14}{15xx + 30x + 15} = 15\cdot261584 \times 60 = 915.69504''$ (per pa. 111 of Emer. Flux. 1st edit.); from which x is found = 4 feet, and the top and bottom diameters = 5 and 3 respectively, and the content = 51·31293 solid feet = 314·42767 ale gallons.*

Mess. *T. Allen* and *C. Hutton*'s answers are nearly the same.——It is also answered by Mess. *W. Barnes*, *W. Rawle*, *Tho. Robinson*, *Alex. Rowe*, *Edw. Smith*, and *W. Spicer*.

XIV. QUES-

* This solution is false, by the wrong application of the theor. in Emers. Flux. above quoted. The conclusion will be very different when that theor. is properly applied, as any one may easily find. But the same will be more easily and directly deduced from the 1st Art. of our Miscel. cor. 1. pa. 7, thus: Using the same notation as above, we have $\frac{2\sqrt{x}}{b\sqrt{2a}} \times \cdot 7854 \times$

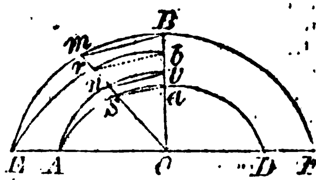
$$\frac{8.\overline{x-1}|^2 + 4.\overline{x-1}.\overline{x+1} + 3.\overline{x+1}|^2}{15} = \frac{288\sqrt{x}}{\sqrt{32\frac{1}{6}}} \times$$

$$\frac{15xx - 10x + 7}{15} = 915\cdot69504; \text{ or } \sqrt{x} \times \overline{xx - \tfrac{2}{3}x + \tfrac{7}{15}} = 18\cdot033;$$

Hence the length x is = 3·4026 nearly; and therefore the top and bottom diameters = 4·4026 and 2·4026.

XIV. QUESTION 548 *answered by* Mr. T. Moss (*the Proposer*).

Let AaD, EBF be two concentric semi-ellipses, in which $Ea = Ba = FD$. Take Ob to OE in the given ratio of Oa to OA, and Ov to $OA :: OB : OE$, and conceive the elliptic arcs Eb, Av, respectively similar to Aa, EB, to be described. Draw the semidiameter Om, cutting the similar elliptic arcs in



s, n, and r, and let the points r, b; n, v; and s, a be joined; then (alternately) $Ob : Oa :: OE : OA$ and $OB : Ov :: OE : OA$; whence, by equality and division of ratios, &c. $OB - Ob : Ob :: Ov - Oa : Oa$, or, alternately, $Bb : va$ ($:: Ob : Oa$) $:: OB : Ov$, by equal. from above. Also, by sim. Δs, &c. arising from the similarity of the ellipses,

$Om : On :: OB : Ov$ ($:: OE : OA$, by supposition)
$Or : Os :: Ob : Oa$ ($:: OE : OA$, by supposition likewise) } and $Ov : Bv ::$

$On : mn$; whence, by equality, division of ratios, and alternation, $mr : ns$ ($:: Or : Os :: Ob : Oa$) and so is $Bb : va$ (by equality, from above): Consequently the flux. (or decrease, supposing Om to be increasing and indefinitely near to OB) of mr, is to the flux. of ns in the constant ratio of Bb to va, and the flux. of Om to the flux. of On as $OB : Ov$ (because, if the fluents of two variable quantities are always in a constant ratio, their fluxions must necessarily be always in the same ratio); that is, alternately,

Bb : flux. of mr $::$ va : flux. of ns,
OB : flux. of Om $::$ Ov : flux. of On, } and, alternately

and by equal. from above, &c. $Ov : va ::$ flux. of $On : \frac{va}{Ov}$ $\times$ flux. of $On =$ flux. of ns; also, for the same reason, $Ov : Bv$ ($On : mn$, from above) $::$ flux. of $On : \frac{Bv}{Ov} \times$ flux. of $On =$ flux. of mn; which, as Bv is always $\sqsubset va$, demonstratively proves the increase of mn to be always $\sqsubset$ the decrease of ns, and consequently that ms is every where $\sqsubset Ba$, or EA (because, if the said increase and decrease were always equal, ms would be a constant quantity $= Ba$, and if the decrease was greatest, ms would be $\sqsupset Ba$; as is very evident).

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ſame by Mr. Rob. Butler (*the Propoſer*).

dinate GK, and, perpendicular thereto, PL, int of interſection of the tangents DP, GP; mG, and $LI \parallel GK$, KD, and GP reſpec- t the abſciſſa $DK = x$, ord. $GK = y$, $w =$ portion DG, and $a =$ the invariable tenſion lt point): Then, n being indefinitely near G, $nm = \dot{y}$, and, by mechanics, GP, GL, and the tenſion at G and D, and weight of the ctively: Whence $w : a$ (: : $PL : GL$) : : $\dot{x} : \dot{y}$, ntly $\frac{w}{a} = \frac{\dot{x}}{\dot{y}}$; which is general, let the curve ll: But, in the preſent caſe, the curve being ſe diam. ſuppoſe $= 2r$), $\frac{\dot{x}}{\dot{y}}$ will be $= \frac{y}{r-x} =$

he weight (w) will be always as the tang. of ding arc DG: Therefore, putting T, $t =$ the 39′ and 77° 32′, the degrees, &c. contained in D (70) and QD (69 feet) reſpectively, $T : t : :$ (per queſt.); whence $w = \frac{T}{T-t} = 10{\cdot}86$ lb. bled gives 21·72 lb. = the whole weight of the red.——Ingenious anſwers to it have likewiſe ived from Meſſ. *J. Chipchaſe* and *W. Spencer*, different methods, find the weight, the law of

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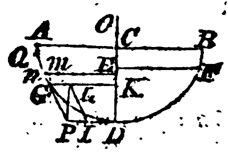
The certainty of this may also in some measure be evinced by considering the ray Om as revolving about the center O; it being easy to perceive that its intercepted part sm will continually vary, as the inclination to the axis EF varies, and consequently that it must pass through a max. or a min. before it can become of the same magnitude again, in E, as it was of, at first, in B.——But it is very easy to demonstrate that it will begin to increase immediately on its departure from the position OB, and be greater than aB the moment before its coincidence with AE ($= aB$).——It therefore must pass through a max. between the positions OB, OE, and be every where greater than its two extreme equal values aB, AE.

Mr. *T. Allen* has given an algebraic solution to it.

XV. QUESTION 549 *answered by Mr.* Cha. Hutton.

Call the abscissa DE, x; the ord. EF, y; the arc DF, z; and its tangent t; also call the tangent of the whole arc DB, T. ——Now, in any curve formed by a perfectly flexible line, the tension or force drawing in direction of the tang. at D (the lowest point) is a constant quantity, which let be denoted by a. Then, the general prop. of any such curve being as 1 (rad.) : tang. $\angle EFD$: : a : weight of the arc DF, or (because the angle at F is, in the circle, measured by the arc DF) $1 : t :: a : at =$ weight of the arc DF, and $1 : T :: a : aT =$ weight of the arc DB, $aT - at$ will be $=$ 1 lb. (per quest.), supposing $DF = 69$ feet; whence $a = \overline{T-t}|^{-1}$, and consequently $2aT = 2T \div \overline{T-t} =$ weight of the whole chain ADB.



Now, since the arc DB is given $= 70$, and the sine $CB = 50$, the rad. OD will be found $= 50{\cdot}99397$, and the length of the semi-circumference $= 160{\cdot}26266$; whence the arcs DB (70) and DF (69) appear to contain $78^\circ\ 39'$ and $77^\circ\ 31'\ 36''$, and their tangents T, t to be $= 4{\cdot}9818813$ and $4{\cdot}5210821$ respectively, and $2T \div \overline{T-t} = 21{\cdot}6227$ lb. $=$ weight of the whole chain required——The law of density, or of the section of the chain, is directly as the flux. of the tang. and inversely as the flux. of the arc; for $a\dot{t} =$ flux. of the weight $=$ weight of the length $\dot{z}$; which, divided by the length $\dot{z}$, gives $a\dot{t}\dot{z}^{-1} =$ the density or section at any point.

COR.

Cor. 1. The law of denſity is as $a\ddot{t}z^{-3}$ in all curves; for the manner of finding it is general, ſuppoſing t to be the tang. of the angle in which the curve cuts the ordinate.——In the catenary, which is formed by a line of a conſtant thickneſs, or whoſe law of denſity is a conſtant quantity, in order to have it as ſuch, in the general expreſſion, $\dot{t}$ muſt be as $\dot{z}$, and thence t as z; which is a known property of that curve.

Cor. 2. The arc DB can never become a quadrant; becauſe then the denſity at B would be infinitely great.

Schol. Hence may be eaſily deduced the requiſites for building the arches of bridges of any known curvature deſirable, either for elegance or to ſuit the ſituation of the place, &c. and yet be equally ſtrong in all their parts, as if they were catenarian arcs.

Anſwer to the ſame by Mr. Rob. Butler (*the Propoſer*).

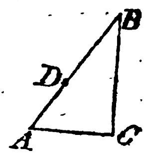
Draw the ordinate GK, and, perpendicular thereto, PL, from P, the point of interſection of the tangents DP, GP; alſo draw nm, mG, and $LI \parallel GK$, KD, and GP reſpectively, and put the abſciſſa $DK = x$, ord. $GK = y$, $w =$ weight of the portion DG, and $a =$ the invariable tenſion at D (the loweſt point): Then, n being indefinitely near G, $Gm = \dot{x}$ and $nm = \dot{y}$, and, by mechanics, GP, GL, and PL will be as the tenſion at G and D, and weight of the part DG reſpectively: Whence $w : a$ ($:: PL : GL$) $:: \dot{x} : \dot{y}$, and conſequently $\frac{w}{a} = \frac{\dot{x}}{\dot{y}}$; which is general, let the curve be what it will: But, in the preſent caſe, the curve being a circle (whoſe diam. ſuppoſe $= 2r$), $\frac{\dot{x}}{\dot{y}}$ will be $= \frac{y}{r-x} = \frac{w}{a}$; that is, (the weight ($w$) will be always as the tang. of its correſponding arc DG: Therefore, putting T, $t =$ the tang. of 78° 39′ and 77° 32′, the degrees, &c. contained in the arcs AD (70) and QD (69 feet) reſpectively, $T : t ::$ $w : w - 1$ (per queſt.); whence $w = \frac{T}{T-t} = 10.86$ lb. which doubled gives 21·72 lb. $=$ the whole weight of the chain required.——Ingenious anſwers to it have likewiſe been received from Meſſ. *J. Chipchaſe* and *W. Spencer*, who, by different methods, find the weight, the law of

 denſity

density or thickness, and the diameter in the middle and at each end.

The PRIZE QUESTION *answered by Mr.* W. Spencer, *of* Stannington, *near* Sheffield, *in* Yorkshire.

Let BA represent the given rod, revolving about the axis BC perpendicular to the horizontal line AC, D the place of the ring at the end of any time t, and v its velocity, there, in the direction DA.



Put AB (= 50 feet) $= r$, BC (= 40) $= p$, AC (= 30) $= b$, $32\frac{1}{6} = g$, 3'' (the time of one revolution) $= s$, 3·1416 &c. $= \frac{1}{2}c$, and $BD = x$; then will $\frac{bccx}{rss} =$ the centrifugal force in a direction parallel to the horizon, and its effect in the direction DA (by the resolution of forces) $= \frac{bbccx}{rrss}$; which added to $\frac{pg}{r}$, the true measure of the force of gravity in the same direction, gives $\frac{bbccx}{rrss} + \frac{pg}{r} =$ the whole force accelerating the ring along the rod, at D: Whence, by the principles of motion, $\frac{bbccx\dot{x}}{rrss} + \frac{pg\dot{x}}{r} = v\dot{v}$, and (putting $\frac{bbcc}{rrss} = m$, and $\frac{pg}{r} = n$) $v = \sqrt{mxx + 2nx}$; consequently $\dot{t} = \frac{1}{\sqrt{m}} \times \frac{\dot{x}}{\sqrt{2qx + xx}}$, putting $q = \frac{n}{m}$; and, taking the correct fluent $t = \frac{1}{\sqrt{m}} \times$ hyp. log. of $\frac{q + x + \sqrt{2qx + xx}}{q}$ = (when x becomes $= BA = 50$) 1·6559'' &c. = 1'' 39''' 21'''' &c. the time required.

COR. 1. The time of descent along the rod in motion, is less than the time of descent along it when at rest, by 18''' 51'''' &c.

COR. 2. When x becomes $= \frac{*grss}{pcc}$, the ring, if at free liberty, would no longer slide along the rod, but fly off.

In

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* Found by making $\frac{bg}{p}$ (the grav. of the ring on the rod) $= \frac{bccx}{rss}$ the horizontal centrifugal force above found.

In this manner, exceedingly near, the answer is given by Mess. *T. Allen*, *R. Butler*, *J. Chipchase*, *Curiosus*, and *C. Hutton*; but the prize of 12 Diaries was won by Mr. *William Spencer*, and that of 8 by *Curiosus*.——Mr. *Da. Kinnebrook* has answered the 6th, 7th, 8th, 15th, and his own quest. (the prize), but his letter came to hand too late to be taken any further notice of.

The Eclipses calculated for 1766.

This year will produce four eclipses, two of which only will be visible in this island.

The first is of the sun, on Sunday the 19th of February, a little after noon: It will be invisible here, but annular and central in the southern parts of the Pacific and Southern Oceans, and may be seen from the most southern parts of South America, Africa, Madagascar, and other Indian islands.

The second is a partial and visible lunar eclipse, on Monday the 24th of February, in the evening, according to the following calculations.

Calculated by	Beg. h.	m.	Mid. h.	m.	End h.	m.	Dur. h.	m.	Dig. °	'
Mr. W. Chapman, for Foxton, Leicestershire	6	32⅓	7	40¼	8	47¾	2	15	4	6
Mr. S. Hodges, for Althorp, Northamptonshire	6	13½	7	21½	8	29⅔	2	16½	4	10⅔
Mr. Isaac Tarratt, for London	6	26⅚	7	36½	8	43	2	16⅙	4	6⅚
Mr. Tho. Harris, for Brington	6	13½	7	21⅔	8	29⅚	2	16½	4	8¾
Mr. J. Metcalfe, for London	6	38¼	7	46⅚	8	55⅙	2	16⅞	4	12¼
Mr. Allen, for Spalding, Lincoln.	6	40	7	37	8	34	1	54	4	10

The moon will be vertical at the middle of this eclipse to the Indian Ocean, between the island of Ceylon and east part of Abissinia, in Africa; it will therefore be visible to all Europe and Asia, and greatest part of Africa, from beginning to end; and at the end may be seen in the Brazils, in South America.

The third is an eclipse of the sun, on Tuesday the 5th of August, in the afternoon, correspondent to these computations.

Cal-

Calculated by	Begin. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° ′
Mr. W. Chapman, for Foxton	5 24$\frac{1}{3}$	6 17$\frac{5}{6}$	7 7$\frac{1}{2}$	1 43$\frac{1}{6}$	5 4
Mr. Steph. Hodges, for Althorp	5 7$\frac{2}{3}$	6 20$\frac{1}{4}$	7 9	1 43$\frac{1}{6}$	4 33$\frac{1}{4}$
Mr. Isaac Tarratt, for London	5 28	6 22$\frac{1}{2}$	7 10$\frac{5}{6}$	1 42$\frac{5}{6}$	4 35$\frac{1}{8}$
Mr. Tho. Harris, for London	5 30$\frac{5}{6}$	6 23$\frac{1}{2}$	7 12$\frac{1}{2}$	1 41$\frac{2}{3}$	4 36$\frac{1}{2}$
Mr. T. Allen, by construc. for Spalding	5 25$\frac{1}{2}$	6 21	7 8$\frac{1}{2}$	1 43	4 24
Mr. J. Metcalfe	5 25$\frac{1}{2}$	6 18$\frac{1}{2}$	7 8$\frac{1}{4}$	1 42$\frac{3}{4}$	4 37$\frac{1}{4}$

This solar eclipse may be seen from all parts of North America, the Atlantic Ocean, north and west parts of Africa, western parts of Europe, and to parts near the north pole.

The fourth and last is of the moon, on Wednesday the 20th of August, in the morning, and visible in America. This clipse will be ended before the moon rises at London, and consequently will be invisible there, and to all the western parts of Europe; but to all parts of Asia, most parts of Africa, and eastern parts of Europe, it will be visible. The digits eclipsed will be a little more than 6 on the moon's lower limb. J. METCALFE.

We have been favoured with other calculations of the eclipses, and particularly from Mess *T. Atkinson*, *W. Baylis*, *Ed. Greensted*, *Rob. Peckham*, *W. Swift*, and *Jos. Webster*; which we are sorry our narrow bounds will not suffer us to publish.

New Questions.

I. QUESTION 550, *by Mr.* Tho. Sadler.

Ye fair, who can with ease unfold,
What puzzled Oedipus of old,
And can, from algebraic art,
Th' abstrusest of all things impart;
From what you see appear below,
John's age and fortune you will know;
Who courts young Susan of the Mill,
(But she is more in love with Will.)
Will's young and spruse, but hath no store—
No mouldy sterling to count o'er—
John boasts of gold, and more than that,—
Which makes Sue's heart go pit a-pat:

She

She begs the ladies' kind advice,
Were they to chuse, and take their choice,
Whether 'tis best, to marry John,
With all his gold,—or Will, with none.

* $x+y=152$, and $\overline{x-y}|^{\frac{2}{3}} \times \overline{x-y}|^{\frac{3}{2}} = 8192$; where y = John's age, and x his fortune.

II. QUESTION 551, *by Miss* Ann Nicholls.

I observed a cloud bearing N. E. by E. and took its altitude 17° 4'; some time after I observed the same cloud S. E by S. at which time its altitude was 45° 23'; from whence the point the wind blew from is required.

III. QUESTION 552, *by Mr.* Isaac Tarratt.

A pleasant village, ladies, is defin'd
From the equation underneath subjoin'd,*
Whose pleasing bowers and delightful shades
Are far superior to the woodland glades.
To these retreats the merry nymphs will hye,
With sudden transports of unbounded joy!
And meet their swains, the shepherds of the plain,
Who tune their pipes with a melodious strain:
The choristers will listen there to hear
What Damon says to his beloved fair.
My residence, dear ladies, pray explore,
And you'll oblige your servant evermore.

$$\left.\begin{aligned} * \; & v+w+x+y+z=64, \\ & v^2+w^2=5\times\overline{x+y+z}+30, \\ & vwx=8yz+6, \\ & x^2+yz=492, \\ & v+w+z=x+y: \end{aligned}\right\}$$

Where v, w, x, y, and z represent the places of the letters in the alphabet, that compose the town's name.

IV. QUESTION 553, *by Mr.* Wm. Spicer.

There is a conical spire steeple, whose slant side is = 4 times its base; from the vertex of which, two heavy bodies were let fall at the same time, the one down the perpendicular, and the other along the slant side: Now it was observed that the sound of the body which fell down the perpendicular upon the center of the base, arrived at the extremity of the slant side just at the same moment of time with the other body; required the steeple's altitude?

V. QUESTION 554, *by Mr.* Stephen Hodges.

Near the renown'd Lord Viscount Spencer's seat,
Where shady groves project a cool retreat,
Two children at one birth appear'd in view;
From these equations * pray their ages shew.

$$\left.\begin{array}{l} {}^{*}\,xy^2 + z^3v^3 = 290304, \\ x^2u + x^3y^4 = 12521472, \\ y = u,\ z = v, \text{ and } v = 2y; \end{array}\right\}$$ Where x represents the year,

y the month, z the day, v the hour, and u the minute A. M. when the eldest was born; and the other was born 4 m. after.

VI. QUESTION 555, *by Mr.* Richard Gibbons.

A tradesman owed his dealer 150l. who agreed to take a certain sum yearly on being allowed lawful interest upon balancing each year's accompt, as the same became due, until the whole was paid, when the debt and interest amounted to 179l. 9s. 10$\frac{1}{2}$d. required the yearly payment, and the time the whole took paying off?

VII. QUESTION 556, *by Miss* Ann Nicholls.

There is, in lat. 54° north, a town, and in lat. 64° north, a mountain, both under the same meridian, and on Dec. 21st the sun was observed to rise at the same moment of time at the town and on the mountain; from which data I demand the height of the mountain.

VIII. QUESTION 557, *by Mr.* Tho. Harris.

Say when, * ye astronomic spies,
Bright Sol in the least time will rise?

* Lat. 51° 32′ N.

IX. QUESTION 558, *by* Curiosus.

So to draw a right line, cutting the two sides of a given plane triangle, that the rectangle of the two lower segments shall be equal to a given quantity, and the rectangle of the two upper ones a maximum.

X. QUESTION 559, *by Mr.* John Chipchase.

A ball being shot, at an elevation of 76°, from a cannon, whose greatest horizontal range is three miles, struck an object standing at such a distance that, if it had been projected

against

against it in a direction perpendicular to its surface, its force would have been twice as great as it then was; required the object's distance from the cannon, supposing it to stand perpendicular to the horizon.

XI. Question 560, *by Mr.* T. Moss.

To determine (à priori) the form of a spheroidical standing cask, whose ullage may be truly found by means of the line of segments on the common sliding rule; its whole content being 100 gallons.

XII. Question 561, *by Mr.* Joseph Fisher.

Admit the earth and moon two perfectly spherical bodies, and the sun's parallax to be 10″; and granting also, that the sun and moon, when conjoined, and both at their mean distances from the earth, elevate the waters of the ocean 12 feet perpendicular, thereby causing the tides: Quære, whether a pendulum clock would gain or lose, and how much per day, by transporting from the earth to the moon; and what length a pendulum should be to make 60 vibrations on the moon's surface, supposing one 39·2 inches to make 60 in a minute on the earth, in lat. 55°.

XIII. Question 562, *by Mr.* Rob. Butler.

A current of water is discharged by three equal arches or sluices: The 1st (in shape) a rectangular parallelogram, the 2d a semicircle, and the 3d a parabola, having their altitudes equal, and their bases (which are downwards) situated in the same horizontal line, and the water level with the tops of the arches; on this supposition let be shewed the proportion of the quantities discharged by these sluices

XIV. Question 563, *by Mr.* Paul Sharp.

If a vessel, in form of a cone whose base diameter and depth are equal, stand on its base and be filled with water, it will empty itself, through a hole in its base, in the same time that a sphere, whose internal diameter is equal to that of the cone, will empty itself through an equal orifice in its bottom: Quære the demonstration?

XV. Question 564, *by Mr.* W. Toft.

To find the time of shortest twilight in any given latitude, and its duration, by stereographic projection.

XVI. Ques-

XVI. QUESTION 565, *by Mr.* Rob. Butler.

Required the content, in ale gallons, of a cask formed by the rotation of a cycloid about its base or longest diameter, and having its head diameter 32, bung 40, and length = 48 inches.

The PRIZE QUESTION, *by Mr.* Tho. Allen, *of* Spalding *in* Lincolnshire.

Supposing a heavy body to descend freely by the force of gravity upon a common cycloidal curve, the radius of whose generating semicircle is 18 feet; and let its distance from the vertex of the cycloid be 12 feet at the commencement of motion. It is required to determine where the body will quit the curve, its distance from the axis when it impinges upon the horizon, and time of its whole descent.

1767.

Questions answered.

I. QUESTION 550 *answered by Mr.* Edward Bayley.

PUT $x-y=z^6$, and then the 2d given equation becomes z^{13} ($= z^4 \times z^9$) $= 8192$; whence $z = 2$, and $x-y = 64$; from which, and the first given equation ($x+y = 152$), is readily found $y = 44$, John's age, and $x = 108$ l. his fortune.——And in this manner, exactly, the answer is likewise given by Mess. *J. Bennett, R. Gibbons, R. Gray, J. Mason, W. Rawle, P. Sharp, Is. Tarratt*, and *S. Vince*.

It is also answered by Mess. *Josh. Adams, T. Adams, T. Atkinson, T. Barker, W. Baylis, W. Baily, W. Cole, J. Coulthred, Is. Dalby, R. Dening, W. Dent, W. Godbe, J. H. R. Hale, W. Hubbard, J. Probert, P. De Quir, T. Robinson, B. Rotherham, Alex. Rowe, B. Sikes, W. Sewell, R. Smetham, Edw. Smith, W. Spicer, W. Stoker*, Mrs. *Eleanor Suggett, T. Vessey, Ja. Urmstow, Marshal Wroot*, and *Ja. Young*.

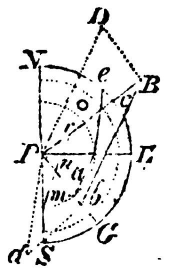
Mr.

Mr. James Mills *answers it in the following Advice to fair* Susan.

My charming fair, John's age is forty-four,
His money one hundred eight pounds, and no more.
If my advice from these few hints you'll take,
Oh! marry William, for your own dear sake:
Better for John will broths and flannels be,
Than in a bridal bed with lovely thee——
He'll clasp thy charms as misers hug their gold,
But cannot use the treasure they behold!

II. QUESTION 552 *answered by Mr.* Tho. Vessey, *at Mr.* Allen's *School, in* Spalding.

Let P be the place of observation. Draw $Pb \perp PB$, and let BD, bd be perpendicular to the horizon in the points B and b. Suppose D the place of the cloud at the 1st, and d that at the 2d observation. Put t = tang. of $\angle BPD = 17° 4'$, T = that of $\angle dPb = 45° 23'$, rad. = 1; then (per trigonom.) $BD = t \times PB$, and $bd = T \times Pb$; whence, the height of the cloud above the horizon being supposed to continue the same, $t \times PB = T \times Pb$, or $PB : Pb :: T : t$; therefore (the points B, b being joined) the $\angle PBb = 16° 52'$, and consequently the wind blew from N. E. ½ N.



And in this manner, nearly, it is also answered by Mess^{rs}. *Josh. Adams*, *T. Barker*, *J. Bennett*, the Rev. Mr. *J. Cross*, *J. Mason*, *J. Probert*, *W. Rawle*, *T. Robinson*, *Alex. Rowe*, *P. Sharp*, *B. Sikes*, *R. Smetham*, *W. Spicer*, and Mrs. *Eleanor Suggett*.

If right lines PC, PG be conceived to be drawn, making $\angle$s NPC, NPG, with the meridional line NPS, equal to $56\frac{1}{4}°$ and $146\frac{1}{4}°$, the given bearings of the cloud from the north, and the distances Pr, Pn be taken thereon equal to 3·25729 and ·98671 &c. the cotangents of 17° 4' and 45° 23' (the cloud's altitude), at the two observations, to rad. 1, and the points r, n be joined, and the right line rn produced till it meets NS in m, the $\angle rmN$ will measure the point

 of

of the compass required, which is too evident to need a demonstration, the height of the cloud above the plane of the horizon being the same at each observation; and hence, by trigonom. 4·244 ($Pr+Pn$) : 2·27058 ($Pr-Pn$) :: 1 (tang. of 45°) : ·53500 = tang. of $\frac{\angle Pnr - \angle Prn}{2}$; ∵ the angle $rmN = 39° 24'$; the same as before.

According to this method, very nearly, it is answered by Mess. *T. Adams*, *Felix M'Carthy*, *R. Hale*, *B. Rotherham*, *W. Sewell*, and *W. Stoker*.

But Mr. *R. Gibbons*, supposing the direction of the cloud to form the arc of a great circle, gives the following solution. Lay down $NC = 5$ and $SG = 3$ points, on the horizontal projection NES, whereof N, E, and S represent the north, east, and south points, and draw the two right circles PC, PG, cutting the parallels of the given altitudes (17° 4′ and 45° 25′) in o and a; through which points the arc of a great circle aoe being described, cutting the horizon in e, it will shew the direction of the cloud, and the arc NE, the point which the wind blew from, which is found, * at one operation in spherics, to be = 39° 24′, answering to N.E. by N. ½E.

And upon this supposition it is likewise answered by Mess. *W. Bayly*, *Is. Dalby*, *R. Gray*, and *Edward Smith*.

* Viz. Sine Po : tang. Pa :: sine Co : tang. Ce = 16° 51′, which subtracted from CN (= 56° 15′) leaves Ne = 39° 24′.

III. QUESTION 551 *answered by Miss* Anna Nicholls; *To the Proposer.*

May you of ev'ry joy partake,
  That EPSOM fair can give;
And, when you shall this earth forsake,
  May you for ever live.

Answer to the same by Mr. B. Sikes.

From the 1st and 5th given equations, $x+y=32$; and, since v and w are concerned exactly alike in every equation where they enter, let $s = \frac{1}{2}$ their sum, and $r = \frac{1}{2}$ their difference; then, w being $= s+r$ and $v = s-r$, those values substituted in the given equations, by reduction, &c. will found $ss = \frac{32^2 - 64z + zz}{4} = \frac{5zx + 190x + 16yz + 12}{4x}$;

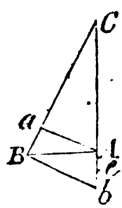
Whence

Whence, z and y being exterminated, &c. $x^5 - 53x^4 + 1034x^3 - 10928x^2 + 514320x - 8073216 = 0$; from which x comes out $= 18$, and consequently $y = 14$, and $z = 12$. Hence $w = 15$ and $v = 5$, and the place of Mr. *Turratt's* residence is EPSOM.——Mr. *E. Bayley's* and Mr. *W. Cole's* answers are the same as this, exceedingly near.

Amicus, from the nature of the given equations, very readily determines $x = 18$, and consequently $z = 12$, and $y = 14$; therefore the 1st, 2d, and 3d given equations become $v + w = 20$, $v^2 + w^2 = 250$, and $vw = 75$; from which $v \backsim w = 10$, &c.

Mess. *Josh. Adams*, *T. Adams*, *T. Atkinson*, *T. Barker*, *W. Bayly*, *W. Baylis*, *W. Cole*, *Is. Dalby*, *R. Dening*, *W. Dent*, *R. Gibbons*, *M. Gordon*, *R. Gray*, *W. Hubbard*, *R. Hale*, *J. Mason*, *J. Mills*, *J. Probert*, *Pet. De Quir*, *W. Rawle*, *T. Robinson*, *B. Rotherham*, *Alex. Rowe*, *Abby Sadler*, *Paul Sharp*, *W. Sewell*, *Edw. Smith*, *W. Spicer*, *W. Stoker*, *Eleanor Suggett*, *Sam. Vince*, *Ja. Urmston*, *Marshall Wroot*, and *J. Young*, have also answered it.

IV. QUESTION 553 *answered by Mr.* W. Cole.



Let AC and BC represent the perpendicular and slant heights of the steeple, respectively. Draw $Aa \perp BC$, and $Bb \parallel Aa$, meeting CA produced in b; then will A, a, and also B, b, be cotemporary positions of the two falling bodies, supposing the body at A to continue in motion till the other body arrives at B (as is well known by the laws of descending bodies); and, BC being to BA as 8 to 1, per quest. AB is easily found to be to Ab as $\sqrt{63}$ to 1. Therefore, as $\sqrt{63} : 1 :: 1142$ (the velocity of sound per second) : $1142 \div \sqrt{63} =$ the velocity of the body at e; whence the distance eC will be $= 321\frac{3151}{4053}$, and (per quest.) Ab is found $= \frac{1}{64}Cb$, and $eA = \frac{1}{127}eC$; therefore, as $127 : 126 :: 321\frac{3151}{4053}$ (eC) : $319\frac{125473}{514751}$ (AC) $=$ the perpendicular height of the steeple required.

It is also answered by Mess. *Josh. Adams*, *T. Barker*, *J. Dalby*, *W. Dent*, *R. Grey*, *R. Hale*, *Felix M'Carthy*, *J. Probert*, *W. Rawle*, *T. Robinson*, *B. Rotherham*, *Alex. Rowe*, *W. Sewell*, *P. Sharp*, *B. Sikes*, *R. Smetham*, *E. Smith*, *W.*

 Spicer,

Spicer (the proposer), *W. Stoker*, *J. Tarratt*, *T. Todd*, *Sam. Vince*, *Jos. Webster*, and others.*

V. QUESTION 554 *answered by Mr.* Isaac Tarratt.

By substituting the values of z, v, and n, in the terms of x and y, we get $xy^2+64y^6=290304=a$, and $yx^2+8y^7=12521472=b$; from the 1st of which equations x is found $=\overline{a-64y^6}\div y^2$, and this value substituted in the latter of them, it becomes $\overline{a-64y^6}|^2\div y^3+8y^7=b$: Solved, $y=4$. Consequently the eldest was born anno 1760, April the 8th, at 4m. past 8 in the morning, and the other 4m. after that.

Much after the same manner it is also answered by Mess. *Josh. Adams*, *T. Adams*, *T. Barker*, *Edw. Bayley*, *W. Baylis* *W. Bayly*, *W. Cole*, *J. Coultbred*, *R. Dening*, *J. Dalby*, *W. Dent*, *R. Gibbons*, *R. Gray*, *R Hale*, *Stephen Hodges* (the proposer), *J. Mason*, *James Mills*, *J. Probert*, *W. Rowle*, *T. Robinson*, *Alex. Rowe*, *W. Sewell*, *B. Sikes*, *Edw. Smith*, *W. Spicer*, *W. Stokes*, *Ja. Urmston*, *and Ja. Young*.

If it be considered that x and y must be whole positive numbers, and the latter of them under 12, by the quest. the answer may be easily obtained from the given equations as they stand at first, without any further reduction, &c.

VI. QUESTION 555 *answered by Mr.* Richard Gibbons (*the Proposer*).

The present worth 150l. ($=p$) and the rate of interest 1·05l. ($=r$) being given, put $a=$ the annuity, or yearly payment required, and $t=$ the time it was paying off; then there is further given $ta=179·4925$l. to find t and a separately.

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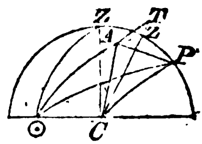
* An algebraic solution to this question may be thus: Put $s=16\frac{1}{12}$, $a=1142$, and $x=AB$: Then $8x=BC$, and $x\sqrt{63}=AC$. By the laws of gravity, $\sqrt{s}:\sqrt{AC}::1'':\sqrt{\frac{x\sqrt{63}}{s}}=$ time of falling through CA, and $AC:BC::\sqrt{\frac{x\sqrt{63}}{s}}:\sqrt{\frac{64x}{s\sqrt{63}}}$ $=$ time in CB, also $\frac{x}{a}=$ time of sound's moving through AB; therefore we have $\frac{x}{a}+\sqrt{\frac{x\sqrt{63}}{s}}=\sqrt{\frac{64x}{s\sqrt{63}}}$: Hence $x\sqrt{63}$ $=a^2\times\frac{127-16\sqrt{63}}{s}=319·249=AC$ required.

rately. By the doctrine of annuities and the question, $atr^t - at = ptr^{t+1} - ptr^t$, or, in numbers, &c. $179{\cdot}4925r^t - 7{\cdot}5tr^t = 179{\cdot}4925$: Whence $23{\cdot}93233r^t - tr^t = 23{\cdot}93233$: Solved, $t = 6{\cdot}527$ years, and $a = 27$l. 10s. the annual payments.

In this manner, nearly, it is likewise answered by Mess. *Josh. Adams, T. Adams, T. Barker, J. Bennet, R. Gray, Felix M'Carthy, J. Mason,* Miss *Anna Nicholls, J. Probert, W. Rawle, Tho. Robinson, W. Sewell, B. Sikes, Edw. Smith, W. Spicer,* and *T. Todd.*

VII. QUESTION 556 *answered by Mr.* Felix M'Carthy, *of* Aberdeen.

In the annexed projection, let P represent the north pole, Z the zenith of the place in lat. 54°, z that of the place in lat. 64°, and ⊙ the sun in the horizon in the former of them, on the day proposed; then, in the spherical triangle $Z\odot P$, is given all the sides to find the angle $ZP\odot = 53° 17' 27''$. Through ⊙ and z describe the arc of a great circle ⊙z; also describe the arc AP, making ⊙A = ⊙Z, and then a spectator at A will, it is evident, see the sun rising at the same time with the other two spectators at Z and T, the point where a tang. to the arc Az, at the point A, meets Cz (the semidiameter of the earth) produced, *i. e.* at the top of the mountain zT required; to determine the height of which, in the spherical triangle ⊙Pz, are given the sides ⊙P, Pz, together with the included angle, to find ⊙$z = 96° 45' 36''$, from which subtracting $A\odot = 90°$, there remains Az (the measure of the $\angle ACz$) $= 6° 45' 36''$: And thence CT is found $= 4009{\cdot}887$ miles (per plane trig.), from which deducting 3982 (the miles contained in Cz, the earth's rad.) leaves $zT = 27{\cdot}887$ miles, the required height of the mountain.



This question we received from the ingenious proposer in the year 1761, and now find that, through mistake, it has been since sent to, and published in, another place.

Answers to this quest. have also been received from Mess. *T. Adams, T. Barker, W. Baylis, J. Chipchase, J. Dalby, J. Probert, W. Rawls, Abby Sadler, W. Sewell, B. Sikes, Edw. Smith, W. Spicer, Eleanor Suggett, J. Webster,* and *J. Young.*

Mr. *Joshua Adams*, allowing 33′ for the ⊙'s horizontal refraction, finds the height = 30·2 miles.

VIII. QUESTION 557 *answered by* JUVENIS.

Put a and b = sine and cosine of 51° 32′ (the given lat.), — c = cosine of 90° 32′ 12″ (the dist. of the sun's lower limb from the zenith the moment his upper limb touches the horizon), and x = cosine of the declin. required, rad. = 1; then, per spherics,

$$\overline{\overline{ax+c}\times\sqrt{bb-xx}-ax\sqrt{bb\times\overline{1-xx}-\overline{ax+c}|^2}}\div$$

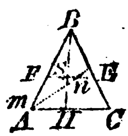
$bb\times\overline{1-xx}$ = sine of the angle measuring the time the sun is in rising, which (per quest.) is to be a min. In fluxions, &c. and reduced, x comes out = ·0036441 &c. answering to 12½′ nearly, the declination required.

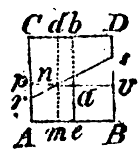
Mess. *T. Adams*, *T. Barker*, *R. Gibbons*, *J. Probert*, *Alex. Rowe*, and *W. Spicer*, have likewise answered it.

Mathematicus easily reduces this prob. to that of describing the circumference of a circle to touch two circles, given in magnitude and position, and cut the circumference of another circle given in magnitude and position likewise, in a given angle; and from thence readily infers, nearly, the same conclusion as above.

Mr. *Tho. Harris* (the proposer) and Mr. *Isaac Tarratt* observe that 'the sun will rise quickest when his azimuth 'increases the slowest, and that will be when he ascends 'the horizon in a vertical direction:" and upon this principle they solve it independent of fluxions, and find the declination sought to be 12′ 40″ south; from whence they conclude that the sun rises the quickest the morning before the vernal, and the morning after the autumnal equinoxes.

IX. QUESTION 558 *answered by* Mr. T. MOSS.





Let ABH (fig. 1st) represent the given plane triangle; produce AH till the right line BC, joining its extreme and the point B, becomes $= BA$, and upon AB (= one of those equal sides) let a square AD (fig. 2d) be constituted, and conceive a right line rs to be drawn through the same, so that $Ar\times Bs$ may be to the given parallelogram of the lower segments as BH : BA (fig. 1st) and the area of the part $ArsB$ = a min. which, having bisected AB in m and drawn $md \parallel AC$, it is manifest will be when $Ar + Bs$ is a min. because ma (=

$(=\frac{Ar+Bs}{2}) \times AB$ is the measure of the space $ArsB$, and AB is a constant quantity. But it is well known, and may be easily proved geometrically, that the sum of two right lines, under a given rectangle, will be the least possible when they are equal to each other: Whence, taking $Ap =$ the side of a square, whose magnitude is a fourth proportional to BH, BA and the given parallelogram of the lower segments, and drawing $pv \parallel AB$, upon the side AB, of the given $\triangle ABH$, set off $AF = Ap$ (Bv), and draw $FE \parallel$ the base AC, cutting BH in s, and the thing is done.

DEMONS. Because, by construction, the $\square ApvB$ (fig. 2) is a min. when the $\square$ of the lower segments is a given magnitude, thence any rectangular part (Aa) thereof will also be a min. and the $\square$ of the said segments remain the same. Let, therefore, $pa = Cp$, and draw $eab \parallel AC$; then because $Apae$ is a min. the $\square pb$, which is a multiple of the $\triangle FBE$, will be a max. and consequently FBs, being a part of FBE, will also be a max. when $AF \times EC$ $(= Ap \times Bv)$ is a given magnitude. Now, seeing that EF must be $\parallel AC$, in order that a multiple of the $\triangle mBn$ may be a max. and the $\square$ under the lower segments of the $\triangle ABC$ a given magnitude (*i. e.* to the given rectangle as $BA : BH$ or $FA : sH$), it thence evidently follows, that the $\triangle FBs$ (being as $BF \times Bs$) will be the greatest possible (and the $\square$ of the lower segments of the $\triangle ABH =$ the given rectangle) when Fs is $\parallel AH$.

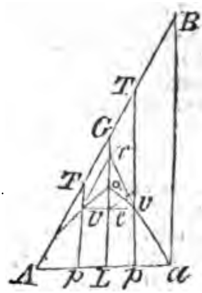
A Fluxionary Answer to the same, by Mr. Samuel Vince.

Suppose AB (fig. 1st) to be given $= a$, $HB = b$, the $\square$ of the lower segments, AF, Hs, $= rr$ (supposing Fs to be the position of the line required), and put $AF = x$, $Hs = y$; then will $xy = rr$, and $\overline{a-x} \times \overline{b-y}$ be a max. per quest. In fluxions, $x\dot{y} + y\dot{x} = 0$, and $x\dot{y} + y\dot{x} - a\dot{y} - b\dot{x} = 0$, i. e. $-a\dot{y} - b\dot{x} = 0$, and consequently $a\dot{y} = b\dot{x}$; whence x (AF) $: y$ $(Hs) :: a$ $(AB) : b$ (HB), and $Fs \parallel AH$, by the Elem. of Geometry.

Mess. *T. Adams, T. Barker, W. Bayly, J. Bennett, J. Dalby, R. Gray, Felix M'Carthy, J. Mason, Step. Ogle, J. Probert, W. Rawle, T. Robinson, Alex. Rowe, W. Sewell, P. Sharp, W. Spicer, T. Todd,* and *Ja. Young* have likewise answered it.

X. QUESTION 559 *answered by Mr.* T. Todd.

By the nature of projectiles in a non-resisting med. 1 (rad.) : ·4694714, (s. 2 $\angle BAa$, the given elevation) :: 3 miles (the given, greatest rand.) : 1·4084142 miles $= Aa$, and, by trig. $Ba =$ 5·64884077 miles $= n$. Now, if $t =$ ·249328 (the nat. tang. of $\angle B = 14°$), $x = Tv$, and $y = vp$; then, by trig. $tn = Aa$, $\frac{tn}{2} = AL = La$, and therefore, by conics, $\frac{1}{4}n$ $(Go = oL)$: $\frac{1}{4}\overline{ttnn}$ $(\overline{Laa})$ :: $\frac{1}{4}n - y$ (ve) : $\frac{1}{4}ttn \times \overline{n-4y} = ev^2$; ∵ by 47 Euc. 1, the tang. rv at v, is $= \sqrt{\frac{1}{4} \times \overline{n-4y}|^2 + \frac{1}{4}t^2 n \times \overline{n-4y}}$; which (by the resolution of forces and



the quest.) is to $\frac{1}{4}t\sqrt{nn-4ny}$ $(= ve)$ :: 2 : 1; from whence, by multiplying extremes and means and reducing, &c. y is found $= \frac{1}{4}n \times \overline{1-3tt} =$ 1·148842. Moreover, by Simp. Exercises, cor. 1 pa. 188, $Ba \times Tv = Tpp$, i.e. nx $(= \overline{x+y}|^2) = x + \frac{1}{4}n \times \overline{1-3tt}|^2$: Solved, $x = \frac{1}{4}n \times \overline{1+3tt} \pm \frac{1}{2}nt\sqrt{3} =$ 2·89530048, or ·45585554; and consequently Tp $(= x+y) =$ 4·044142, or 1·604697, and Ap $(= t \times \overline{x+y})$ $=$ 1·008318, or ·4000962 miles, either of those distances answering the question.——The answer by Mr. *J. Chipchase* (the proposer) is the same as this, exceedingly near.

It is also answered by Mess. *T. Barker*, *Step. Ogle*, *J. Probert*, *W. Rawle*, *Alex. Rowe*, *W. Sewell*, and *P. Sharp*.

XI. QUESTION 560 *answered by Mr.* B. Sikes.

Let $b =$ the bung diameter, $l =$ the semi-length, $x =$ the wet inches, $y =$ the diameter at x dist. from the head on which the cask stands, $c =$ the circular divisor for gallons, and the ratio of the bung to the head diam. as $1 : n$. Then, by the nature of the ellipsis, $yy = nnbb + \frac{2 \cdot \overline{1-nn} \cdot bbx}{l}$ $- \frac{\overline{1-nn} \cdot bbxx}{ll}$, which multiplied by $\frac{\dot{x}}{c}$, and the fluent taken, &c. the ullage (or quantity of liquor in the cask, putting $2lv = x$) will be found to be $2lbb \times \frac{2+nn}{3c} \times$ $\frac{3nnv + 6 \cdot \overline{1-nn} \cdot vv - 4 \cdot \overline{1-nn} \cdot v^3}{2+nn}$ where the factor

$2lbb$

$2lbb \times \frac{2+nn}{3c}$ is known to be $=$ the whole content of the cask; therefore $\frac{3nnv + 6.\overline{1-nn}.vv - 4.\overline{1-nn}.v^3}{2+nn}$ will express the fractional part of the ullage, the whole content being the integral quantity, or 1.——If now the whole length of the cask be supposed $= 1$, and to be divided into 100 equal parts, v will truly represent the number of parts contained in the wet inches; and, to make the foregoing expression agree with the multipliers, as found, on the sliding-rule, set 100 on the slider to 100 on the line of segments S. S. on the rule, then against any given number v of parts on the slider, is the proper multiplier, or fractional part of the ullage on the rule, or line S. S; which, being put $= m$, and made $=$ the last expression, the value of n will be determined $= \sqrt{\frac{2m - 2vv.\overline{3-2v}}{3v - m - 2vv.\overline{3-2v}}}$, which will give the form of the cask required.

Let the wet inches divided by the length, or v, be $= \cdot 28$; then m will be found $= \cdot 26$, nearly, on the segments S. S. and consequently $n = \cdot 8343$: But if v be taken $= 72$, m will come out $= \cdot 743$ on the line S. S. and $n = \cdot 8097$; which shews that the line of segments will not give the ullage truly, in any part of the spheroid except the middle: But if n be taken $= \cdot 822$ ($=$ the mean between these two ratios), the line of segments will give the ullage of a spheroid, formed by this ratio, nearer the truth than any other.

Mr. *T. Moss* (the proposer) founds his solution on p. 207, 208, &c. of his Treatise of Gauging, where it is proved, in a general manner, that the ullages of two upright spheroidical casks, having their bung and head diameters in the same ratio, and their wet inches in the ratio of their lengths, will be to each other in the ratio of their whole contents, let their lengths be what they will: Whence it follows that the ullages of every standing spheroidical cask, having some one certain proportion of bung and head diameters, may be truly determined by the sliding-rule, let the lengths of such casks be what they will. Thus, if the ratio of the head and bung diameters of the required spheroidical cask be supposed to be as x to 1 (that being all that, in the present case, is required), $1 =$ its content, $y =$ its length, $\frac{1}{4}y =$ the wet inches, and $p = \cdot 7854$; then, by pa. 206 and 207 of the Theory of Gauging, will be found $\overline{1 - \overline{1-xx} \times \cdot 48 \div 2+xx}$ $= \cdot 316$ (the ullage, in this case, by the sliding-rule; whence $x = \cdot 82$, very nearly.——If the content of the cask be denoted by

by a, and every thing else as before, then will $\overline{a-a-axx}\times{}^{\cdot}48\div 2+xx=316$, $31\cdot6$, $3\cdot16$, or $\cdot316$, according as a (or the content of the cask on the segment line) is supposed to be divided into 1000, 100, 10, or 1 equal parts; from each of which x comes out $=\cdot82$, the same as before.

XII. QUESTION 561 *answered by Mr.* Jos. Fisher, (*the Proposer*).

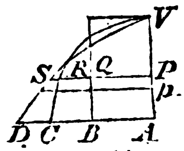
The whole solar force to move the waters of the ocean may be found, by a proper calculus of the earth's centripetal force towards the sun, to be to the force of gravity with us as 1 is to 12868200, and Sir Isaac Newton, in his Principia, has shewn that the earth's centrifugal force, arising from its vertiginous motion, is to the force of gravity at the equat. as 1 is to 289; which force, being calculated, should make the waters under the equat. exceed those under the poles in alt. 90640 feet: And hence the elevation of the waters by the sun appears to be $2\frac{1}{18}$ feet; consequently the moon raises them the other $9\frac{17}{18}$ feet, and the solar force to elevate the waters is to the lunar force as 1 : 4·88.—— The moon's force is to the force of gravity on the earth's surface as 1 is to 2634837, and the moon's density is to the sun's as 1 : 0·191, and the sun's is to the earth's as ·191 is to ·75, both computed from the triplicate proportion of their true diameters, and the simp. propor. of gravity towards each at equal distances, conjunctly; whence the earth's density is to the moon's as ·756 is to 1. Now, the moon's diameter being to the earth's as 1 : 3·65, and the density as 1 is to ·756, the accelerating gravity or force with which a pendulum would be actuated on the moon, is to the same on the earth, under the poles, as 1 is to 2·8; whence the gravity in lat. 55° appears to be 2·7968136, and consequently 34736″, or 9 h. 38′ 56″, the time a clock would lose per diem on the moon's surface; and hence the length of the pendulum required may be easily known.

NOTE, The moon's centrifugal force, arising from its rotation, is here omitted, being so small as could cause little variation.

XIII. QUESTION 562 *answered by Mr.* Cha. Hutton.

Let VB represent half the parallelogram, AVC half the semicircle, and AVD half the parabola, *i. e.* the halves of the respective sluices or arches, $a=AV$ (the common alt.) $p=$

$p = \cdot 7854$ &c. then paa is $=$ the area of each of them, $pa = AB$, $a = AC$, and $AD = \frac{3}{2}pa$. Put $x = VP$, and $\dot{x} = Pp$; then, the water discharged, at any depth x, being as the velocity and aperture, and the velocity as $\sqrt{x}$,



we shall have $\dot{x}\sqrt{x} \times PQ$, $\dot{x}\sqrt{x} \times PR$, and $\dot{x}\sqrt{x} \times PS$, or $pax^{\frac{1}{2}}\dot{x}$, $x\dot{x}\sqrt{2a-x}$ and $\frac{3}{2}p\sqrt{a} \times x\dot{x}$ as the fluxions of the quantity of water discharged by the said □, semicircle, and parabola, respectively; the correct fluents of which (when $x = a$) are $\frac{2}{3}paa\sqrt{a}$, $\frac{2}{15}aa\sqrt{a} \times \overline{8\sqrt{2}-7}$ and $\frac{3}{4}paa\sqrt{a}$, and the quantity of water discharged by the □, the semicircle, and parabola, as 1, 1·09847 &c. and $1\frac{1}{8}$ respectively.

The solutions by Mess. *T. Allen*, *Rob. Butler* (the proposer), *J. Chipchase*, and *δί Φίλον*, are the same, exceedingly near.

XIV. QUESTION 563 *answered by Mr.* Paul Sharp, (*the Proposer*).

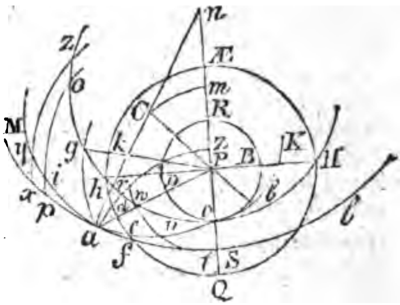
Put $a =$ the diam. of the sphere (which is also $=$ the diam. and alt. of the cone, per quest.), $e = \cdot 7854$, $t =$ the time of a cylinder's, of equal base diam. and altitude, emptying itself, at an equal orifice, when full, with its first or greatest velocity, and $x =$ any variable alt. of the surface of the water above the base of the cone, &c. at the end of any time y, then will $\dot{y} = -t\dot{x} \times \frac{\overline{a-x}|^2}{aa\sqrt{ax}}$ in the cone, and $= -4t\dot{x} \times \frac{ax - xx}{aa\sqrt{ax}}$ in the sphere; the correct fluents of which (when $x = a$) will, in each case, be $y = \frac{16t}{15}$, *i. e.* they will be the same, &c.——Mr. *T. Allen* (substituting, nearly, the same as above) derives the very same conclusion.

Much after the same manner the answer is given by Mess. *C. Hutton*, *Edw. Smith*, and *T. Todd*.

XV. QUES-

XV. QUESTION 564 *answered by* Mr. W. Toft, (*the Proposer*).

PROJECT. Having described, about the centers P, m, and n, the primitive circle $ÆHQ$, representing the equator, zOH the horizon, and pSl a parallel thereto, at 18° dist. below it, describe, from the centers P, n, with the radii Pm and $nS - mO$, two arcs intersecting each other in C; then, producing the right line nC, joining those points it will meet the parallel of depress. pSl in a, the point through which the parallel of the ☉'s declin. will pass on the day the proposed phænom. happens, and aP, the right line joining those points, will be the distance of the parallel from the visible pole.



DEMONST. Describe the circ. of perpetual apparition, $RBOD$, and the arcs tw, ag, po, xz, &c. of the parallels of declin. intercepted between pSl (the parallel of depress.) and zOH (the horiz.); also about the center C, with the rad. Ca ($= mO$), let the circ. KM be described, intersecting the parallels of declin. &c. in u, e, a, i, y, &c. and it will touch the parallel circ. pl in a (by 11 E. 3): it will also touch the circ. $RBOD$ (by the same), because $CP = mP$ (by construc.), and make an angle (aeb) with the equator $= Ohe$, the angle which the horizon makes with the same; and hence the parallel arcs uw, eb, ag, io, yz, &c. intercepted by them, (by prop. 6th on pa. 121 of Emerson's Trigonom. 1st edit.) will be similar, and consequently passed over by the ☉ in the same time, which will, manifestly, be less than the time in which the corresponding twilight arcs tw, fb, po, xz, &c. will be passed over in, except the twilight arc ag, which will be the same, and therefore will will be the shortest, &c.

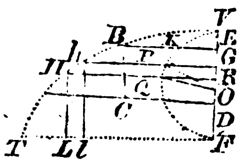
CALCULA. Describe the vertical circle Za, cutting the horizon and equator in r and d respectively, and the meridional arc Pg, intersecting the equator in k; then the right-

right-angled spherical triangles ead, hrd, being equiangular, will be mutually equilateral, and consequently $ad = dr = 9°$; whence (hr being $= ea = hg$, by prop. 6. on pa. 121 of Emer. Trigonom. aforesaid) 1 (rad.) : tang dr ($\frac{1}{2}$ the depress.) : : cos. $\angle gbk$ = sine of lat. of the place) : sine of gk (the required declin.)——Also, in the spheric Δaed, sine of $\angle aed$ (or cos. of lat.) : sine of ad ($= 9°$, $\frac{1}{2}$ the depress.) : : 1 (rad.): sine of ed, the arch of the equat. measuring $\frac{1}{2}$ the duration of the shortest twilight required.*

Mess. *T. Allen*, *Felix M'Carthy*, *C. Hutton*, *R. Gibbons*, *Step. Ogle*, *T. Robinson*, *Alex. Rowe*, and *B. Sikes* have also projected it.

XVI. QUESTION 565 *answered by Mr.* Cha. Hutton.

Let VKF be the generat. semicircle, and suppose half the cask to be generated by the revolution of the cycloidal arc VB about DC (= the ord. EB). Put $VF = D$, $VD = 20 = m$, DF ($= d - m$) $= n$, $p = 3{\cdot}14159$ &c. $DC = x$, and $BC = y$; then, by the prop. of the cycloid, &c. $\dot{x}$ will be found $= -\dot{y}\sqrt{\overline{n+y} \div \overline{m-y}}$



and $py y\dot{x}$ (the flux. of the solid.) $= -py^2\dot{y}\sqrt{\overline{n+y} \div \overline{m-y}}$, and the correct fluent (when $y = 16 = a$) is

$$\overline{15m^2 + 4mn - 3n^2 + 2a \times 5m + n + 8a^2} \times \tfrac{1}{15}p \times \overline{m-a}\sqrt{\overline{n+a} \div \overline{m-a}} + \overline{5m^2 - 2mn + n^2} \times \tfrac{1}{4}p \times \overline{m+n} \times \text{arc, whose rad. is 1 and cotang. } \sqrt{\overline{n+a} \div \overline{m-a}},$$

= the solidity of $\frac{1}{2}$ the cask required.

But to find VF, let it be now called z, and VE ($= m - a = 4$) $= c$; then, by the prop. of the cycloid, &c. is found

$$24\ (= BE) = 2\sqrt{cz} \times : 1 - \frac{c}{3z} - \frac{c^2}{2.4.5z^2} - \frac{3c^3}{2.4.6.7z^3} - \frac{3.5c^4}{2.4.6.8.9z^4} - \text{\&c.}$$

and hence $z = 37{\cdot}34394 = d$, and $n = 17{\cdot}34394$. Hence the above fluent comes out $26434{\cdot}3196$ inches $= \frac{1}{2}$ the content of the cask, and consequently the whole content is $187{\cdot}4774$ ale gallons.

Answer

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* Other solutions may be seen at Quest. 471.

Answer to the same by Mr. Rob. Butler (*the Proposer*).

Put $2b = VF$ ($\frac{1}{2}$ the bung diameter) $= 20$, $VR = 4$ ($\frac{1}{2}$ the given difference of the bung and head); then, by prop. of the circle, &c. $RQ = 8$; and $QH = 16 = d$ (Gh and RH being $\perp VF$, and cutting the semicircle VKF in P and Q). Also the $\angle QOV$ (at the center of the generating circle) will be found $= 53°13°$, and the length of its corresponding arc $QV = 9°273$ inches, which put $= n$, and let $p = 3°1416$, $C =$ the content, in inches, of the solid generated by the revolution of the space $lhVF$, about LF, as an axis (hl and HL being $\perp TF$), the variable quantities x, y, and $z = OG$, GP and PV respectively: Then, per the nature of cycloids, $n : d :: z : \frac{dz}{n} = Ph = mz$ (putting $\frac{d}{n} = 1°72544 = m$), and $mz + y =$ the ord. Gh; the flux. of which drawn into $\overline{b+x}^2 = hl^2 = GF^2$), viz. $\overline{m\dot{z}+\dot{y}} \times \overline{b+x}^2$, will be $= \frac{\dot{C}}{p}$. But, by the prop. of the circle, $b\dot{y} = x\dot{z}$, $bb - yy = xx$ and $-x\dot{y} = \dot{s}$ (s being the area of the circular seg. PVG); from which, and the preceding equation, (when the fluents are taken, &c.) will be had $mbbz + 2bb \times \overline{m+1} \times y + b \times \overline{m+2} \times xy - \frac{1}{3}y^3 + b \times \overline{m+2} \times s = \frac{C}{p}$: Whence the whole content required appears to be $187\frac{1}{2}$ ale gallons.—— It is observable, that though the cask in the 1st of these solutions is supposed to be generated from the revolution of part of a common cycloid about an ord. $\parallel TF$, and in the last from the revolution of part of an inflected cycloid around the longest semidiam. TF itself, yet they exactly agree in their contents.

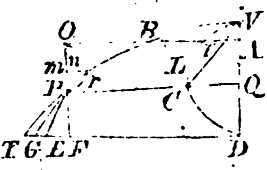
In this manner, nearly, it is also answered by Mess. *Wm. Sewell* and *T. Todd.*

But if the cask be supposed to be generated from the revolution of part of a common cycloid round the greatest semidiam. TF, then its length need not be given, being determinable from the nature and other given dimensions of the generating cycloid, as Mess. *T. Allen*, *Edw. Smith*, and *T. Todd justly* observe, and find, by elegant processes, the content, in this case, to be $135°2$ ale gallons.

The

The PRIZE QUESTION *answered by Mr.* Tho. Allen (*the Proposer*).

It is evident that the body will quit the curve when its velocity in direction of the ord. becomes the greatest possible.



Let B be the place of the body at the commencement of motion. Put VD the diam. of the generating semicir. $= 36$ feet) $= 2a$, and $VQ = x$; then, since BV is given $= 12$ feet, we have, by the properties of the cycloid, $VA = 1$, and therefore $AQ = x - 1$. Now, since the velocity of the body along the curve, is to the velocity in direction of the ord. as the fluxion of the former is to that of the latter, $\sqrt{\frac{\overline{2a-x} \times \overline{x-1}}{2a}}$ will be $=$ the velocity in direction of the ordinate; by making the fluxion of which $= 0$, will be had $x = a + \frac{1}{2} = 18{\cdot}5$ feet.

Put $s = 16\frac{1}{12}$ feet, and $t =$ the time of describing BP; then, per the laws of falling bodies, $\dot{t} = \frac{1}{2}\sqrt{2as^{-1}} \times x^{-\frac{1}{2}}\dot{x} \times \overline{x-1}|^{-\frac{1}{2}}$, whose fluents give $t = \sqrt{\frac{a}{2s}} \times$ hyp. log. of $\overline{\sqrt{x} + \sqrt{x-1}}|^{2}$; which, when $x = 18{\cdot}5$, will be $3{\cdot}1989''$.

Lastly, when the body ceases to touch the cycloid, as at P, it will describe a parabolic curve, suppose PG. Now, having given the velocity at $P = 34{\cdot}5$ feet per second, very nearly (from what is found above), and likewise the position of the tang. TP, or the $\angle PTF = 45° 47'$, the dist. GF (supposing $FP \perp TD$) will be determined $= 12{\cdot}22$ feet, and the time of the body's describing $PG = {\cdot}508''$. Whence GD (the dist. the body will impinge upon the horizontal line TD from the axis VD) will be $= 58{\cdot}62$ feet, and the time of the whole descent $= 3{\cdot}7069''$.

The same answered by Mr. Cha. Hutton.

Let $VBPE$ be the cycloid, and VLD the generating semicir. Suppose the body to commence motion at B, and quit the cycloid at P, going off, there, in the curve of a parab. PG. Draw $ED \perp VD$, and PQ and $BA \parallel$ thereto, meeting the semi-circumf. VLD in C and I, producing the latter

 of

of them till it meets FP, $\perp DE$, produced in O, and upon OP describe the semi-cir. OrP. Draw nT a tang. to both the given cycloid VBE, and parab. PG, at P, meeting the semi-circumf. OrP in n, and DE, produced, in T, and let nm be $\perp OP$, and the points V, I and V, C be joined.—— Put $VA = a$, $VD\ (= 36) = d$, $16\frac{1}{12} = s$, and $AQ = x$. Then, by the laws of descending bodies, $2\sqrt{sx} =$ the velocity per second, at Q or P, in the direction PT or VC, and consequently $2\sqrt{d^{-1}sx \times \overline{d-a-x}} =$ the velocity at P in the horizontal direction. Now, since the body goes off at P in a parab. and the horizontal velocity in a parab. is a constant quantity, the last expression will be constant, and consequently its fluxion $= 0$; whence x comes out $= \frac{1}{2} \times \overline{d-a}\ (\frac{1}{2}AD)$. Also, $a = VI^2 \div d = \frac{1}{4}BV^2 \div d = 1$, and $VP\ (= 2VC) = \sqrt{2d \times \overline{d+a}}\ (= 2\sqrt{d \times \overline{a+x}}) = 51'613951$. Again, per sim. triangles (putting $\frac{1}{2} \times \overline{d+a} = \frac{1}{2}c$, and $\frac{1}{2} \times \overline{d-a} = \frac{1}{2}g$) will be found $FT = \frac{1}{2}g\sqrt{c^{-1}g}$, and $mn = \frac{1}{4}d^{-1}g\sqrt{cg}$: Then, per conics, $d^{-1}gg\ (= 4mO = 4QD \times PO \div VD =$ the parameter of the parabola's axis, and $\frac{1}{2}d^{-1}g\sqrt{g \times \overline{2d+c}}\ (= \sqrt{Fm \times d^{-1}gg}) =$ an ordinate of the axe terminated at the point G of the curve; also, $FG = g\sqrt{g} \div \overline{\sqrt{c} + \sqrt{2d+c}}\ (= \overline{2mn + \text{ord. to point } G}^{-1} \times 4mn \times FT)$: Whence, (putting $p = 3'14159$ &c.) DG $(= FG + Q\ + PC = FG + QC +$ the circ. arc $VC) =$ $g\sqrt{g} \div \overline{\sqrt{c} + \sqrt{2d+c}} + \frac{1}{2}\sqrt{cg} + \frac{1}{180}dp \times$ degr. in $\angle VCQ = 59'299337$, the distance from the axis when the body impinges on the horizon.——Lastly, to find the whole time of descent to the horizon. Suppose $t =$ the time of descending through the cycloidal arc BP, then will $\dot{t} = \frac{1}{2}\dot{x}\sqrt{ds^{-1}x^{-1} \times \overline{a+x}^{-1}}$, and, taking the correct fluent when $x = \frac{1}{2}g$), $t = \frac{1}{2}\sqrt{ds^{-1}} \times$ hyp. log. of $\overline{d + \sqrt{cg}} = 3'199035''$: But, for the time in the parab. the horizontal velocity being constant, and $= g\sqrt{sd^{-1}}$ feet per second (as found above), $\sqrt{gds^{-1}} \div \overline{\sqrt{c} + \sqrt{2d+c}}\ (= FG \div g\sqrt{sd^{-1}}) = \sqrt{\frac{1}{2}dgs^{-1}} \times \overline{d + c + \sqrt{c \times \overline{2d+c}}}^{-1} = '535681'' =$ the time in the said parab. and the sum of these two, or $3'734716''$, is $=$ the whole time of descent required.

Ingenious

Ingenious answers have likewise been received from Mess. *J. Chipchase*, *Phil. George*, *R. Hale*, Miss *Anna Nicholls*, *T. Robson*, and the Rev. Mr. *Wm. Smith*; but the 1st prize of 12 Diaries, for the solution thereof, is fallen to the lot of Mr. *Cha. Hutton*, and that of 8 to Mr. *Tho. Allen*.

The Eclipses calculated for 1767.

There will happen only two eclipses this year, and both of the greater luminary, the sun; and both likewise invisible in Great Britain.

The first will be on Friday the 30th of January, near four o'clock in the morning; it will be central and total in the Indian Ocean, and will be a very great eclipse in most of the East Indian islands.

The second will happen on Saturday the 25th of July, about seven o'clock in the afternoon; it will be annular, and may be seen in most of the British colonies in North America, Gulf of Mexico, West Indian islands, Terra-Firma, in South America, and from part of both the Atlantic and Pacific Oceans.

John Metcalfe.

New Questions.

I. Question 566, *by Mr.* Tho. Atkinson.

Near Lincoln's city a worthy knight doth dwell,
For wit and valour few can him excel:
In the late war his honour there did shew,
How he was to his king and country true.
The poor to him do cry in their distress—
He rights the indigent and fatherless;
A friend to arts and sciences is he;
An ornament to all society.
Ladies, from hence * declare his worthy name,
And it record in Diary of fame.

* viz. $\left\{\begin{array}{r} w+x+yz=61 \\ x^2+y+zw=501, \\ y+z+wx=381, \\ w-4=x; \end{array}\right\}$ Where the values of w, x, y, and z shew the places, in the alphabet, of the letters composing this gentleman's name.

II. QUESTION 567, *by Miss* Anna Nicholls.

Walking on shore, I was surprized by the flash of a gun, at sea, bearing S. E. by E. seven seconds after the flash I heard the report, and 4 seconds after that I heard the echo from a castle bearing from me S. W. by W: the distance of the gun and castle are required.

III. QUESTION 568, *by Mr.* John Chipchase.

There is a town in lat. 50°, and another in lat. 30°, north; and, when it was noon, on the 4th of May, at the 1st of of them, the sun was just rising at the second; 'tis required to find the lat. of a port that is equally distant from them both, and 40° W. of the first.

IV. QUESTION 569, *by Mr.* Rd. Gibbons.

An annuity being forborn five years, at compound interest, amounted to 500 l. and now the same hath been forborn ten years it amounts to 1100 l. Required the annuity, and the rate of interest per cent. per annum.

V. QUESTION 570, *by Mr.* Rob. Butler.

Given the radii of two circles = 6 and 8, and the distance of their centers = 10 inches; 'tis required to draw a right line 19 inches long, through their point of intersection, so as to terminate in their peripheries, and to determine the length of each segment thereof, intercepted between the said point and peripheries.

VI. QUESTION 571, *by the late Mr.* W. Toft.

Admit a ship can make her way good, when close hauled, within $6\frac{1}{2}$ points of the wind, being then at N. N. W. and, when she had sailed on the larboard tack 62, and on the starboard tack 75 miles, she is found, by observation, to have altered her latitude 25 miles, northing; required the variation of the compass.

VII. QUESTION 572, *by Mr.* Paul Sharp.

There are two places, *A* and *B*, in north latitude, whose least distance from each other is = $3405\frac{1}{10}$ miles; *A* the westermost's latitude is more than *B*'s by 10°, and the sum of both their latitudes is = their difference of longitude; required the latitude of each place, with the investigation.

VIII. QUES-

VIII. QUESTION 573, *by Mr.* Wm. Spicer.

Given $\left(\frac{0\cdot6}{x^{\frac{1}{3}}}\right)^{\frac{x^{\frac{1}{3}}}{0\cdot5}} = y$, where x multiplied by 100000*l.* is the fortune of an agreeable young lady; required her fortune when y is a maximum.

IX. QUESTION 574, *by Mr.* Samuel Vince.

Given $ax^7y^3 = bx^8y^2 + cx^4y^6 - x^{11} + dy^{10}$, the equation of a curve; to find the area and subtangent.

X. QUESTION 575, *by Mr.* Tho. Barker, *Teacher of Mathematics, and Land-Surveyor, at* Wiſſett *in* Suffolk.

Given $ax^n = y^9$, the equation of a curve; 'tis required to find the exponent n, and the length of the curve, when the diſtance of the center of gravity of a ſolid formed thereby, from the vertex, is $= \frac{25x}{34}$.

XI. QUESTION 576, *by Mr.* Wm. Sewell.

Required the dimenſions of a cone, which, being ſuſpended by its vertex, ſhall vibrate as many times in a minute, as it is inches in altitude, its content being 1728 ſolid inches.

XII. QUESTION 577, *by Mr.* T. Moſs.

To inveſtigate a general expreſſion that will exhibit all the different factors, whereby any quantity of ſpirits, of any ſtrength above hydrometer proof (* as 1 to 1, 1 to 2, 1 to 3, &c. to 1 to 20) being multiplied, the reſpective products will ſhew what quantity of water will be neceſſary to reduce ſuch ſpirits to any required ſtrength under that proof († as 1 in 2, 1 in 3, &c. to 1 in 20).

* One gallon of water and one of ſpirits make 2 gallons of hydrometer proof.

† One gallon of water and one of hydrometer proof make 2 gallons of 1 in 2 under that proof, &c.

XIII. QUES-

XIII. QUESTION 578, *by Mr.* Joseph Walker.

To determine the ratio of the axes of a ſpheroid, to which if the common diagonal rod be applied, in the ſame manner as in the gauging of a caſk, it ſhall exhibit its true content.

XIV. QUESTION 579, *by Mr.* T. Moſs.

To determine the number of fifteens that can be made out of a pack of cards, with the inveſtigation.

XV. QUESTION 580, *by Mr.* Cha. Hutton.

Let AIB, $RISO$ [*ſee the fig. to the ſolution*] be two right lines, any-how interſecting each other within the circle $ARBWO$, whoſe diameter is ASW; then, putting $AW = d$, and p, q, r = the coſines of the $\angle$s I, S, A reſpectively, the general equation of the circle, defined by AI (x) and RI (y), conſidered as an abſciſſa and an ordinate, is $\left\{ y^2 \begin{array}{l} +dq \\ +2px \end{array} \right\} y \left. \begin{array}{l} -drx \\ +x^2 \end{array} \right\} = 0$: Required the demonſtration or inveſtigation.

XVI. QUESTION 581, *by Mr.* Tho. Allen, *of* Spalding.

Let DGB be a ſemi-circle [*ſee the fig. to the ſolution*] P a given point in its diameter; to find the nature of the curve DEA, whoſe ordinate EC ſhall be always proportional to the area DGP, bounded by the right lines DP, GP, and the circular arc DG.

The PRIZE QUESTION, *by Mr.* Tho. Moſs.

The longeſt ſide of a trapezium (being the diameter of a circle in which it may be inſcribed), the diſtance intercepted between the extremity thereof and the point of interſection made by that ſide and that which is oppoſite, both produced, being given; alſo the angle formed at the interſection of the diagonal: To conſtruct the trapezium.

Queſtions

1768.

Questions answered.

I. QUESTION 566 *answered by Mr.* James Mills.

LONG may you dwell on earth, immortal WRAY,*
And, when departed, shine in endless day.

Pamphagus observes that this gent.* is a descendant of Sir *Cecil Wray*, who was created a baronet Nov. 25th, 1612.

Answer to the same, by Mr. Paul Sharp.

By substituting for w its equal $x+4$ in the 1st given equation, the value of x will, from thence, be found $= \frac{1}{2} \times \overline{57 - yz}$, and consequently $w = \frac{1}{2} \times \overline{65 - yz}$; and these values substituted in the 2d and 3d given equations, they become $57 - yz + 65z - yz^2 + 2y = 2 \times 501$, and $y + z + \frac{1}{2} \times \overline{57 - yz} \times \overline{65 - yz} = 381$; from which, by pa. 82 of Simp. Essays, $y = 1$ and $z = 23$; and thence $x = 17$ and $w = 21$, and the honourable gentleman's name is *Wray*.

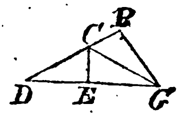
Mess. *G. Lodge*, *E. Jones*, *L. Ker*, *J. Probert*, and *J. Roper* find the values of w, x, y, and z the same as above, by means of equations of the 5th power with all the terms (to which dimension the quest. seems naturally to rise).—— It is also answered by Mess. *J. B. Ashton*, *T. Atkinson* (the proposer), *T. Barker*, *E. Bayley*, *N. Brownell*, *J. Buddle*, *W. Cave*, jun. *W. Dent*, Miss *Ann Nicholls*, *J. Nordon*, *T. Robinson*, *W. Spicer*, *W. Stoker*, Mrs. *E. Suggett*, *E. Smith*, *W. Smith*, *S. Vince*, and *J. Young*.

Mess. *T. Bennett*, *R. Dening*, *W. Dixon*, *H. Fry*, *R. Gibbons*, *T. Hague*, *J. Osborn*, *Pamphagus*, *J. Paty*, *W. Rawle*, *E. Reed*, *A. Rowe*, *W. Sewell*, and *Watson* of *Crawcrook*, have likewise answered it.

II. QUES-

II. QUESTION 567 *answered by Mr.* W. Spicer.

CONSTRUC. From the ΔGPD, having its $\angle P = 112^\circ\ 30'$ (the sum of the given bearings of the gun and castle), and the containing sides PG, PD equal to 7×1142 and $\overline{7+4} \times 1142$ respectively (1142 being the number of feet found passes over in 1″), and draw the right line GC, making the $\angle CGD = \angle CDG$; then, P being the place of observation, and G the gun, C will be the place of the castle required: For, CG being $= CD$ (per construc.), $GC + CP$ will be $= PD$ (the given distance, by construction).



CALCULA. In the ΔGPD, the two sides PG, PD, and included angle being given, the $\angle D$ is found $= 25^\circ\ 18'\ 15''$, and its double, viz. the $\angle PCG$, is $= 50^\circ\ 36'\ 30''$; then, in the ΔPCG, all the angles, and the side PG, are given, to find $GC = 9556{\cdot}4844$ and $PC = 3005{\cdot}5156$ feet.—And in this manner, nearly, it is also constructed by *Nugo Dargnas*; but Mess. *J. Chipchase*, *W. Cole*, *J. Osborne*, *Pamphagus*, *J. Paty*, *E. Reed*, and *W. Wales*, after describing the ΔDPG nearly as before, bisect DG with the perpendicular EC meeting PD in the point C, the place of the castle required.

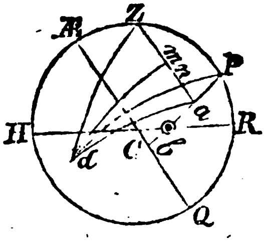
Mr. *W. Sewell* observes that the point C aforesaid, will be in the periphery of an ellipse, having its foci in P, G, and transverse $= PC + GC$, as from the well known property of that curve is manifest.

It is likewise answered by Mess. *J. Addison*, *J. Ainsworth*, *Tho. Barker*, *W. Cave*, *W. Dent*, *W. Dixon*, *R. Gibbons*, *T. Hague*, *E. Jones*, *G. Lodge*, Miss *A. Nicholls* (the proposer), *J. Probert*, *W. Rawle*, *T. Robinson*, *J. Roper*, *P. Sharp*, *C. Smith*, *E. Smith*, *W. Stoker*, Mrs. *E. Suggett*, *T. Todd*, *S. Since*, *H. Watkins*, jun. *Watson* of *Crawcrook*, and *J. Young*.

III. QUESTION 568 *answered by Mr.* John Roper.

Let P represent the north pole, Z the zenith of the place in lat. 30°, a that of the place in lat. 50°, ☉ the sun in the horiz. HR of the former, and the merid. Pb of the latter of them, on the day proposed, d the port required, $Æ Q$ the equat. intersecting HR in the cent. of projec. C, Za, Pd two arcs of great circles intersecting each other in n, and

dm



dm another arc $\perp Za$, which will bisect it, as Zd is $= ad$, per quest. Then, in the right-angled spherical $\triangle Cb\odot$, are given $b\odot$ (the $\odot$'s declin.) $= 15° 58'$, and the $\angle bC\odot$ (the comp. of lat. of the place Z) $= 60°$, to find $Cb = 9° 30\frac{1}{2}'$; whence $Æb$ (or the $\angle ÆPb$ or ZPa) is $= 99° 30\frac{1}{2}'$: And then, in the spherical $\triangle ZPa$, the sides ZP, aP with their included angle being given, Za will be found $= 73° 4' 39''$, and the $\angle PaZ = 63° 13\frac{1}{2}'$. Now, the $\angle aPn$ (the westward bearing of the merid. of the required port d from that of the place a) being given $= 40°$ (per quest.), in the $\triangle aPn$ are given two angles with the interjacent side aP; whence the sides nP, na, and the $\angle Pna$ ($= \angle dnm$) are found $= 35° 12' 4''$, $24° 31' 18''$, and $84° 34' 25''$ respectively; and consequently in the right-angled spherical $\triangle dmn$, mn ($= \frac{1}{2}Za - na$), and the $\angle mnd$ being now known, dn is, from thence, found $= 66° 2' 49''$; to which adding np, and subtracting $90°$ from their sum, the remainder $11° 14' 53''$ south, will be the lat. of the port required.

And in this manner, nearly, the answer is also given by Mess. *T. Barker*, *J. Chipchase* (the proposer, *W. Dixon*, *J. Dymond*, *R. Gibbons*, *E. Jones*, Miss *Ann Nicholls*, *J. Nordon*, *Pamphagus*, *J. Probert*, *R. Pulman*, *T. Robinson*, *A. Rowe*, *C. Smith*, and *W. Wales*.

Mess. *J. Ainsworth*, *J. Buddle*, *J. Dalby*, *W. Rawle*, *P. Sharp*, *W. Spicer*, and *J. Young* answer it algebraically.

IV. QUESTION 569 *answered by Masters* J. Paty *and* J. Osborne, *Youths of about 13 Years of Age, at the Mathematical Academy*, Bristol.

Let $x =$ the annuity, $y =$ the amount of 1l. in one year, $a = 500$, $m = 1100$; then, by the doctrine of compound interest and annuities (see Note, p. 350 of Donn's Arithmetic) $\frac{ay - a}{y^5 - 1} = \frac{my - m}{y^{10} - 1}$ ($= x$), and consequently (putting $\frac{m}{a} - 1 = c$, and $\frac{m}{a} = 2n$) $y^{10} - 2ny^5 = -c$; whence (com-

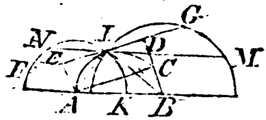
pleating

pleating the ſquare, &c.) * $y = \sqrt[5]{n \pm \sqrt{nn - c}} =$ 1·037137 &c. and the rate of intereſt = 3 l. 14 s. 3¼ d. per cent. per annum, and the annuity required = 92 l. 16 s. 10½ d. ——And thus it is alſo anſwered by Meſſ. *J. Addiſon, T. Barker, J. Bennett, T. Boſworth, N. Brownell, J. Buddle, J. Dalby, Nugo Dargnas, W. Dent, W. Dixon, J. Dymond, T. Hague, E. Jones, G. Lodge, J. Nordon, J. Probert, W. Rawle, T. Robinſon, J. Roper, A. Rowe, W. Sewell, P. Sharp, C. Smith, W. Spicer, W. Stoker, T. Todd, S. Vince,* and *J. Young*, very nearly.

Pamphagus obſerves that 'ſince the annuity forborn 5 'years amounts to 500 l. and when forborn 10 years to '1100 l. it is manifeſt that the intereſt of 500 l. for 5 years 'muſt be = 100 l. Therefore the rate per cent. will be eaſily 'found = 3 l. 14 s. 3 d. ¼·19; and thence, by a well known 'theorem, the annuity = 92 l. 16 s. 10 d. ¼·76.'——And, from this conſideration of the matter, Meſſ. *Jer. Ainſworth* and *W. Wales* have likewiſe anſwered it.

V. QUESTION 570 *anſwered by Mr.* Wm. Cole.

CONSTRUC. Upon AB (the right line joining the centers of the given circles interſecting each other in I) conceive a ſemi-circle to be deſcribed, and apply therein, from either A or B, the chord $AC = 9{\cdot}5$ (half the given line), and parallel thereto let FG be drawn through I, terminating in their peripheries, and it will be the line required.



DEMONST. Through C draw BD, and parallel to it, AE, meeting FG at right angles (by 31. III, and 29. I. Euc.) and biſecting

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* The value of y is here brought out by means of a quadratic equa. without any neceſſity, for it naturally becomes a ſimple one, thus: Divide the numerators of the terms of the given equation $\left(\frac{ay - a}{y^5 - 1} = \frac{my - m}{y^{10} - 1}\right)$ both by $y - 1$, and the denominators by $y^5 - 1$, &c. and you have $y^5 + 1 = \frac{m}{a}$; hence $y = \sqrt[5]{\frac{m}{a} - 1} = \sqrt[5]{\frac{m - a}{a}} = \sqrt[5]{\frac{6}{5}} = 1{\cdot}037137.$

bisecting the chords IG, IF (by 3. III. Euc.), and then $FE + DG$ will be $= ED = AC$ ($= 9{\cdot}5$, per construction), and consequently $FG = 2\,AC = 19$.

CALCULA. Join the points A, I and B, I: Then, in the $\triangle AIB$ the sides being given, the $\angle ABI$ is found $= 36°\ 52'\ 12''$; which taken from the $\angle ABC$ (found $= 71°\ 48'\ 18''$, from the right-angled $\triangle ACB$) leaves $34°\ 56'\ 6'' = \angle IBD$: Whence, from the right-angled $\triangle IDB$, is found $ID = 4{\cdot}5811$; and therefore the segment in the greater circle is $= 9{\cdot}1622$ &c. and that in the lesser $= 9{\cdot}8377$ &c. inches.——If the chord AC be inscribed the other way from B, the construction and method of calculation will be the same, and the segment in the greater circle will, in that case, be found $= 15{\cdot}1681$ &c. and that in the lesser $= 3{\cdot}8318$ &c.

Mess. *J. Ainsworth*, *R. Butler* (the proposer), *T. Bosworth*, *J. Buddle*, *J. Dalby*, *Nujo Dargnas*, *J. Dymond*, *R. Gibbons*, *C. Hutton*, *E. Jones*, Miss *Ann Nicholls*, *Pamphagus*, *J. Paty*, *R. Pulman*, *A. Rowe*, *W. Sewell*, and *W. Wales* construct it in this manner likewise, very nearly.

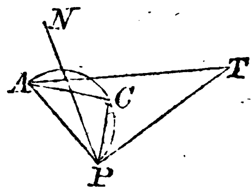
Algebraic Solution to the same, by Mr. John Addison.

Put $FG\,(= 19) = a$, $AI\,(= 6) = \frac{1}{2}b$, $BI\,(= 8) = \frac{1}{2}c$, and $x = FI$; then will $IG = a - x$, and (per 3 Euc. III.) $EI = \frac{1}{2}x$, and $ID = \frac{1}{2} \times \overline{a - x}$. Then, the sides of the triangle AIB being in the ratio of 3, 4, and 5 (per quest.), the angle AIB, opposite the longest of them, will therefore be a right angle, and consequently the right-angled $\triangle$s AEI, BDI similar; whence $\frac{1}{2}b\ (AI) : \frac{1}{2}x\ (EI) :: \frac{1}{2}c\ (BI) : \frac{cx}{2b} = BD$, and (per 47 Euc. I.) $\overline{\frac{cx}{2b}}\big|^2 + \frac{1}{4} \times \overline{a - x}^2 = \overline{\frac{1}{2}c}|^2$, or (in numbers), $x^2 - 13{\cdot}68x = -37{\cdot}8$; solved, $x\ (FI) = 9{\cdot}8375$, or $3{\cdot}8423$; and consequently $IG = 9{\cdot}1625$, or $15{\cdot}1577$, &c.

In this manner, nearly, it is also answered by Mess. *T. Barker*, *W. Dixon*, *J. Probert*, *W. Rawle*, *T. Robinson*, *J. Roper*, *S. Vince*, and *J. Young*.

VI. QUESTION 571 *answered by Mr.* J. Chipchase.

CONSTRUCTION. From P, the port sailed from, let the right line PN be drawn to represent the magnetical or erroneous meridian. Make the $\angle NPT =$ $4\frac{1}{2}$ and the $\angle PTA = 3$ points, taking $PT = 62$ miles (the dist. run on the larboard tack), and $TA = 75$ miles, the distance on the starboard tack), and on the right line joining the points A, P describe a semicircle, and inscribe therein the chord $PC = 25$ miles (the given difference of lat.), and PC will be the direction of the true meridian, AC the departure, and the $\angle NPC$ the variation required; the demonstration of which is evident from the nature of turning to windward, &c.



CALCULATION. In the $\triangle APT$ are given PT, TA together with their included angle, whence AP is found $=$ $41\cdot6694$, and the $\angle APT = 90^\circ\ 29\frac{3}{4}'$, nearly; from which take the $\angle APC$ (found $= 53^\circ\ 8'$, from the right-angled $\triangle ACP$) and the remainder $37^\circ\ 21\frac{3}{4}'$ ($= \angle CPT$) taken from the $\angle NPT$ ($= 50^\circ\ 37\frac{1}{2}'$, per construc.) leaves the $\angle NPC$ $= 13^\circ\ 15\frac{1}{4}'$ the variation required, which is westerly.

In this manner, nearly, it is also constructed by Mess. *J. Addison*, *T. Barker*, *T. Bosworth*, *J. Buddle*, *J. Dalby*, *W. Dixon*, *R. Gibbons*, *C. Hutton*, *E. Jones*, Miss *Ann Nicholls*, *Pamphagus*, *J. Probert*, *W. Rawle*, *T. Robinson*, *J. Roper*, *A. Rowe*, Mrs. *E. Suggett*, *H. Watkins*, jun. *J. Young*, and *W. Wales*.

Mess. *W. Spicer* and *W. Stoker* have given neat algebraic solutions to it.

VII. QUESTION 572 *answered by Mr.* W. Wales.

Put $c =$ cosine of 10° (the given diff. of lat. of the two places), $a =$ cosine of $56^\circ\ 45'\ 2''$ (their given nearest dist.) and $x =$ cosine of their difference of longitude required (rad. $= 1$); then, the sine or cosine of the sum of two arcs being every where equal to the sine or cosine of the sum of their complements, the cosine of the sum of the co-latitudes of the said places will be $= x$ (per quest.); and (per Simp. Trig. prop. 27 p. 74) $\frac{2c - 2a}{c + x} = 1 - x$ (the versed sine of

their

their diff. of long.): Whence, by reduction and putting $1-c=2s$, there results $x=\sqrt{ss+2a-c}+s='34199$, the nat. cos. of 70°; and so the latitudes are 40° and 30°. ——But x may be either affirmative or negative, and consequently the sum of the lat. of the two places either 70°, or its supplement to 180°, &c.

This question is also ingeniously answered by Mess. *J. Ainsworth*, *T. Barker*, *W. Dixon*, *R. Gibbons*, Miss *Ann Nicholls*, *Pamphagus*, *W. Rawle*, *T. Robinson*, *P. Sharp* (the proposer), *W. Spicer*, Mrs. *E. Suggett*, *S. Vince*, and *J. Young*.

VIII. QUESTION 573 *answered by* Pamphagus.

Would you the lady's fortune know,
Look down, and you will see't below.*

* 937l. 13s. 1d.

The same answered by Mr. N. Brownell.

The given equation being $\overline{\frac{0{\cdot}8}{x^{\frac{1}{3}}}}\Big|^{2x^{\frac{2}{3}}}=y$, put $n=\frac{2}{3}$, $z=$ hyp. log. of $\frac{0{\cdot}8}{x^{\frac{1}{3}}}$, and $m=0{\cdot}43429448$, and then $2zx^n$ ($=$ log. of y) is a max. (per question). In fluxions, $2\dot{z}x^n+2nzx^{n-1}\dot{x}=0$, or $-\frac{2}{3}\times x^{n-1}\dot{x}+2nzx^{n-1}\dot{x}=0$ (because $\dot{z}=\frac{-\dot{x}}{3x}$, by the nature of logs.); whence z ($=$ hyp. log. of $\frac{0{\cdot}8}{x^{\frac{1}{3}}}$) $=\frac{4}{3}$, and $0{\cdot}5790593$ $(\frac{4m}{3})$ is $=$ tabular log. of $\frac{0{\cdot}8}{x^{\frac{1}{3}}}$, or the log. of $x=\overline{\text{log. of } 0{\cdot}8-{\cdot}5790593}\times 3=7{\cdot}9720921$, and the natural number answering thereto is $0{\cdot}0093777608$; which multiplied by 100000, gives 937l. 15s. 2½d. $=$ the lady's fortune.

Mess. *J. Ainsworth*, *T. Barker*, *J. Bennett*, *J. Buddle*, *J. Dalby*, *W. Rawle*, *T. Robinson*, *J. Roper*, *W. Sewell*, *P. Sharp*, *E. Smith*, *W. Spicer* (the proposer), and *S. Vince* have also answered it in this manner nearly, and bring out the same conclusion.

IX. QUESTION 574 *anfwered by Mr.* J. Leader.

Subftitute vx for y in the given equation, and it becomes $a\dot{v}^3 \times x^{10} = bv^2x^{10} + cv^6x^{10} - x^{11} + dv^{10}x^{10}$; and thence $x = dv^{10} + cv^6 - av^3 + bv^2$, $y\ (= vx) = dv^{11} + cv^7 - av^4 + bv^3$, and confequently $y\dot{x}$ (the fluxion of the area $= \overline{dv^{11} + cv^7 - av^4 + bv^3} \times \overline{10dv^9\dot{v} + 6cv^5\dot{v} - 3av^2\dot{v} + 2bv\dot{v}}$; and the fluent $\frac{10}{21}d^2v^{21} + \frac{16}{17}cdv^{17} - \frac{13}{14}adv^{14} + \frac{1}{13} \times \overline{12bd + 6c^2} \times v^{13} - \frac{9}{10}acv^{10} + \frac{8}{9}bcv^9 + \frac{3}{7}a^2v^7 - \frac{5}{6}abv^6 + \frac{2}{5}b^2v^5 - f$, is $=$ the area required (f being the fum of the coefficients of the powers v).——Then, from the given equation in fluxions, (by tranfpofition, divifion, and multiplying by y) is had $\frac{y\dot{x}}{\dot{y}} = \frac{2bx^8y^2 - 3ax^7y^3 + 6cx^4y^6 + 10dy^{10}}{11x^{10} - 8bx^7y^2 + 7ax^6y^3 - 4cxy^6}$, the fubtangent required.

Anfwers to this queftion have alfo been received from Meff. *J. Ainfworth, T. Allen, T. Barker, Pamphagus, T. Robinfon, W. Sewell, P. Sharp,* and *S. Vince* (the propofer).

X. QUESTION 575 *anfwered by Mr.* Pamphagus.

Since $y^9 = ax^n$, y^2 will be $= a^{\frac{2}{9}}x^{\frac{2n}{9}}$, and (by page 204 of Simp. Flux.) the $\dfrac{\text{flu. of } x^{\frac{2n}{9}+1}\dot{x}}{\text{flu. of } x^{\frac{2n}{9}}\dot{x}} = \frac{2n+9}{2n+18}x =$ the diftance of the center of gravity of the faid folid from its vertex, which (per queft.) is $= \frac{25}{34}x$; and confequently $n = 8$.——Having thus obtained the numerical value of n, we fhall, in the next place, have (putting $\frac{81}{64a^{\frac{1}{4}}} = c$) $\dot{y}\sqrt{1 + cy^{\frac{1}{4}}} =$ the flux. of the curve's length; and the correct fluent

fluent * thereof $\frac{8 \times \overline{1 + cy^{\frac{1}{4}}}|^{\frac{3}{2}}}{9c} \times \overline{y^{\frac{3}{4}} - \frac{6y^{\frac{1}{2}}}{7c} + \frac{24y^{\frac{1}{4}}}{35c^2} - \frac{16}{35c^3}}$ $+ \frac{128}{315c^4}$ will be the length of the curve required.

Mess. *J. Ainsworth*, *T. Allen*, *T. Barker* (the proposer) *J. Bennett*, *T. Bosworth*, *N. Brownell*, *T. Todd*, *W. Rawle*, *W. Sewell*, *W. Spicer*, and *S. Vince* answer it in the same manner, very nearly.

* See Simpson's Flux. art. 84.

XI. QUESTION 576 *answered by Mr.* T. Todd.

Put $c = 3{\cdot}1416$ (the circumference of the circle whose diameter is 1), $s = 1728$ (the given solidity of the required cone), $x =$ its altitude, $y =$ the rad. of its base, and $p = 39{\cdot}2$ inches (the length of a pendulum vibrating seconds); then (per p. 225 of Simp. Flux.) $\frac{4xx + yy}{5x} =$ the dist. of the center of oscillation from its vertex (or point of suspension), and (by the doctrine of pendulums, their lengths being inversely as the squares of the number of vibrations made by them in the same time) $\frac{60^2 \times 5px}{4xx + yy}$ (the square of the number of vibrations performed by the cone in 1′) $= xx$ (per quest.); from which, and the equation $\frac{1}{3}cxyy = s$, is found x $(= \overline{4500p - \frac{3s}{4c}}|^{\frac{1}{3}}) = 56{\cdot}039$ &c. and y $(= \sqrt{\frac{3s}{cx}}) = 5{\cdot}426$ &c. inches.

Mr. *Ja. Young* puts $a = 39{\cdot}2$ inches, $s = 1728$, $p = 3{\cdot}1416$, $x =$ the height, and $y =$ rad. of the base of the required cone, in inches; then $\frac{1}{3}pxyy = s$, and (per pa. 239 of Emer. Flux. 1st edit. $\frac{4xx + yy}{5x} =$ dist. of the cent. of oscillation from its vertex, or the length of a pendulum isochronal to the cone: Whence, $\sqrt{\frac{4xx + yy}{5ax}}$ being $=$ the time of one vibration, $\sqrt{\frac{4xx + yy}{5ax}} : 1 :: 60'' : x$, or $x\sqrt{\frac{4xx + yy}{5ax}} = 60$ (per quest.); from whence, and the preceding equation, x comes out $= 56{\cdot}0394$ &c. and $y = 5{\cdot}4264$ &c. inches.

In the same manner as above, nearly, the answer is also given by Mess. *J. Addison*, *J. Ainsworth*, *T. Allen*, *T. Barker*, *T. Bosworth*, *N. Brownell*, *J. Buddle*, *W. Dawes*, *W. Dent*, *W. Dixon*, *E. Jones*, *Pamphagus*, *W. Rawle*, *T. Robinson*, *W. Sewell* (the proposer), *C. Smith*, *E. Smith*, *S. Vince*, and *W. Wales*.

XII. QUESTION 577 *answered by Mr.* T. Moss, (*the Proposer*).

Let such a quantity of the strong spirits (*i. e.* 1 to 1, 1 to 2, 1 to 3, &c.) as one gallon of water will reduce to hydrometer proof be $= a$, and the quantity when so reduced $= b$ (viz. $b = a + 1$): Moreover, let such a quantity of hydrom. proof as one gall. of water will reduce to the lower strength (*i. e.* 1 in 2, 1 in 3, &c.) $= c$, and the quantity when so reduced $= d$; also, let the required quantity of water necessary to reduce one gallon of the strong to any given strength under hydrom. proof $= x$:——Then, $d : c :: 1 + x : \frac{c + cx}{d}$ = the quantity of dydrom. proof spirits in. $1 + x$ quantity (or in one gallon of strong); but $a : b :: 1$ (*i. e.* one gall. of strong spirits): $\frac{b}{a}$; whence $\frac{c + cx}{d} = \frac{b}{a}$, and $x = \frac{bd}{ac} - 1$.

GEN. RULE. Let such a quantity of the strong as one gallon of water will reduce to hydrometer proof, be multiplied by such a quantity of the strong as one gallon of water will reduce to the given strength under hydrometer proof: Then let the product of the two quantities so reduced be divided by the former product; from the quotient subtract unity, and the remainder will be the required factor for multiplying any given quantity of spirits above hydrometer proof, in order to shew the quantity of water necessary to be added to make them of any given strength below that proof.

Suppose, for example, spirits of 1 to 3 were to be reduced to 1 in 7 (*i. e.* to export strength): Here $a = 3$, $b = 4$, $c = 6$, and $d = 7$; therefore, by the above rule, $\frac{bd}{ac} - 1$ $= \frac{4 \times 7}{3 \times 6} - 1$ ($= 1{\cdot}5555$ &c. $- 1$) $= {\cdot}555$ &c. the required factor.

NOTE, This prob. will be of great utility to persons concerned in trying the strength of spirits by the hydrometer.

Mess. *T. J. Ainsworth*, *T. Barker*, and *E. Jones* have likewise sent answers to this question.

XIII. QUES-

XIII. QUESTION 578 *answered by Mr.* Rd Gibbons.

As this quest. refers to the common diagonal rod, I shall take my example therefrom, where, against 22 inches, is 29 wine gallons or 6699 cubic inches. Then, putting $2x =$ the diameter and $2z =$ the axis of the required spheroid, $xx + zz$ will be $= 22^2$ (per 47 Euc. I.), and $4{\cdot}1888\,xxz$ (the solid content of the spheroid, in inches) $= 6699$ (per quest.). Whence (xx being exterminated) $484z - z^3 = 1599{\cdot}2647$: Reduced, $z = 20{\cdot}112$; and thence $x = 8{\cdot}917$, and their ratio (or the ratio required) is as $2{\cdot}25$ to 1, or as 9 to 4.

Mess. *J. Addison* and *J. Young* have also answered this quest. very ingeniously, and bring out the same conclusion.

The same answered by Mr. Jos. Walker (*the Proposer*).

Let the content of a spheroid (in inches) be denoted by a, its corresponding diagonal (on the gauging rod) by d, and the semi-axis, on which the spheroid is supposed to be generated, by x ($p = 3{\cdot}14159$ &c. Then, by the known theorem, $\overline{dd - xx} \times \frac{4px}{3} = a$; which is a general equation. Now, let a, for instance, be interpreted by 60 ale gallons, or 16920 cubic inches, and d the corresponding diagonal (nearly on the rod), by 30 inches; then, the above equation becomes $900x - x^3$ ($= 16920 \div \frac{1}{3} \times 4p$) $= 4040{\cdot}1165$: Which having 2 affirmative roots, we therefore have $x = 27{\cdot}425$ and $4{\cdot}61$, nearly; being respectively the semi-lengths of the oblong and prolate spheroid: Whence ($\sqrt{dd - xx}$) the semi-diamenters thereof will be $12{\cdot}16$ and $29{\cdot}64$. Hence it appears that the axes of every oblong spheroid, whose content may be truly obtained by the common diagonal rod, must be in the ratio of $12{\cdot}16$ to $27{\cdot}425$, but those of the prolate one, as $4{\cdot}61$ to $29{\cdot}64$, very nearly: Which ratios agree with those in Moss's Gauging, pa. 197, and are a confirmation of the truth of the author's principles, with respect to the vast comprehensiveness and utility of the diagonal rod.

Pamphagus thinks this question is solved in a manner sufficiently scientifical in Moss's Gauging, pa. 197, he says, 'excepting to such as are resolved to join in opinion with 'the doughty reviewer of that excellent treatise, and pronounce such a thing impossible.

XIV. QUES-

XIV. QUESTION 579 *answered by Mr.* H. Brown (*of the* Tower), *Mr.* T. Moss (*the Proposer*), *and Mr.* J. Wore.

Cards.	*Combinations.*		*Cards.*	*Combinations.*	
10,5	16.4	64	7,3,2,2,1	4.4.6.4	384
10,4,1	16.4.4	256	7,3,2,1,1,1	4.4.4.4	256
10,3,2	16.4.4	256	7,2,2,2,2		4
10,3,1,1	16.4.6	384	7,2,2,2,1,1	4.4.6	96
10,2,2,1	16.6.4	384	7,2,2,1,1,1,1	4.6	24
10,2,1,1,1	16.4.4	256	6,6,3	6.4	24
9,6	4.4	16	6,6,2,1	6.4.4	96
9,5,1	4.4.4	64	6,6,1,1,1	6.4	24
9,4,2	4.4.4	64	6,5,4	4.4.4	64
9,4,1,1	4.4.6	96	6,5,3,1	4.4.4.4	256
9,3,3	4.6	24	6,5,2,2	4.4.6	96
9,3,2,1	4.4.4.4	256	6,5,2,1,1	4.4.4.6	384
9,3,1,1,1	4.4.4	64	6,5,1,1,1,1	4.4	16
9,2,2,2	4.4	16	6,4,4,1	4.6.4	96
9,2,2,1,1	4.6.6	144	6,4,3,2	4.4.4.4	256
9,2,1,1,1,1	4.4	16	6,4,3,1,1	4.4.4.6	384
8,7	4.4	16	6,4,2,2,1	4.4.6.4	384
8,6,1	4.4.4	64	6,4,2,1,1,1	4.4.4.4	256
8,5,2	4.4.4	64	6,3,3,3	4.4	16
8,5,1,1	4.4.6	96	6,3,3,2,1	4.6.4.4	384
8,4,3	4.4.4	64	6,3,3,1,1,1	4.6.4	96
8,4,2,1	4.4.4.4	256	6,3,2,2,2	4.4.4	64
8,4,1,1,1	4.4.4	64	6,3,2,2,1,1	4.4.6.6	576
8,3,3,1	4.6.4	96	6,3,2,1,1,1,1	4.4.4	64
8,3,2,2	4.4.6	96	6,2,2,2,2,1	4.4	16
8,3,2,1,1	4.4.4.6	384	6,2,2,2,1,1,1	4.4.4	64
8,3,1,1,1,1	4.4	16	5,5,5		4
8,2,2,2,1	4.4.4	64	5,5,4,1	6.4.4	96
8,2,2,1,1,1	4.6.4	96	5,5,3,2	6.4.4	96
7,7,1	6.4	24	5,5,3,1,1	6.4.6	144
7,6,2	4.4.4	64	5,5,2,2,1	6.6.4	144
7,6,1,1	4.4.6	96	5,5,2,1,1,1	6.4.4	96
7,5,3	4.4.4	64	5,4,4,2	4.6.4	96
7,5,2,1	4.4.4.4	256	5,4,4,1,1	4.6.6	144
7,5,1,1,1	4.4.4	64	5,4,3,3	4.4.6	96
7,4,4	4.6	24	5,4,3,2,1	4.4.4.4.4	1024
7,4,3,1	4.4.4.4	256	5,4,3,1,1,1	4.4.4.4	256
7,4,2,2	4.4.6	96	5,4,2,2,2	4.4.4	64
7,4,2,1,1	4.4.4.6	384	5,4,2,2,1,1	4.4.6.6	576
7,4,1,1,1,1	4.4	16	5,4,2,1,1,1,1	4.4.4	64
7,3,3,2	4.6.4	96	5,3,3,3,1	4.4.4	64
7,3,3,1,1	4.6.6	144	5,3,3,2,2	4.6.6	144

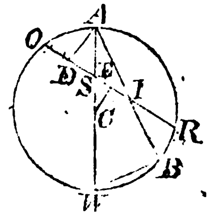
Cards.	Combinations.		Cards.	Combinations	
5,3,3,2,1,1	4.6.4.6	576	4,3,3,2,2,1	4.6.6.4	576
5,3,3,1,1,1,1	4.6	24	4,3,3,2,1,1,1	4.6.4.4	384
5,3,2,2,2,1	4.4.4.4	256	4,3,2,2,2,2	4.4	16
5,3,2,2,1,1,1	4.4.6.4	384	4,3,2,2,2,1,1	4.4.4.6	384
5,2,2,2,2,1,1	4.6	24	4,3,2,2,1,1,1,1	4.4.6	96
5,2,2,2,1,1,1,1	4.4	16	4,2,2,2,2,1,1,1	4.4	16
4,4,4,3	4.4	16	3,3,3,3,2,1	4.4	16
4,4,4,2,1	4.4.4	64	3,3,3,3,1,1,1		4
4,4,4,1,1,1	4.4	16	3,3,3,2,2,2	4.4	16
4,4,3,3,1	6.6.4	144	3,3,3,2,2,1,1	4.6.6	144
4,4,3,2,2	6.4.6	144	3,3,3,2,1,1,1,1	4.4	16
4,4,3,2,1,1	6.4.4.6	576	3,3,2,2,2,2,1	6.4	24
4,4,3,1,1,1,1	6.4	24	3,3,2,2,2,1,1,1	6.4.4	96
4,4,2,2,2,1	6.4.4	96	3,2,2,2,2,1,1,1,1		4
4,4,2,2,1,1,1	6.6.4	144			
4,3,3,3,2	4.4.4	64			
4,3,3,3,1,1	4.4.6	96			

The number sought 17264.

Mr. *J. Pike* has also answered it very ingeniously by a different method.——Mr. *T. Barker* and *Pamphagus* answer it from p. 520, 521, of Birks' Arithmetic, where, they inform us, it is calculated by Major Watson, and the number found to be 17264, or 33528 holes.

XV. QUESTION 580 *answered by Mr.* W. Wales.

Draw AD, $CE \perp OR$ (C being the cent. of the circ.), and let the points W, B be joined.



Then, d being $=$ the diameter AW, p, q, r $=$ the cosines of the angles I, S, A (to rad. 1), $x = AI$, and $y = IR$ (per quest.), $1 : d :: r : dr = AB$; $\because BI = dr - x$. Also, $1 : x :: p : px = DI$, and $y \pm px = DR$. Lastly, (because of the parallel lines AD, CE, and by composition of ratios) $1 : \frac{1}{2}d :: q : \frac{1}{2}dq = DE$, and $DR \mp DE = ER = EO = \frac{1}{2}RO$ (cor. 2, 3. Simp. Geom. 2d edit.) $= y \mp \frac{1}{2}dq \pm px$; $\because IO = \overline{y \mp dq \pm 2px}$, and (21.3) $y \times \overline{y \mp dq \pm 2px} = x \times \overline{dr - x}$; which, properly ordered, gives $yy \mp dqy \pm 2pxy - drx + xx = 0$, a general equation; the upper signs of the 2d and 3d terms obtaining when the $\angle S$ is obtuse and the lower ones when the $\angle I$ is obtuse, and when they are both acute, the two affirmative signs take place. But, if AB be drawn on the other

other side of the diam. AW, the contrary must be observed in every case.

COR. 1. If AB coincides with AW, $p=q$, and $r=$ radius; ∵ the equation, in this case, is] $yy \mp dqy \pm 2qxy - dx + xx = 0$.

COR. 2. When OR is $\perp AW$, q vanishes, and the equation becomes $yy \pm 2pxy - drx + xx = 0$.

COR. 3. Lastly, when both these suppositions take place, the expression becomes $yy - dx + xx = 0$; the common equation.——And in this manner, nearly, the answer is also given by Mess. *J. Ainsworth*, *Pamphagus*, *W. Sewell*, and *T. Todd.*

Answer to the same by Mr. Cha. Hutton (*the Proposer*).

Put $a = AB$, and s, n, and $c =$ the sines of the $\angle$s I, S, and A (the rest the same as above); then (by trigonom.) $cxn^{-1} = IS$ and $sxn^{-1} = AS$; and (by the prop. of the circle, OS being $= AS \times SW \times RS^{-1} = sxn^{-1} \times \overline{d - sxn^{-1}} \times \overline{y + cxn^{-1}}|^{-1}$ and $OI \times IR = AI \times IB$) $snx^{-1} \times \overline{d - sxn^{-1}} \times \overline{y + cxn^{-1}}|^{-1} + cxn^{-1} \times y = x \times \overline{a - x}$: But $a = dr$, $s = cq + nr$, and $p = cn - qr$; whence, by substitution, $\left.\begin{matrix} yy + dq \\ + 2px \end{matrix}\right\} y \left.\begin{matrix} - drx \\ + xx \end{matrix}\right\} = 0$.

COR. 1. Hence, $y = -px - \frac{1}{2}dq \pm \sqrt{\overline{px + \frac{1}{2}dq}|^2 + drx - xx}$.

COR. 2. If $y = 0$, then $x = 0$, or $= dr (= AB)$ as it ought.

COR. 3. If $x = 0$, then $y = 0$, or $= -dq$.

COR. 4. If $x = AB = dr$, then $y = 0$, or $= -d \times \overline{2pr + q}$.

COR. 5. If either S or I be an obtuse angle, the sign of q or p must be changed wherever it is found. A can never be an obtuse angle, and therefore r is always affirmative.

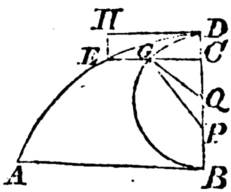
COR. 6. If AB be on the other side of AW, the signs of p and q will be changed.

COR. 7. If AB coincides with AW, then $r = 1$, $p = q$, and the general equation is $yy + dqy + 2qxy - dx + xx = 0$; which, when $S =$ a right angle (q being then $= 0$) becomes barely $yy = dx - xx$, the common equation to the circle.

XVI. QUES-

XVI. QUESTION 581 *answered by Mr.* T. Allen (*the Proposer*).

Let DGB be any curve meeting the axis DB in B, and let P be a given point in that axis. Then, putting the abscissa $DC = x$, ord. $CG = y$, and $DP = a$, $\frac{y\dot{x} + a\dot{y} - x\dot{y}}{2}$ will truly exhibit the fluxion of the space DGP. Wherefore, in the present case, putting DQ (rad.) $= r$, $QP = q$, and the arc $DG = z$, we shall have $\frac{y\dot{x} + a\dot{y} - x\dot{y}}{2} = \frac{\dot{x}\sqrt{2rx - xx}}{2}$



$- \frac{rx\dot{x} - x^2\dot{x}}{2\sqrt{2rx - xx}} + \frac{a\dot{y}}{2} = \frac{r\dot{x}x}{2y} + \frac{a\dot{y}}{2}$; whose fluent, $\frac{rz + qy}{2}$, * is the area of the space DGP. If therefore this area be to the ordinate EC in the constant ratio of 1 to m, the ordinate itself will be $= \frac{m}{2} \times \overline{rz + qy}$.

CORROL. Suppose $m = 2$, and draw DH parallel and $EH \perp CE$, then will $rx\dot{z} + qx\dot{y} = \frac{arx\dot{x} - qx^2\dot{x}}{y}$ be the flux. of the space EDH; whose fluent is $ar \times \overline{z - y} + \frac{3qry + qyx - 3qrz}{2}$: And therefore the area of EDC will be $= rzx + \frac{1}{2}qyx + ar \times \overline{y - z} + \frac{1}{2}qr \times \overline{z - y}$; which, when $x = 2r$, becomes $= \frac{3.1416rr}{2} \times \overline{4r - 2a + 3q}$, the true area of the whole curve DAB.

Answer

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* This expression is no more than the sum of the two values of the sector $DGQ = \frac{1}{2}rz$, and $\triangle QPG = \frac{1}{2}qy$, as given by the common known rules; and therefore all the fluxionary process from the beginning of the solution to this expression, is quite needless, and may be omitted.

Answer to the same by Mr. Cha: Hutton.

Let $\mathfrak{Q}$ be the center of the semicircle DGB, and draw $\mathfrak{Q}G$. Put $r = D\mathfrak{Q}$ $(= \mathfrak{Q}G = \mathfrak{Q}B)$, $d = \mathfrak{Q}P$, and $n = \dfrac{\text{the semicir. } DGBD}{AB} = \dfrac{\text{the area } DGPD}{EC}$. Then, it is evident that $EC = (\dfrac{DGPD}{n} =)\ \frac{1}{2}rn^{-1} \times$ the arc $DG \pm \frac{1}{2}dn^{-1} \times GC$, according as P is below or above $\mathfrak{Q}$.

COR. 1. If P coincides with $\mathfrak{Q}$, $d = 0$, and then $EC = \frac{1}{2}rn^{-1} \times DG$.

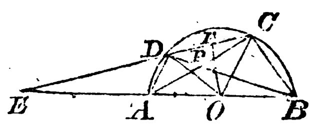
COR. 2. If P coincides with B, then $d = r$, and $EC = \frac{1}{2}rn^{-1} \times \overline{DG + GC}$.

COR. 3. If P coincides with D, $d = -r$, and then $EC = \frac{1}{2}rn^{-1} \times \overline{DG - GC}$.

This question is likewise answered by Mess. *T. Barker*, *J. Bennett*, *J. Buddle*, *Pamphagus*, *Plus Minus*, *W. Sewell*, *T. Todd*, and *S. Vince*.

The PRIZE QUESTION *answered by Mr.* W. Wales.

CONSTRUC. From O, the center of the circumscribing semicircle, draw the radii OC, OD, making with each other an angle = twice the complement of that formed by the diagonals at the point of intersection, and through O draw BE, meeting the semi-circumference in B, A and the right line, joining the points C, D, produced in E, so that AE may = the given intercepted distance; then, join the points B, C and A, D, and the thing is done.



DEMONST. Let the diagonals AC, DB be drawn, intersecting each other in P. The $\angle ACB$ (PCB), being in a semicircle, is a right angle; whence the $\angle PBC$ is the complement of the $\angle P$, = half the $\angle DOC$ (Simp. Geom. 10. 3. 2d edit.).——The method of calculation, from this construc. is extremely obvious and easy.

Construction

CONSTRUCTION *to the same by Mr.* Tho. Bosworth.

Describe the $\triangle ECO$, having the $\angle C =$ the given angle to be formed at the intersection of the diagonals, the side $OC = \frac{1}{2}$ the given side of the trapezium, and the side $OE = OC +$ the given distance to be intercepted, and with the radius OC describe a semicircle meeting EO, produced in B, and cutting EC in D; join A, D and B, C, and then will be formed the trapezium required.——For, letting fall $OF \perp DC$ and drawing the diagonals, intersecting each other in P, it is evident (from the properties of the circle, &c.) that the $\angle DBC = \angle FOC$, and the $\angle ACB = \angle OFC$ (being both right angles); wherefore the $\angle CPB = \angle ECO$, the given angle, by construction.

Exceedingly neat and concise constructions to this question have also been received from Mess. *T. Adams, J. Ainsworth, T. Barker, J. Bennett, D. Bolton, J. T. Bersham, J. Buddle, R. Butler, J. Chipchase, W. Cole, J. Dalby, Nuso Dargnas, J. Davidson, W. Dixon, J. Dymond, P. George, R. Gibbons, C. Hutton, E. Jones, L. Ker, Pamphagus, Plus Minus, J. Powle, R. Pulman, Don Quixote, W. Rawle, J. Roper, W. Sewell, E. Smith, W. Smith,* and *S. Vince.*——The construction by Mr. *T. Moss* (the proposer) is very ingenious, and depends on a new prop. of the circle not yet publicly known.

The two prizes of 12 and 8 Diaries, for the solution of the prize question, are fallen to the respective lots of Mr. William Wales *and Mr.* Tho. Bosworth.

The Eclipses calculated for 1768.

There will happen six eclipses this year; three of the sun, invisible, and three of the moon, visible in Great Britain.

The first is of the moon, on Monday the 4th of January, in the morning, according to the following calculations.

Calculated by	Begin. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Digits ° ′
Mr. Chapman, for Foxton	3 12½	4 23	5 33½	2 21	4 54
Mr. T. Allen, for Spalding	3 17	4 24	5 32		4 34
Mr. Metcalfe, for London	3 13⅓	4 24½	5 35½	2 22¼	4 57⅚

This eclipse will be visible to all Europe, America, and great part of Africa.

The ſecond is of the ſun, on Tueſday the 19th of January, about 6 o'clock in the afternoon, viſible in North America.

The third is a total lunar eclipſe, on Thurſday the 30th of June, in the morning, correſponding to the following calculations.

Calculated by	Beg. of Eclipſ.	Beg. of to. Da.	Mid.	End to. D.	End Eclip.	Dur. Eclip	Digits eclipſ.
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	° ′
W. Chapman	2 $4\frac{3}{4}$	3 $17\frac{1}{2}$	3 $51\frac{1}{13}$	4 $24\frac{1}{2}$	5 $37\frac{1}{2}$	3 $32\frac{2}{3}$	14 27.
T. Allen	2 7	3 $22\frac{1}{3}$	3 $50\frac{2}{3}$	4 19	5 $34\frac{1}{2}$		
J. Metcalfe	2 $17\frac{1}{20}$	3 $23\frac{1}{11}$	3 54	4 $24\frac{5}{6}$	5 $30\frac{5}{6}$	3 $13\frac{5}{6}$	14 $26\frac{1}{6}$

This eclipſe may be ſeen from the beginning to the end in moſt known parts of America, and the weſtern parts of Africa: But in the weſtern parts of Europe, only part of it can be ſeen; for the moon ſets before the eclipſe ends.

The fourth is a ſolar eclipſe, on Thurſday the 14th of July, near 2 o'clock in the morning. It will be viſible in the remote ſouthern parts of the earth, beyond New Holland.

The ſifth eclipſe is alſo of the ſun, upon Friday the 9th of December, near 8 in the morning, but utterly inviſible to all known parts of the globe.

The ſixth, and laſt, this year, is another total eclipſe of the moon, and will happen upon Friday the 23d of December, in the afternoon. The moon riſes totally eclipſed a little before the ſun ſets; and, if the air proves clear, they may both be ſeen above the horizon, at the ſame time.

Calculated by	Beg. Eclip.	Beg. to. D.	Mid.	End to. D.	End Eclip.	Dura. Eclip.	Dig. eclip.
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	° ′
Mr. W. Chapman	1 $18\frac{1}{3}$	2 $16\frac{1}{3}$	3 $5\frac{1}{3}$	3 $54\frac{1}{3}$	4 $52\frac{1}{3}$	3 34	20 45
Mr. T. Allen	1 $21\frac{1}{6}$	2 $18\frac{1}{3}$	3 $6\frac{1}{2}$	3 $54\frac{3}{4}$	4 $51\frac{5}{6}$		
Mr. J. Metcalfe	1 $18\frac{2}{3}$	2 $16\frac{5}{6}$	3 $6\frac{1}{2}$	3 $56\frac{1}{4}$	4 $54\frac{1}{2}$	3 $35\frac{5}{6}$	20 50

At the beginning of this eclipſe the moon will be vertical to the eaſtern ocean, at the middle of the Chineſe ſea, and at the end, to the ſouth coaſt of China; it will therefore be ſeen, from the beginning to the end, in all parts of Aſia and the Eaſt Indies, the eaſtern parts of Europe and Africa, and to parts near the pole; but only viſible in part to the weſtern kingdoms of Europe, becauſe the moon riſes not in thoſe countries till after the eclipſe begins.

JOHN METCALFE.

Meſſ.

Meſſ. *H. Andrews*, *T. Atkinſon*, and *J. Denham* have alſo obliged us with calculations of theſe eclipſes, but we could not find room for their inſertion.

Mr. *W. Chapman*, of *Foxton*, in *Leiceſterſhire*, has favoured us with a curious table of all the ſolar eclipſes that will be viſible from the year 1769 to the year 2000; which we are ſorry our narrow limits will not this year permit us to publiſh.

New Queſtions.

I. Question 582, *by Mr.* Geo. Lodge.

Diarian ſages, you will find,
From the * equation here ſubjoin'd,
The age and fortune of a fair,
Whoſe life is blameleſs, heart ſincere.

$$\text{* viz.} \left\{ \begin{array}{l} xxy + \dfrac{\sqrt{x}}{y} = 16386, \\ xyy + \dfrac{x}{\sqrt{y}} = 1056; \end{array} \right\}$$ where x repreſents the number of 10*l.* notes that make up the fortune, and y the ſquare root of the age of this amiable fair one.

II. Question 583, *by Mr.* Wm. Spicer.

A heavy body deſcending freely by the force of gravity on an inclined plane, whoſe length is 400 feet, deſcends 111 feet of the ſaid length in the laſt ſecond; required the altitude of the ſaid plane and the time of deſcent.

III. Question 584, *by Maſter* J. Paty, *at the Mathematical Academy*, Briſtol.

Suppoſing a cow to bring forth a ſhe calf at the age of two years, and then to continue yearly to do the ſame, and every one of her brood to bring forth a ſhe calf at the age of two years, and afterwards yearly likewiſe; how many may ſpring from the old cow and her brood in 40 years?

IV. Question 585, *by Mr.* Jer. Ainſworth.

Required to find a fraction ſuch, that being taken from its reciprocal the remainder ſhall be a ſquare.

V. Question 586, *by Miss* Ann Nicholls.

I made observation, last spring, at two places under the same meridian, differing in latitude 4°, and found the sun to rise exactly at 5 o'clock in the one and at 4 in the other, and that the difference of his meridian altitude at the said places, at the times of observation, was 2°; the latitude of each place and the days of observation are required.

VI. Question 587, *by Mr.* C. Smith.

A designer, having occasion to delineate in perspective a longest half of an ellipse coinciding with the horizontal plane, desires to know the true breadth of its projection, when projected on a perspective plane touching (both it and its vertical) the nearest end of the transverse diameter, and making an angle with the said diameter of 50°; the distance of the eye from the perspective plane being 100, the transverse diameter 32·92, and the semi-conjugate 11 feet.

VII. Question 588, *by Mr.* Jos. Dymond.

Given the right line bisecting the vertical angle of a plane triangle, and terminating in its base, and the perpendiculars falling thereon (produced) from the extremities of its base; to determine and construct the triangle.

VIII. Question 589, *by* Pamphagus.

Given the base, one of its adjacent angles, and the line bisecting the vertical angle of any plane triangle; to determine the said triangle.

IX. Question 590, *by* Nujo Dargnas.

To determine a point, from which if right lines be drawn to the three angular points of a given plane triangle, the sum of their squares shall equal a given space (aa), and if from the said point lines be drawn to the three sides of the triangle (produced if necessary) making given angles therewith, the rectangle contained under the line drawn to one of the sides and a given line (b) shall be equal to the sum of the rectangles under the other drawn lines and given lines (c, d).

X. Question 591, *by Mr.* Paul Sharp.

Required the area of the common parabola, whose abscissa is = 10, and radius of curvature, at the bounding ordinate,

ordinate, equal to three times a tangent drawn from thence and terminating in its axis produced.

XI. Question 592, by Mr. J. Chipchase.

There is a bason, in form of semi-globe, filled with spring water and placed on horizontal ground, and a person can see a piece of money laid at the bottom of it to the distance of nine feet from its axis, when quite full; but, when filled to only half the whole depth, he can see it no further than to the distance of 6 feet therefrom: Required the height of the person's eye and the diameter of the bason.

XII. Question 593, by Mr. J. Bennett.

To determine the nature of the curve, whose tangent terminated every where by it and an indefinite right line CD, is a constant quantity $= 100$; also, to determine the length of the part thereof intercepted between its highest point and that point whose height, above the said line CD, is $= 20$.

XIII. Question 594, by Mr. S. Vince.

To find the sum of the infinite series $\frac{1}{1.5.v^5} + \frac{1}{1.2.3.7.v^7}$ $+ \frac{1.3.3}{1.2.3.4.5.9.v^9} + \frac{1.3.3.5.5}{1.2.3.4.5.6.7.11.v^{11}} +$ &c.

XIV. Question 595, by Mr. T. Moss.

To draw a right line from the given point R, [*see the fig. to the solution*] in the base FA, produced, of the given triangle ACF, intersecting AC in n, and meeting CF in E; so that, drawing the right line AE, the triangle AnE may be the greatest possible.

XV. Question 596, by Mr. Cha. Hutton.

Let NO be the fixed and MP the semi-revolving axe of any spheroid [*see the fig. to the solution*] $AGBQA$ any section of it by a plane inclined to the axes, and NPO another section through its axes and perpendicular to the former section: It is required to find the figure of the section AGB and the solidity of the part cut off the spheroid by it, both in the case of the oblong and oblate spheroid, supposing the distances DM, ME, of the perpendiculars BD, AE, from the center M, to be 5 and 12, and the axes 50 and 30.

The PRIZE QUESTION, *by Mr.* Tho. Allen.

If a chain 40 feet long, consisting of exceedingly small links of equal density, be put over a pulley (void of friction) with the two ends *A* and *B* hanging down, to the respective distances of 21 and 19 feet from the pulley, and in these circumstances, suppose the chain to put itself in motion. It is required to determine in what time the end *A* will arrive at an horizontal plane 200 feet below the pulley.

1769.

Questions answered.

I. QUESTION 582 *answered by Mr.* Rd Gibbons.

IT is manifest (from the data) that x and y (and consequently xxy and xyy) are whole square numbers; whence $xxy + \frac{\sqrt{x}}{y}$ being $=$ (16386 $=$) $128^2 + 2$ and $xyy + \frac{x}{\sqrt{y}}$ $=$ (1056 $=$) $32^2 + 32$ (per quest.), $\frac{\sqrt{x}}{y}$ will be $= 2$, and $\frac{x}{\sqrt{y}}$ $= 32$, and consequently $x = 64$ and $y = 4$. Whence the age of this amiable fair one is 16 years, and her fortune is 640l.——All vastly pretty!

In this manner, nearly, it is also answered by Mess. *J. B. Ashton*, *T. Atkinson*, *T. Barker*, *J. Coulthred*, *Curiosus*, *H. Curtis*, *R. Dening*, *W. Dent*, *G. Lodge* (the proposer), Miss *Ann Nicholls*, *J. Nordon*, Master *Jer. Osborne*, *Pamphagus*, Master *J. Paty*, *E. Reed*, *W. Reynolds*, *Reub. Robbins*, *T. Robinson*, *Alex. Rowe*, *P. Sharp*, *R. Snowball*, *W. Spicer*, *W. Stoker*, *T. Walker*, and *J. Young*.

Mr. *Ja. Mills*, of *Brixworth*, answers it in the following manner.——To the proposer.

The age, kind sir, of your most charming fair,
Is just SIXTEEN——th' equations make appear;
Her fortune too, more charming to behold,
Exact SIX HUNDRED FORTY POUNDS in gold.

At

At Hymen's altar I with joy could ſtand,
And gladly take this maiden by the hand;
Haſte then, dear Lodge, and crown the nuptial day,
Fatal may prove one moment's ſhort delay!

II. QUESTION 583 *anſwered by Mr.* Paul Sharp.

By the laws of deſcending bodies and the diviſion of ratios, $\sqrt{400}-\sqrt{289} : \sqrt{400} :: 1'' : \sqrt{400} \div \overline{\sqrt{400}-\sqrt{289}}$ $= (\frac{20}{3} = 6\frac{2}{3}''$, the whole time of deſcent; and the diſtance that would be perpendicularly deſcended in the ſame time is $714\frac{22}{27}$ feet: Whence the altitude of the plane required is readily found $= (400^2 \div 714\frac{22}{27} =)$ $223\frac{161}{193}$ feet.

Much after the ſame manner the anſwer is alſo given by Meſſ. *J. Addiſon*, *T. Barker*, *J. Bartlett*, *J. Chipchaſe*, *J. Clough*, *W. Dent*, *J. Edwards*, *R. Gibbons*, Maſter *Jer. Oſborne*, *Pamphagus*, *E. Parnal*, Maſter *J. Paty*, *R. Reed*, *W. Reynolds*, *T. Robinſon*, *Alex. Rowe*, *R. Snowball*, *W. Spicer* (the propoſer), *W. Stoker*, *S. Vince*, and *J. Young*.

III. QUESTION 584 *anſwered by Mr.* W. Spicer.

From the nature of the queſtion, it appears that the increaſe in the firſt year will be 0, in the 2d year 1, in the 3d year 1, in the 4th year 2, in the 5th year 3, in the 6th year 5, and ſo on to 40 years or terms (each term being = the ſum of the two next preceding ones); whence, the two laſt terms are 39088169 and 63245986, and the ſum of them all (or the whole ſeries) $= 2 \times 63245986 + 39088169 - 1 =)$ 165580140, the increaſe required.

And thus it is alſo anſwered by Meſſ. *J. Brown Aſhton*, *T. Barker*, *H. Brown*, *Curioſus*, *R. Dening*, *W. Dent*, *J. Nordon*, Maſter *Jer. Oſborne*, *Pamphagus*, Maſter *J. Paty* (the propoſer), *E. Reed*, *Alex. Rowe*, *P. Sharp*, *R. Snowball*, *W. Stoker*, and *J. Young*, very nearly.

IV. QUESTION 585 *anſwered by Mr.* Tho. Barker, *of* Wiſſett.

Let $\frac{1}{x}$ be the required fraction; then will $x - \frac{1}{x} = aa$, a ſquare number (per queſt.), and conſequently $x = \frac{1}{2}\sqrt{a^4+4}$ $+ \frac{1}{2}aa$; whence it appears that if ſuch a fraction a be found

as that its biquadrate added to 4 may be a square number; the conditions of the question will be answered, &c.

The solutions by *Pamphagus* and Mess. *T. Robinson*, *Alex. Rowe*, *R. Snowball*, and *S. Vince*, are nearly the same.

V. QUESTION 586 *answered by Mr.* Wm. Gawith.

Since the difference in latitude of two places situate under the same meridian is universally equal to the difference of the sun's meridional altitudes at those places for the same day, it is manifest that during the interval of the two observations, the sun had changed his declination 2°. This being premised, let a and b represent the sines of the two given ascensional differences, c and d the tangents of 2° and 4° respectively, and x the tangent of the sun's declination at the first time of observation: Then (by Crakelt's translation of Mauduit's Trigonometry, p. 31) $\frac{x+c}{1-cx}$ will be the tangent of the declination at the second time of observation; and (by spherics) $\frac{a}{x}$ and $\frac{b-bcx}{c+x}$ the tangents of the less and greater latitudes respectively; the difference of which being 4° (per quest.), $\frac{a+dx}{x-da}$ will be $=\frac{b-bcx}{c+x}$, or $xx\times\overline{d+bc}+x\times\overline{a+cd-b-dabc}=-ca-dab$, and $x=$ tang. 7° 54′ 50″, corresponding to April 10th: Whence the other day of observation will be found to be April 15th; and the two latitudes 48° 48′ 19″ and 52° 48′ 19″.

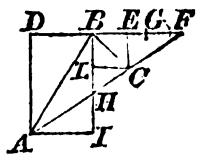
This question is also ingeniously and concisely answered by Mess. *T. Barker*, *J. Chipchase*, *W. Crakelt*, *E. Parnel*, *T. Robinson*, *W. Spicer*, *W. Stoker*, and *S. Vince*.

The direction of the visual line from the eye to the perspective plane not being given in proposing QUESTION 587, has prevented any of our ingenious correspondents answering it; but were it sufficiently limited, nothing more would be required than To find the breadth of the section of a given elliptical cone, made by a plane passing through the extremity of the greater diameter of its base in a given direction; which may be easily affected, as *Pamphagus* justly observes.

VII. QUES-

VII. QUESTION 588 *answered by Mr.* J. Chipchase.

CONSTRUC. On the right line DG take DB and BE respectively equal to the greater and the less given perpendiculars, and erect thereon, at B, the perp. $BH =$ the given bisecting line: Produce DG till $DF : EF :: DB : BE$ (*i. e.* in the ratio of the given perpendiculars), and through the points F, H let a right line be drawn, meeting EC, DA (perpendicular to DF) in C and A; join the points A, B and C, B, and ABC will be the triangle required.



DEMONSTRA. Draw AI and $CL \perp BH$ (produced) in I and L; then, DF being to $EF :: DA : EC$ (per sim. Δs) $:: DB : BE$ (per construc.) and the $\angle$s D and E equal (being both right), the Δs BDA, BEC will be similar (Euc. 6. 6.), and consequently, taking the equal $\angle$s DBA, EBC from the right $\angle$s DBH, EBH, their remainders, viz. the $\angle$s ABH and CBH will be equal; whence BH manifestly bisects the $\angle ABC$, and AI, CL are respectively $= DB$, BE (the given perpendiculars, per construction).

The method of calculation from this construction is exceedingly easy.

Neat and elegant constructions to this question have also been received from Mess. *J. Addison*, *T. Barker*, *W. Crakelt*, *H. Curtis*, *J. Dy ond* (the proposer), *T. Moss*, Master *Jer Osborne*, Master *J. Paty*, *T. Robinson*, *Alex. Rowe*, *R. Snowball*, *W. Spicer*, and *S. Vince*.

VIII. QUESTION 589 *answered by Master* Jer. Osborne, *at the Mathematical Academy*, Bristol.

Suppose the base given $= a$, the bisecting line $= b$, the sine and cosine of the given angle at the base $= s$ and c, and the sine of half the vertical angle $= x$ (rad. $= 1$). Then, (per trigonom.) the sine of the angle the line bisecting the vertical angle makes with the base will be $= s\sqrt{1 - xx} + cx$, and that of the vertical angle itself $= 2x\sqrt{1 - xx}$; also $\dfrac{bs\sqrt{1 - xx} + bcx}{s}$ will be found $=$ the side adjacent

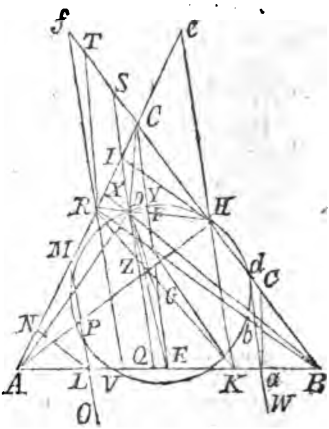
to

to the given angle, $\frac{bx}{s}$ = the fegment of the bafe (made by the faid bifecting line, and) adjacent to the fame, $\frac{sa}{2x\sqrt{1-xx}}$ = the other fide, and $\frac{as-bx}{s}$ = the other fegment. Whence (Euc. 3. 6.) $abss\sqrt{1-xx} + abscx - bbsx\sqrt{1-xx} - bbcxx = \frac{abss}{2\sqrt{1-xx}}$, or (when properly reduced) $x^6 \times 4b^2 - x^5 \times 8abs + x^4 \times \overline{4aass - 4bbss - 4bb} + x^3 \times \overline{4abs^3 + 8abs} + xx \times \overline{4bbss - 4aass} - 4abs^3x + a^2s^4 = 0$; from whence x may be determined.——Mafter *J. Paty*'s anfwer is not greatly different.

It is alfo anfwered by Meff. *J. Addifon*, *T. Barker*, *Pamphagus* (the propofer), *T. Robinfon*, *Alex. Rowe*, *R. Snowball*, *W. Spicer*, *S. Vince*, and *T. Walker*.

IX. QUEST. 590 *anfwered by* Nujo Dargnas (*the Propofer*).

CONSTRUC. Let ACB be the given triangle; the given lines b, c, d are referred to AB, AC, and BC refpectively. From any point L in AB drawn LM, LN making the angles ALM, ANL equal to the given angles to be made with the fides AB, AC refpectively; take $OL : LN :: c : b$ and $LP : PM :: OL : LM$, and draw AP meeting BC in H. In like manner from any point a in AB draw ad, ac making the angles Bad, Bca = the given angles to be made with the fides AB, BC refpectively: Take $Wa : ac :: d : b$ and $ab : bd :: Wa : ad$; join B, b, meeting AC in R, and join R, H. Again, bifect AB in E; join E, C and take $EG = \frac{1}{2}GC$, and defcribe a circle with a radius GY fuch that $3\,GY^2 = aa - AB \times AE - 2\,EC \times EG$, meeting RH (produced if neceffary) in D, the point required.



DEMONS.

DEMONS. From R and H draw RV, $HK \parallel DQ$ or ML; and RT, $HI \parallel DS$ or ac; and DX or LN respectively: Join D, E; D, G and R, K meeting DQ (produced if necessary) in Z, and draw $DF \perp CE$; produce, if necessary, HK, RV to meet AC, CB produced in e and f.——Then $AD^2 + BD^2 = AB \times AE + 2ED^2$ (Simp. Geom. II. 3.) $= AB \times AE + 2EF^2 + 2DF^2$, and adding to each $CD^2 = CF^2 + FD^2$, $CD^2 + AD^2 + BD^2 = AB \times AE + 2EF^2 + CF^2 + 3FD^2$. But $CF^2 + 2EF^2 = CD^2 - 3DF^2 + 2ED^2 = DG^2 + CG^2 - 2CG \times GF$ (CD^2, Euc. 13. 2.) $+ 2DG^2 + 2EG^2 + 2CG$ $(4EG) \times GF$ ($2ED^2$ (Euc. 12. 2.) $- 3DF^2 = 3FG^2$ $(3DG^2 - 3DF^2)$ $+ 2EC \times EG$ $(CG^2 + 2EG^2)$; whence $AD^2 + BD^2 + CD^2 = AB \times AE + 3FG^2 + 2EC \times EG + 3FD^2 = AB \times AE + 2EC \times EG + 3GT^2$ $(3DG^2) = aa$, the given space (per construc.). Again, $KH : He :: LP : PM :: OL : LM$, and $IH : He :: LN : LM$; whence, by equality, $KH : IH :: OL : LN :: c : b$ (per construction), and consequently $KH \times b = IH \times c$. Also, $VR : Rf :: ab : bd :: Wa : ad$, and $RT : Rf :: ac : ad$; by equality, $VR : RT :: Wa : ac :: d : b$ (per construction); whence $VR \times b = RT \times d$. Again, $DZ \times b : KH \times b :: ZR : RK :: DR : RH :: DX \times c : IH \times c$: But $KH \times b = IH \times c$; whence $DZ \times b = DX \times c$. Also, $QZ \times b :: RV \times b :: KZ : RK :: DH : RH :: DS \times d : RT \times d$: But $RV \times b = RT \times d$; whence $QZ \times b = DS \times d$, and therefore $DS \times d + DX \times c = (QZ \times b + DZ \times b =) DQ \times b$. Q.E.D.

If the point D (required) falls without the $\triangle ACB$, the preceding conclusion will become $DS \times d - Dx \times c = DQ \times b$, or $DX \times c - DS \times d = DQ \times b$, according as the said point is situate on the left or right hand side of the line EC; the demonstration whereof is exactly the same as above.——*Pamphagus* has solved this question algebraically.

X. QUESTION 591 *answered by Mr.* J. Addison.

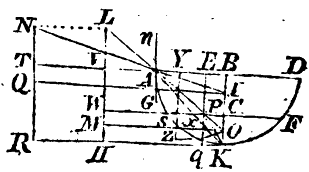
Put the given abscissa $(= 10) = a$, and the corresponding ordinate $= y$; then will $2a$ be the subtangent, $\sqrt{4aa + yy}$ the tangent and $\frac{yy}{a}$ the latus-rectum of the parabola required, and (per Simp. Flux. pa. 75, 2d edit.) $\frac{\overline{4aa + yy}|^{\frac{3}{2}}}{2ay}$ = the radius of curvature at its bounding ordinate; whence

whence $\frac{\overline{4aa+yy}|^{\frac{3}{2}}}{2ay} + y = 3\sqrt{4aa+yy}$ (per queſtion); ſolved, $y = 9.5$, and the area required $= 126.66$, &c.

And according to this method, nearly, the anſwer is alſo given by Meſſ. *T. Barker*, *Pamphagus*, *T. Robinſon*, *Alex. Rowe*, and *S. Vince*; but a preſs error having unluckily happened in propoſing this queſtion, viz. *and* inſtead of *at* before "the bounding ordinate, &c." it is become neither ſo neat nor ſimple as was intended; for, according to the propoſer's ſubſtitution and meaning, a is $=$ the given abſciſſa, $4x =$ the parameter, $2\sqrt{aa+ax}$ the tangent $\frac{2a+2x}{x} \times \sqrt{ax+xx}$ the radius of curvature at the ordinate, and $\frac{2a+2x}{x} \times \sqrt{ax+xx} = 3 \times 2\sqrt{aa+ax}$, or $a+x = 3\sqrt{ax}$; whence $x = 1.459$, the parameter $= 5.836$, the bounding ordinate $= 15.27874$, and the area $= 101.85826$.

XI. QUEST. 592 *anſwered by the Rev. Mr.* C. Wildbore.

Let ABK repreſent half the baſon, and join A, K; then ſhall the $\angle GAK = BAK$ $(45^\circ) =$ the angle of refraction at the 1ſt ſtation: Make $AI = \frac{3}{4}AK$, ſo ſhall GAI $=$ the correſponding angle of incidence, their ſines being to each other as $3:4$; whence $BI = \frac{1}{2\sqrt{2}}AB = .35355333\,AB$.



Now, let CS be the ſurface of the water, $BY = \frac{\sqrt{7}}{4} \times AB$, $xz = \frac{3}{8}AB$, and from the center Y, on the line Cx, and with the line (or diſtance) xz, deſcribe the conchoid of Nicomedes zO, and from the point O, where it cuts BK, draw OL through A; join K, P; ſo ſhall OPq be the angle of incidence and KPq that of refraction, at the 2d ſtation. For the common equation of the conchoid is $\frac{CO \times BY}{BO}$ $(= \frac{\sqrt{7}}{4}CP) = \sqrt{xz^2 - CO^2}$, or $28CP^2 = 9AB^2 - 64GO^2$,

or

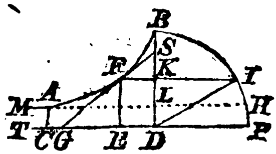
or $64OP^2$ ($= 64CP^2 + 64CO^2$) $= 9AB^2 + 36CP^2 = 36CK^2 + 36CP^2 = 36KP^2$, or $8OP = 6KP$: Whence, as $8:6$, or $4:3 :: KP:OP$, and consequently the sines of the angles OPq, KPq are in the same ratio, and must be the required angles.——The equation of the conchoid gives $CO = \cdot 287087953 \times AB$. Also, by sim. Δs, $1 : \frac{1}{2} + \cdot 287087953 :: 6\ (OM) : LM = 4\cdot 7225274$, and $AB : BI$, or $1 : \frac{1}{2\sqrt{2}} :: QI\ (9) : NQ = 3\cdot 1819800$; whence $LM - NQ = 1\cdot 5405474 = IO = \overline{\cdot 287087953 + \frac{1}{2} - \frac{1}{2\sqrt{2}}} \times AB$, and consequently $AB = 1\cdot 553456$ (the radius of the bason), and $IO - CO = IC = \cdot 5203914$; which give $NQ + IC + CK = 5\cdot 4790694$ feet, the height of the spectator's eye.

Mess. *J. Addison*, of *Hexham*, *J. Chipchase* (the proposer), and *W. Crakelt* have likewise answered this question very ingeniously.

Mr. *T. Barker*, of *Wissett*, and Mr. *S. Vince*, of *Harleston*, have also sent answers to it; which are all the solutions we have received.

XII. QUESTION 593 *answered by* Mr. T. Allen.

Let the abscissa $CE = x$, ordinate $EF = y$, curve $AF = z$; and let $GF = DB = 100 = a$, $AC = 20 = c$. Then will the subtangent $GE = \frac{y\dot{x}}{\dot{y}}$, and $\frac{y^2\dot{x}^2}{\dot{y}^2} + yy = aa$; whence



$\sqrt{\dot{x}^2 + \dot{y}^2}\ (= \dot{z}) = \frac{a\dot{y}}{y}$, whose fluent, corrected, gives $z = a \times$ hyp. log. $\frac{y}{c}$; which when $y = a = 100$, becomes $= 100 \times$ hyp. log. of $5 = 160\cdot 94379$, the length of the curve AFB.——From the equation $\frac{y^2\dot{x}^2}{\dot{y}^2} + yy = aa$, we also have $\dot{x} = \sqrt{aa - yy} \times \frac{\dot{y}}{y} = \frac{aa\dot{y}}{y\sqrt{aa - yy}} - \frac{y\dot{y}}{\sqrt{aa - yy}}$; where

where taking the fluents, properly correcting them, and putting $d = \sqrt{aa-cc}$, will be had $x = \frac{a}{2} \times$ hyp. log. of $\overline{\frac{a-\sqrt{aa-yy}}{a+\sqrt{aa-yy}} \times \frac{a+d}{a-d}} + \sqrt{aa-yy} - d$, the equation of the curve required.

COROLLARY. Upon the center D, with the radius DB, describe the circular arc BIH. Make $DL = AC$, and draw $LH \perp BD$. Then will the area $LBIHL =$ the area of the whole curve $AFBDCA$. For, putting $DK = y$, $KI = \sqrt{aa-yy}$, and the flux. of $LHIK$ will be $\sqrt{aa-yy} \times \dot{y} = y\dot{x}$, from what is given above; therefore, &c.

Mr. *W. Crakelt*'s and Mr. *S. Vince*'s solutions differ very little from the above.

Answer to the same by the Rev. Mr. Cha. Wildbore.

Let TD be the indefinite right line, MFB the required curve, and GF the given tangent at F; then at B the vertex (from the nature of the curve) $BD = FG = 100 = a$. Produce GF till it meets BD in S, and let fall $FK \perp BD$; then, putting $BK = x$ and $FK = y$, SK will be $= \frac{y\dot{x}}{\dot{y}}$, and $SK : FS :: FE\ (KD) : GF\ (a)$: Whence $\dot{y} = \frac{\dot{x}\sqrt{2ax-xx}}{a-x}$ $\left(= \frac{u^2\dot{u}}{aa-uu} = \frac{aa\dot{u}}{aa-uu} - \dot{u}\right.$, by putting $GE = \sqrt{2ax-xx} = u$); and, restoring x, $y = \frac{1}{2}a \times$ hyp. log. of $\frac{a+\sqrt{2ax-xx}}{a-\sqrt{2ax-xx}} - \sqrt{2ax-xx} = 131{\cdot}263155$ when $FE = 20$. Now, let $z =$ the curve FB; then $\dot{z} = \left(\sqrt{\dot{x}^2+\dot{y}^2} =\right) \frac{a\dot{x}}{a-x}$, and z correct $= a \times$ hyp. log. $\frac{a}{a-x} = 160{\cdot}94379$, as required. ——Whence these consectaries.

1. Because, when x is taken $= a$, the expressions for y and z become infinite, therefore the curve will continually approach nearer to the line TD, but can never meet with it.

2. A.

2. A value which this infinite area can never exceed, may be found from the above expression for $\dot{y}$, thus; TE's flux. is $= -\dot{y}$, which multiplied by FE ($= KD = a - x$) gives $-\dot{x}\sqrt{2ax - xx}$ for the flux. of the infinite area $ETMF$. With GF (DB) radius describe the quadrant BIP, and produce FK to meet it in I; then the area of the circular segment $DKIP$ will be the fluent of this expression, or $=$ the area $ETMF$; and, when FE becomes $= BD$, the infinite area $MFBDT =$ the whole quadrant BDP; which is also known from other principles, by considering the invariable line GF, as moving from an horizontal position to a vertical one.

3. Because $y\dot{x}$ is the fluxion of the area FBK, twice the area of the quadrant BDP will be $=$ the fluent of $\frac{1}{2}a\dot{x} \times$ hyp. log. of $\frac{a + \sqrt{2ax - xx}}{a - \sqrt{2ax - xx}}$, when $x = a$; a conclusion not very easy to be derived another way.

This question is also ingeniously answered by Mess. *T. Barker*, *J. Bennett* (the proposer), *J. Chipchase*, and *T. Todd*.

XIII. QUEST. 594 *answered by the Rev. Mr.* C. Wildbore.

In the given series substitute x for $\frac{1}{v}$ and take the fluxion; which will give $x^3\dot{x} \times : x + \frac{x^3}{1.2.3} + \frac{1.3.3.x^5}{1.2.3.4.5} + \frac{1.3.3.5.5.x^7}{1.2.3.4.5.6.7}$ + &c. $= x^3\dot{x}z$ (z being the arc of the circle whose sine is x $(\frac{1}{v})$ and rad. 1): Whence, taking the fluents and restoring $\frac{1}{v}$, $\frac{1}{1.5.v^5} + \frac{1}{1.2.3.7.v^7} + \frac{1.3.3}{1.2.3.4.5.9.v^9} + \frac{1.3.3.5.5}{1.2.3.4.5.6.7.11.v^{11}}$ + &c. $= \frac{8z + 2 + 3vv \times \sqrt{vv-1} - 3zv^4}{32v^4}$, the sum of the series required.

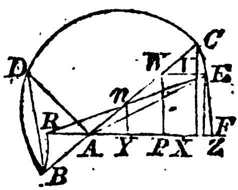
COROLLARY. This method of solution may be extended to any series of the form $\frac{Ax^n}{n} + \frac{Bx^{n+m}}{m+n} + \frac{Cx^{n+2m}}{2m+n} +$ &c. by means of artifices similar to those made use of in Simpson's Fluxions, part 2d, sec. 3d and 6th, where $A + B + C +$ &c. expresses any known series; and whereof the

fummation of the extremely difficult feries $x + \frac{x^2}{4} + \frac{x^3}{9} + \frac{x^4}{16} +$ &c. propofed by the very fagacious Mr. *L*—— in the Ladies' Diary for 1760, is given as an example; but, for want of room, we are obliged to omit it.

Ingenious folutions to this queftion have alfo been received from Meff. *T. Allen*, *T. Barker*, *W. Crakelt*, *C. Hutton*, and the propofer *S. Vince*.

XIV. QUESTION 595 *anfwered by Mr.* Cha. Hutton.

CONSTRUCTION. Parallel to CF draw RB, meeting CA, produced, in B; $\perp CA$ draw AD, meeting the circ. on CB in D; make $Bn = BD$, and n will be the point through which RnE muft pafs.



DEMONS. Draw $EW \parallel$, and EZ, WP, $nY \perp AF$.——Then, by conftruction, $BA \times AC = (DA^2 = DB^2 - BA^2 = Bn^2 - BA^2 = \overline{Bn + BA} \times \overline{Bn - BA} =) \overline{nB + BA} \times An$, or $An : AB :: AC : nB + BA :: nC (AC - An) : nB (\overline{nB + BA} - AB) ::$ (by fim. Δs) $CE : BR$, or $An : CE :: AB : BR ::$ (by fim. Δs) $WC : CE$; whence $An = WC$.

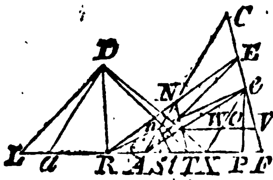
Again, by fim. Δs, $WC (An) : Cn (AW) :: nY : ZE (WP)$, or $WC : CE :: Cn \times nY : CE \times EZ$. But when RnE moves about the center R, and cuts the lines AC, CF, it is known that the flux. of EC : the flux. of $Cn :: CE \times ER : Cn \times nR ::$ (by fim. Δs) $CE \times EZ : Cn \times nY$; whence the flux. of CE : flux. of $Cn :: EC : CW$ (by equality of ratios), and confequently the flux. of EZ = the flux. of nY; in which cafe, it is well known, that their diff. $(EZ - nY)$ is a max. But this difference drawn into $\frac{1}{2}$ the given line AR is = the area of the ΔAnE; and hence the faid triangle is a maximum.

COROLLARY. It appears from the demonftration that $An = WC$; whence $nY = CV$, and $nY + EZ =$ the whole perpendicular CX.

CON-

CONSTRUCTION *to the same by the Rev. Mr.* **C. Wildbore.**

At the given point R erect the perpend. DR (p. 13 Euc. 6) a mean proportional to RA, RF, and take LR and $RT = DR$: Draw $NT \parallel CF$, and through N draw RE; join A, E, and the thing is done.



For, let Re be any other position of the required line, intersecting AC in n. Draw $nt \parallel CF$, eX to CA, nV to RF, and let fall the perpendiculars nS, eP: Then, the Δs RnA, ReA, on the common base AR, are in the ratio of their perpendiculars nS, eP; consequently the $\Delta\, Ane$ (their diff.) will be as $eP - nS = eO$, which must therefore be a maximum. But the $\Delta\, VeW$ being similar to the given one ACF, the ratio of eO to WV is given; therefore $WV = nV\,(tF) - nW\,(AX) = RF - Rt - RX + RA$ (per fig.) must be a maximum: But $RF + RA$ is invariable; therefore $Rt + RX$ must be a minimum. By sim. Δs, $RA : Rt :: RX : RF$; whence DR must be likewise a mean proportional to Rt, RX. Take $Ra = Rt$, and join D, a; then, the $\angle$s aDL, TDX being equal, DT greater than Da, and the $\angle DTX$ greater than DaL, TX is evidently greater than aL, and consequently aX than LT; wherefore $Rt + RX$ is a minimum when each of them becomes equal to RT, as per construction. *Q.E.D.*

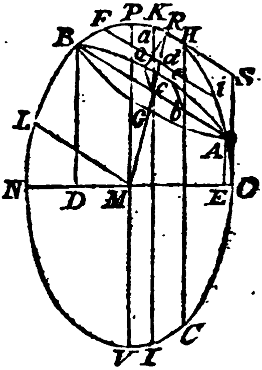
COROLLARY. If $Rt + RX$ instead of a minimum, or the $\Delta\, Ane$ instead of a maximum, had been supposed $=$ a given quantity, from what is shewn above the prob. is reduced to this, viz. In any right-angled $\Delta\, aDX$, are given the perpendicular DR and the base aX, to construct the triangle; the method of doing which is well known: And the construction will be equally easy, when, instead thereof, the ratio of the $\Delta\, RnA$ to the $\Delta\, ReA$ or of nS to eP or of An to eF is given; for, in all these cases, the ratio of $aR : RX$ is given.

We have also received a very curious geometrical construction to this question from Mr. *Moss*, the proposer, which we are extremely sorry our narrow limits will not permit us, this year, to insert.

Mess. *T. Allen*, *T. Barker*, *W. Crakelt*, *Steph. Ogle*, *T. Robinson*, and *S. Vince* have given simple, easy, and concise fluxionary solutions to it.

XV. QUESTION 596 *answered by* Mr. W. Crakelt.

If IK, CH be two ordinates to the fixed axis ON; MR a semidiameter to the ordinate BA, and OS, RS two tangents at the points O, R. Then (by Deidier's Géométrie, p. 401) will be had $Bc \times cA : Ic \times cK :: RS^2 : OS^2 :: Bb \times bA : Cb \times bH$: But (per prop. of the circle) $Ic \times cK = cQ^2$, and $Cb \times bH = be^2$; consequently $Bc \times cA : Bb \times bA :: cQ^2 : be^2$, the property of the ellipse.



Having thus discovered the figure of the section $AGBQ$, let 3·14159 &c. be put $= p$, the semi-fixed axe $NM = m$, the semi-revolving axe $PM = n$, the semi-diameter $MR = r$, its semi-conjugate $LM = s$, the semi-ordinate $Bc = t$, its corresponding abscissa $Rc = u$ (all which may, it is evident, be looked upon as given quantities), and any variable part Rd of $Rc = x$. Then (per prop. of the ellipse) rr (MR^2) : ss (LM^2) :: $2rx - xx$ ($\overline{2MR - dR} \times dR$) : $\frac{ss}{rr} \times \overline{2rx - xx} = dF^2$, and thence $dF = \frac{s}{r}\sqrt{2rx - xx}$. Also (by trig.) rad. (1) : sin. $\angle Rda$ $(= \frac{mn}{rs})$ :: x (Rd) : $Ra = \frac{mnx}{rs}$ (Ra being perpendicular to Fv); whence the fluxion of the solidity (as the semi-conjugate of the section made by the plane Fv is $= \frac{n}{r}\sqrt{2rx - xx}$) will be $\frac{pmnn}{r^3} \times \overline{2rx\dot{x} - x^2\dot{x}}$: the fluent whereof $(\frac{pmnn}{r^3} \times \overline{rxx - \frac{1}{3}x^3})$, when $x = u$, gives $\frac{pmnn}{r^3} \times \overline{ru^2 - \frac{1}{3}u^3}$ for the solidity required, equal in the oblong spheroid (when $m = 25$, $n = 15$, $r = 15·0028901$, and $u = ·6418766$) 31·88508, and equal in the oblate spheroid (when $m = 15$, $n = 25$, $r = 24·9938494$, and $u = 5·3957569$) 1272·97097 &c.

COR. When $x = 2r$, the solid BRA becomes $= \frac{4pmnn}{3}$ = the whole spheroid.

The

The solution by Mr. *Cha. Hutton* (the proposer) is exceedingly elegant and concise; who, after his demonstration of the figure of the section, refers to his Treatise on Mensuration, now publishing in monthly numbers, for other properties of sections, &c.

Ingenious answers to this question have likewise been received from Mess. *T. Allen*, *T. Barker*, *T. Todd*, *S. Vince*, and *C. Wildbore*.

The PRIZE QUESTION *answered by Mr.* Tho. Allen (*the Proposer*).

Let $c = 40$ feet, the whole length of the chain, $a = 21$, $x =$ any variable part passed over the pulley in the time t, $v =$ the velocity of the chain at the end of that time, and $s = 32\frac{1}{6}$ feet.——Then will $a + x$ be as the gravity of the descending, and $c - a - x$ that of the ascending part; therefore $a + x - \overline{c - a - x} = d + 2x$ (by putting $d = 2a - c$) will be as the force acting on the chain, and $c : s :: d + 2x : s \times \frac{d + 2x}{c} =$ the velocity that would be generated in $1''$, by that force. Therefore $1''$ (time) : $s \times \frac{d+2x}{c}$ (velocity) :: $\frac{\dot{x}}{v}(\dot{t}) : \dot{v}$, the velocity generated in the time $\dot{t}$; $\therefore s \times \frac{d\dot{x} + 2x\dot{x}}{c} = v\dot{v}$; whence, taking the fluents, &c. $v = \sqrt{\frac{2s \times \overline{dx + xx}}{c}}$. Therefore $\dot{t} = \sqrt{\frac{c}{2s}} \times \frac{x^{-\frac{1}{2}}\dot{x}}{\sqrt{d + x}}$, whose corrected fluent gives $t = \sqrt{\frac{2c}{s}} \times$ hyp. log. of $\frac{\sqrt{x} + \sqrt{d + x}}{\sqrt{d}}$; which, when $x = 19$ (or the chain quits the pulley) will be $= 2{\cdot}908137$ seconds.

To find the remaining part of the time, put $p = 25{\cdot}33$ (the value of v $(= \sqrt{\frac{2s \times \overline{dx + xx}}{c}})$ when $x = 19$), $z =$ the space descended, after quitting the pulley, in the time T, and $V =$ the velocity at the end of that time. Then, if the chain descended from rest, V^2 would be $= 2sz$; therefore, in the present case, $V^2 = 2sz + pp$: $\therefore V = \sqrt{pp + 2sz}$, and

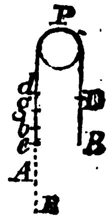
and $\dot{T}\left(\frac{\dot{z}}{V}\right)=\frac{\dot{z}}{\sqrt{pp+2sz}}$; whose corrected fluent gives $T=\frac{\sqrt{pp+2sz}-p}{s}$; which, when $z=160$, becomes $2{\cdot}46342''$. And hence the whole time required will be $= 5''\ 22{\cdot}3'''$, very nearly.

The same answered by Mr. W. Crakelt.

If 40 feet (the length of the whole chain) $=l$, 2 feet (the diff. of the two ends A and B hanging down from the pulley) at first) $=d$, $32\frac{1}{6}$ feet (the velocity generated at the earth's surface in $1''$, by gravity) $=s$, $x=$ a small part of the chain ascended or descended in any variable time t, and $v=$ the corresponding velocity of the point A at the end of that time (in seconds). Then, let the weight of the chain be what it may, $d+2x$ will represent the motive force acting thereon; and therefore (since $l:d+2x::s:\frac{s}{l}\times\overline{d+2x}$, and $1''$ $:\frac{\dot{x}}{v}(=\dot{t})::\frac{s}{l}\times\overline{d+2x}:\frac{s\dot{x}}{l}\times\frac{d+2x}{v}=\dot{v})$ $v\dot{v}=\frac{s}{l}\times\overline{d+2x}\times\dot{x}$; or, by taking the fluents, &c. $v=\sqrt{\frac{2s}{l}\times\overline{dx+xx}}$ $=$ (when $x=19$) $25{\cdot}3322916$ &c. and consequently $\dot{t}\left(=\frac{\dot{x}}{v}\right)$ $=\overline{\frac{2s}{l}\times\overline{dx+xx}}^{-\frac{1}{2}}\times\dot{x}$; and, by taking the correct fluents, $t=\sqrt{\frac{l}{2s}}\times$ hyp. log. of $\frac{\frac{1}{2}d+x+\sqrt{dx+xx}}{\frac{1}{2}d}=$ (when $x=19$) $2{\cdot}90825683$ &c. seconds, the time elapsed when the chain quits the pulley. Now, the height from which a heavy body must fall freely from rest, to acquire the foregoing velocity, will (by Simpson's Select Exer. pa. 184) be $=\frac{vv}{2s}=9{\cdot}975$ feet, and the corresponding time $=\frac{v}{s}=\cdot78753238''$: Then $200-40+9{\cdot}975=169{\cdot}975$ feet, and the time in which a heavy body would descend through that dist. $=3{\cdot}25090661''$; whence $2{\cdot}90825683''+3{\cdot}25090661''-0{\cdot}78753238''=5{\cdot}37163106''$, the time in which the end A will reach the horizontal distance given; which was required.

Answer

Answer to the same by the Rev. Mr. Cha. Wildbore.



Let APB, at its commencement of motion, and EPD when the end A has descended to E, be positions of the chain; then it is manifest that the motive force at E will be as $PE - PD = dE = Ab + 2AE$, which multiplied into $\dot{AE}$, is known to be as half the fluxion of the square of the velocity at E; wherefore that velocity will be as $\sqrt{2} \times \sqrt{Ab \times AE + AE^2}$, and $\dot{AE}$ divided by this will be as the fluxion of the time, and the time itself (bisecting Ab in e, and taking eg $(=\sqrt{Ab \times AE + AE^2})$ a mean proportional to AD and AE) will be as $\frac{1}{\sqrt{2}} \times$ hyp. log. of $\frac{gE}{eb}$: But the real motive force at E, measured by the distance that might be uniformly gone over in 1″ with the velocity at E, is to dE, as the force of gravity to $AP + PB$, or as $32\frac{1}{6} : 40$, or $193 : 240$; therefore the velocity of the end A per second, when B arrives at P, $= \sqrt{\frac{193 \times 399}{120}}$ $= 25{\cdot}332287$ feet, and the time $= 3{\cdot}6882542 \sqrt{\frac{120}{193}} =$ $2{\cdot}9082566''$; and hence, by the laws of descending bodies, only, (without the help of fluxions) the remaining part of the time is easily found $= 2{\cdot}463375''$, and consequently the whole time of descent required $= 5{\cdot}371631''$.

COROLLARY. If B be supposed to ascend along an inclined plane; then, the sine of the plane's inclination to the horizon, to rad. 1, being called s, the motive force at E will be as $PE - s \times PD = PA + AE - s \times PB + s \times AE$ $= \overline{1+s} \times \overline{\frac{PA}{1+s} - \frac{s \times PB}{1+s} + AE}$; whence it is manifest that the velocity and time may be found with the same ease in this case, as before. Or, if A descends, or A and B descend and ascend along two differently inclined planes, the method of solution will be still the same.

Ingenious and neat solutions to this question have also been received from Mess. *T. Barker, J. Bennett, R. Butler, J. Chipchase, J. Clough, J. Dymond, P. George, R. Holden, C. Hutton, S. Ogle, Alex. Rowe, T. Sanderson, W. Sewell,*

E.

E. Smith, *Stannington[iensis]*, *T. Todd*, *S. Vince*, and *W. Wales*; but the two prizes of 12 and 8 Diaries for the solution thereof, are fallen to the respective lots of Mess. *Wm. Crakelt* and *J. Chipchase*.

Eclipses calculated for 1769.

Five eclipses are expected this year; three of the sun and two of the moon, as follow.

The first is of the sun, on Sunday the 8th of January, in the morning, invisible at London.

The second is a partial and visible solar eclipse, on Sunday the 4th of June, in the morning, according to the following calculations.

Calculated by	Beg.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m.	h.	m.	h.	m.	°	'
Mr. R. Robbins, for Greenwich	6	$38\frac{3}{4}$	7	$33\frac{2}{3}$	8	$23\frac{1}{2}$			6	$9\frac{3}{4}$
Mr. W. Chapman, for Foxton	6	$35\frac{1}{2}$	7	$27\frac{1}{4}$	8	$22\frac{5}{6}$	1	$47\frac{2}{3}$	6	46
Mr. J. Edwards, for Cambridge	6	$40\frac{1}{2}$	7	$36\frac{1}{2}$	8	$28\frac{1}{4}$	1	$47\frac{3}{4}$	6	18
Mr. J. Coates, for London	6	42	7	$32\frac{2}{3}$	8	$23\frac{5}{6}$	1	$41\frac{5}{6}$	6	5
Mr. J. Metcalfe, for London	6	38	7	30	8	25			5	58

This eclipse will be very formidable in the north-eastern parts of North America, Greenland, Hyperborean ocean, and beyond the pole; for the center of the lunar shade passes in its traverse, very little to the east of it.

The third eclipse is a lunar one, on Monday the 19th of June, about 8 o'clock in the morning, and therefore invisible to those parts of the world.—It may be seen in South America and the unknown lands and seas near the south pole. It will be total above an hour, and the whole duration above 3 and $\frac{1}{4}$.

The fourth is of the sun, on Tuesday the 28th of November, near 8 of the clock in the morning; but because of the moon's great southern latitude, increased by its vertical parallax, it cannot be seen in these parts of the globe.—It will be seen a partial eclipse in the unexplored Indian south seas, and an annular and central one in the terra incognita, near the south pole.

The fifth, and last, this year, is of the moon, and will happen on Wednesday the 13th of December, in the morning, and be visible in Great Britain and Ireland, from the beginning

beginning to the end, according to the following computations.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° ′
Mr. W. Chapman, for Foxton	4 53$\frac{5}{6}$	6 14$\frac{5}{6}$	7 35$\frac{5}{6}$	2 42	8 0
Mr. J. Edwards, for Cambridge	5 6	6 25$\frac{1}{4}$	7 51	2 45	8 30
Mr. J. Coates, for London	4 58	6 22$\frac{1}{12}$	7 46$\frac{1}{2}$	2 48$\frac{3}{4}$	8 58$\frac{3}{4}$
Mr. J. Metcalfe, for London	4 53$\frac{1}{2}$	6 18$\frac{1}{10}$	7 42$\frac{2}{3}$	2 49$\frac{1}{5}$	9 2$\frac{1}{2}$

The moon at the beginning of this eclipse will be vertical to the West Indian sea near Cuba, and at the end to the Pacific ocean, near the south coast of California. This eclipse therefore will be visible to the western parts of Europe and Africa, the Atlantic and Pacific oceans, and to all parts of America.

This year both the inferior planets Venus and Mercury will transit the sun's disk, but only part of the first can be seen here.—The times that these unusual phænomena happen are as follow.

Beg. Tran. h. m.	Cent. Ingress h. m.	Total Immer h. m.	Mid. Transit h. m.	Cent. Egress h. m.	End of Transit h. m.	Near ap. of Cent. ′ ″	Diam. of ⊙ & ♀ ′ ″
	7 27$\frac{5}{6}$		10 35$\frac{3}{4}$	13 44$\frac{1}{2}$			⊙ 31 31
7 13$\frac{2}{3}$	7 25$\frac{2}{3}$	7 37$\frac{5}{6}$	10 34$\frac{3}{4}$	1 43	1 55$\frac{5}{6}$	9 51	♀ 0 58
7 3$\frac{2}{3}$	7 14$\frac{1}{4}$	7 26$\frac{2}{3}$	10 21$\frac{1}{3}$	1 27$\frac{3}{4}$	1 39$\frac{1}{10}$		

On Thursday the 9th of November, Mercury will pass over the sun at night, and therefore will be invisible in Europe.

JOHN METCALFE.

*** The above calculation of the transit of Venus is as follows, viz.

The 1st. By Mr. William Chapman, for London.
2d. By Mr. J. Coates, for London.
3d. By Mr. J. Metcalfe, for London.

New Questions.

I. QUESTION 597, *by Mr.* Tho. Sadler.

Dear ladies, you with ease may find*
A matchless hero's name,
Who was beloved by mankind,
And mounted up to fame:

To

To serve his country boldly dar'd
Hot sulphur, smoke, and fire,
And long campaigns fatigue he shar'd,
To conquer proud Monsieur.

* viz. From the equations $\left\{\begin{array}{ll} w+x+y+z & =52 \\ wx+yz & =360 \\ wz+xy & =280 \\ wy+xz & =315 \end{array}\right\}$: Where w, x, y, and z denote the places of the letters in the alphabet composing the gentleman's name.

II. QUESTION 598, *by Master* J. Paty, *at the Mathematical Academy at* Bristol.

A gentleman erecting a house, whose breadth was 28 feet and back wall 6 feet 9 inches higher than the front, had by him a sufficient quantity of rafters for the front, each 24 feet long, which, to save timber, he was unwilling to cut, and therefore orders his builder to make the back rafters of such a length as will make the declivity of them and front alike, for uniformity; but being at a loss, is desirous of having it proposed in the Diary, that he may know how to proceed.

III. QUESTION 599, *by Mr.* Wm. Spicer.

Suppose *A* lends *B* 2000 l. at 5 l. per cent. per ann. simple interest, which *B* is to pay again in the following manner, viz. 1 l. immediately down at the end of the 1st year, 2 l. at the end of the 2d year, 3 l. at the end of the 3d year, and so on, increasing 1 l. every year; required at what time *B*'s debt will be the greatest, and also in what time *A* will be indebted to *B* the sum of 85 l. 15 s.

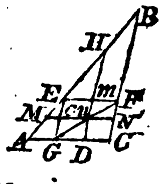
IV. QUESTION 600, *by Mr.* Paul Sharp.

A gentleman having a certain number of guineas and moidores, was asked how many he had; to which he replied, that the square root of the guineas multiplied by the cube root of the moidores when the product is the greatest possible, will be $= 39{\cdot}1918$; required how many he had of each sort?

V. QUES-

V. QUESTION 601, *by Mr.* T. Moss.

If one side AB of any plane triangle ABC, be divided into two parts, and from the point of division (E) two right lines be drawn parallel to, and terminating at the other sides, and the said points of termination be joined by a right line, and there be likewise drawn another right line, as DH or MN, parallel to either of the two aforesaid parallel lines, so as to intersect the other line, and terminate in the sides of the triangle; then the two extreme parts Hm, Dn or Nn, Mc of the three parts into which the line, so drawn, is divided, will always be in the ratio of the two parts BE, AE of the line AB first divided; required the demonstration?



VI. QUESTION 602, *by Mr.* J. Edwards, *of Magdalen College*, Cambridge.

A gentleman has a right-angled triangular garden, at the right angle of which grows a tree 40 feet high, whose shadow, I observed, on the 21st of June, at 2 h. 30 m. P. M. terminated in the hypothenuse of the said triangle in a perpendicular direction, and measuring the garden, I found that the difference of both its sides from the hypothenuse was 15 and 30 yards: Quere the latitude of the place, and area of the garden?

VII. QUESTION 603, *by Mr.* Wm. Gawith.

In a right-angled plane triangle ABC, suppose the two legs AB and $BC = x^5$ and y^5, the hypothenuse $AC = \overline{xx + yy}|^{\frac{3}{2}} \times axy$, and the perpendicular BD upon the hypothenuse $= x^3$; to determine the triangle, by quadratics.

VIII. QUESTION 604, *by Mr.* J. Dymond.

Given the line bisecting the vertical angle of a plane triangle and terminating in the base, the perpendicular falling thereon from one of the angles at the base, and the other angle at the base; to construct the triangle.

IX. QUESTION 605, *by Mr.* J. Turner.

In any ellipsis the transverse axis is the greatest of all the diameters, and the conjugate axis the least; and, of any

other diameters, that which is nearest to the transverse axis is greater than those which are farthest from it. The demonstration hereof is required, geometrically, from the consideration of the solid.

X. QUESTION 606, *by Monf.* Fermat.

Let ANB be a semicircle described upon AB as a diameter, [*see the fig. to the solution*] and AD, BC two perpendiculars, each equal to AN, the chord of 90°. Also, let any point E be assumed in the semi-circumference, and two lines ED, EC be drawn from thence to the points D, C, intersecting the diameter in two points O, V: then $BO^2 + AV^2$ will be, universally, equal to AB^2; required the demonstration?

*** *This question was proposed by Monf.* Fermat *to Dr.* Wallis, *as may be seen at page* 188 *of the Commercium Epistolicum, published at Oxford in the year* 1658, *but was never yet publicly answered.*

XI. QUESTION 607, *by Mr.* Wm. Crakelt.

In a plane triangle there are given one of the angles at the base, the length of a line drawn from the other angle to the middle of its opposite side, and the perpendicular from the given angle upon its opposite side a maximum; to determine the triangle, by construction.

XII. QUESTION 608, *by Mr.* J. Chipchase.

There is a hollow cylinder with a circular hole in its side $\frac{1}{10}$ of an inch in diameter, and at the distance of 3 feet from its bottom, out of which when the cylinder is full, the water will spout to the distance of 5 feet from its bottom on an horizontal plane, but after it has continued running for the space of 15′, it will only spout to the distance of 3 feet: Required the content of the cylinder?

XIII. QUESTION 609, *by Mr.* Wm. Wales.

To determine the declination of that star whose change in azimuth, in a given latitude, is the greatest possible in a given time, reckoned from that of its rising.

XIV. QUES-

XIV. Question 610, *by Mr.* C. Hutton.

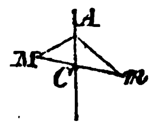
To find the radius of a circle, whose area shall be equal to the surface of an elliptic spindle, or to that of any frustum or segment of it, having given the ellipse from which it is generated, and the distance of the centers of the ellipse and spindle.

XV. Question 611, *by Mr.* Tho. Allen.

The sum of the infinite series $\frac{1}{2.4.2+3} + \frac{1.3}{2.4.6.4+5} + \frac{1.3.5}{2.4.6.8.6+7} + \frac{1.3.5.7}{2.4.6.8.10.8+9} +$ &c. is required, with the investigation.

The Prize Question, *by* Plus Minus.

Let a thread *MAm* be fixed by its ends to the ends *M*, *m* of a ruler, whose middle point is *C*, so as to slide over a tack fixed in the point *A*, whilst the point *C* of the ruler keeps constantly in the line *AC*; then will the ends *M*, *m* of the said ruler describe a certain curve, which revolving about its axis *AC*, will generate a solid, the ratio of whose content to that of its circumscribing cylinder is here required; the length of the thread being to that of the ruler in the sub-duplicate ratio of 4 to 3.



Whoe'er is unable this truth to discover,
Either is *not at all*, or an *unhappy* lover.
Happy lovers do all (tho' not skill'd in the arts)
Know the *solid content* of *two conjugate hearts*.

1770.

Questions answered.

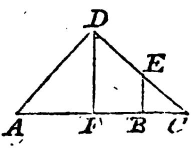
I. QUESTION 597 *answered by Mr.* Wm. Spicer.

FROM the 1st given equation w is $= a\ (52) - x - y - z$; and this value substituted in the other three, they become $ax - xx - xy - xz + yz\ (= 360) = b$, $az - xz - yz - zz + xy\ (= 280) = c$, and $ay - xy - yy - yz + xz\ (= 315) = d$: Whence, by addition, $a \times \overline{x+z} - \overline{x+z}^2 = b + c$, $a \times \overline{z+y} - \overline{z+y}^2 = c + d$, and $a \times \overline{x+y} - \overline{x+y}^2 = d + b$, and compleating the square, &c. $x + z = \frac{1}{2}a - \frac{1}{2}\sqrt{aa - 4 \times \overline{b+c}} = 20$, $z + y = \frac{1}{2}a - \frac{1}{2}\sqrt{aa - 4 \times \overline{c+d}} = 17$, and $x + y = \frac{1}{2}a - \frac{1}{2}\sqrt{aa - 4 \times \overline{b+d}} = 25$, and consequently $x = 14$, $y = 11$, $z = 6$, and $w = 21$; which shew the matchless hero's name required to be WOLF.

In a manner equally ingenious it is also answered by the Rev. Mr. *Crakelt*, Mr. *Gawith*, Mr. *R. Gibbons*, Mr. *Wm. Reynalds*, and Mr. *Tarratt*.

Mess. *T. Barker*, *J. Bartlett*, *E. Bayley*, *B. Brown*, *Wm. Cole*, *J. Coulthred*, *Cumbria*, *R. Dening*, *W. Dent*, *J. Higgins*, *J. Mills*, *E. Reed*, *T. Robinson*, *Alex. Rowe*, *S. Scott*, *P. Sharp*, *Wm. Stephens*, *Wm. Stoker*, Mrs. *E. Suggett*, and Capt. *Williams* have likewise answered it.

II. QUESTION 589 *answered by Master* J. Spencer, *at Mr.* Allen's *Boarding School, in* Spalding.



Put BE, the diff. of the heights of the walls $(= 6{\cdot}75) = a$; BA, the breadth of the building $(= 28) = b$; $AD\ (= DC)$ the length of the front rafters $(= 24$ feet$) = c$; and $BC = x$; then, drawing $DF \parallel EB$, FC will be $= \frac{1}{2} \times \overline{b + x}$, $EC = \sqrt{aa + xx}$, and, per sim. $\triangle$s. $x\ (BC) : \sqrt{aa + xx}\ (EC) :: \frac{1}{2} \times \overline{b+x}\ (FC)$

: c

: c (DC); $\because \sqrt{aa + xx} = \frac{2cx}{b + x}$, and consequently $x =$ 7·345, very nearly; and hence the length of the back rafters are easily found $=$ 14·02288 &c. feet.*

Mess. *J. Addison*, *T. Barker*, *J. Bartlett*, *E. Bayley*, *G. Coughron*, *e Cumbria*, *H. Curtis*, *J. Dalby*, *R. Dening*, *W. Dent*, *R. Gibbons*, Master *Jer. Osborne*, *Pamphagus*, *E. Parnel*, Master *J. Paty* (the proposer), *Plus Minus*, *E. Reed*, *W. Reynolds*, *T. Robinson*, *Alex. Rowe*, *Rusticus*, *P. Sharp*, *W. Spicer*, *W. Stoker*, *J. Tarratt*, and Capt. *Williams* have also answered it.

III. QUESTION 599 *answered by Mr.* Wm. Spicer (*the Proposer*).

Put $p = 2000$, $r = 0{\cdot}05$, $b = 85{\cdot}75$ l. and $x =$ the time in which B's debt will be greatest; then (by progression) $\frac{1}{2} \times \overline{x+1} \times \overline{x+2} + \frac{1}{6} r \times \overline{x^3 + 3x^2 + 2x} =$ the sum of B's yearly payments and their interest, and $p + prx =$ the amount of the principal p in the same time, and consequently $p + prx - \frac{1}{2} \times \overline{x+1} \times \overline{x+2} - \frac{1}{6} r \times \overline{x^3 + 3x^2 + 2x} =$ the sum he will then be indebted, a max. per quest.——In flux. and reduced, $x = 45{\cdot}184086$, &c. which shews that B's debt will be the greatest just before the 46th payment is made. Again, supposing x, now, to denote the time when A is indebted to B the sum of 85·75 l. (the rest remaining the same as before), then will $\frac{1}{2} \times \overline{x+1} \times \overline{x+2} + \frac{1}{6} r \times \overline{x^3 + 3x^2 + 2x} - p - prx = b$ (per quest.); $\because x^3 + 63x^2 - 11818x = 250170$, and $x = 93$ years, the time required.†——*Pamphagus*'s solution is very little different.

It is also answered by Mess. *T. Barker*, *G. Coughron*, *J. Dalby*, *W. Dent*, *R. Gibbons*, *Giles Lacey*, *E. Reed*, *T. Robinson*, *Alex. Rowe*, *S. Scott*, Capt. *Williams*, *R. Williamson*, and *J. Young*.

IV. QUES-

* This is nearly the same as the prize question for the year 1744, to which several constructions were given the year following.

† This solution seems to be false. For, the sum of the payments being $1 + 2 + 3 + \ldots x = \frac{1}{2} x . \overline{x+1}$, and the sum

 of

IV. QUESTION 600 *answered by Mr.* J. Addison.

It is easily proved, that two numbers to have the condition required in this question (viz. that the cube root of the one multiplied into the square root of the other may be a max. &c.) must be in the ratio of 2 to 3, and consequently that if $xx =$ the number of guineas, $\frac{2}{3}xx$ will $=$ that of the moidores; whence $x\sqrt[3]{\frac{2}{3}xx} = 39\cdot1918$ (per quest.), and $xx = 96$, and $\frac{2}{3}xx = 64$, the numbers required.

Mess. *J. Chipchase*, *J. Dalby*, *E. Parnel*, and *Paul Sharp* (the proposer) put x and y for the number of guineas and moidores, and $z =$ the sum of them both; then will $y = z - x$, and $\sqrt[3]{\overline{z-x}} \times \sqrt{x}$ be a max. (per quest.). In flux. and reduced (z being constant), $x = \frac{3}{5}z$: Whence $y = \frac{2}{5}z$; and, $\sqrt{\frac{3}{5}z} \times \sqrt[3]{\frac{2}{5}z}$ being $= 39\cdot1918$ (per quest.), $z = 160$, and consequently $x = 96$, and $y = 64$, the same as before.

Mess. *T. Barker*, *G. Coughron*, *e Cumbriâ*, *W. Dent*, *R. Gibbons*, *Pamphagus*, *E. Reed*, *T. Robinson*, and Capt. *Williams* have likewise answered it.

V. QUES-

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of the interests of these payments $= r.\overline{x-1} + 2r.\overline{x-2} + 3r.\overline{x-3} + \ldots\, \overline{x-1}.r.\overline{x-\overline{x-1}} = rx \times : 1 + 2 + 3 \ldots \overline{x-1} - r \times : 1^2 + 2^2 + 3^2 + \ldots \overline{x-1}^2 = \frac{1}{2}rxx.\overline{x-1} - \frac{1}{6}rx.\overline{x-1}.\overline{2x-1} = \frac{1}{6}r.\overline{x^3-x}$, ∵ the annual payments and their interests together are $\frac{1}{2}x.\overline{x+1} + \frac{1}{6}r\ \overline{x^3-x}$, which taken from $p + prx$, and the flux. of the remainder made $= 0$, we obtain $x = \sqrt{2p + \frac{1}{3} + \frac{1}{rr}} - \frac{1}{r} - \frac{1}{r} = \sqrt{4380\frac{1}{3}} - 20 = 46\cdot18408$, the time when the debt is greatest.

Again, the root of the equation $\frac{1}{2}x.\overline{x+1} + \frac{1}{6}r.\overline{x^3-x} - p - prx = b$, or $x^3 + 60x^2 - 11941x = 250290$, is $x = 94\frac{1}{2}$ years, the time when $85\cdot75$ is due to *B*.

The time when the whole debt is just cleared, will be the root of this equation $\frac{1}{2}x.\overline{x+1} + \frac{1}{6}r.\overline{x^3-x} = p + prx$, or $x^3 + 60x^2 - 11941x = 240000$, where $x = 94\cdot066$, the time when the debt is cleared.

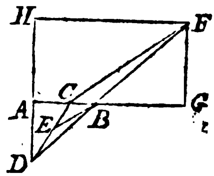
V. QUESTION 601 *anſwered by Mr.* E. Williams, *Captain in the Royal Artillery.*

By the conſtruction of the propoſer's figure (ſee the laſt year's Diary) the following analogies are evident, viz. $BF : Hm :: FE : mE :: EG\ (FC) : cG\ (Dn)$; therefore $Hm : Dn :: (BF : FC) :: BE : AE$. In like manner, $AG : Mc :: GE : cE :: EF\ (GC) : mF\ (Nn)$, and conſequently $Mc : Nn :: (AG : GC) :: AE : EB$.

The demonſtrations by Meſſrs. *T. Barker, W. Cole, G. Coughron,* the Rev. Mr. *Crakelt, J. Dalby, T. Moſs* (the propoſer), *T. Robinſon,* and *W. Spicer* are ſo little different from the preceding, that it is needleſs to repeat them.

VI. QUESTION 602 *anſwered by the Rev. Mr.* Crakelt.

CONSTRUCTION. Let BA (30 yards) repreſent the difference betwixt the hypothenuſe and leſs leg, and CA (15) that betwixt the hypothenuſe and greater; then, at the point A, erect the $\perp AD$, meeting BD, making with AB an angle of 45°, in D, and join the points D, C: This done, from B apply BE (to DC) $= BA$, and from the point F, where CF drawn $\parallel BE$ meets DB produced, let fall the $\perp FG$ upon AB produced; ſo will CFG be the right-angled triangular garden required.



DEMONSTRATION. Produce DA till it meets FH, drawn $\parallel AG$, in the point H: Then, by ſimilarity of Δs, $AB : HF :: DB : DF :: EB : CF$; but (by conſtruc.) $EB = AB$, and conſequently $CF = HF = AG = AC + CG = AB + BG = AB + FG$.

The calculation from this conſtruction is extremely eaſy, whereby the length of the tree's ſhadow, at the given time, comes out $= 36$ yards, and conſequently the ſun's altitude $= 20° 19' 23''$; whence, per ſpherics, the latitude required is readily found to be $37° 33' 38''$ ſouth, &c.

Meſſ. *J. Addiſon, T. Barker, J. Bartlett, J. Chipchaſe, W. Cole,* ? *Cumbrid, J. Dalby, R. Gibbons, J. Haycock, E. Parnel, W. Pearſon, E. Reed, T. Robinſon, Alex. Rowe, W. Spicer, W. Stoker, J. Tarratt,* Capt. *Williams, R. Williamſon,*

son, and *J. Young* have given ingenious algebraic solutions to it.

Mr. *G. Coughron* observes that, as the excess of the hypothenuse above one of the legs is double its excess above the other leg, the sides of the required triangle are in arithmetical progression; and from thence derives every thing (without algebra) nearly the same as above.

VII. QUESTION 603 *answered by Mr.* G. Coughron.

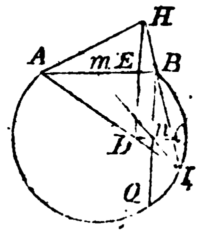
By the data and properties of a right-angled triangle, $x^{10} + y^{10} = \overline{xx+yy}|^{3} \times aaxxyy$, and $x^5y^5 = \overline{xx+yy}|^{\frac{3}{2}} \times axy \times x^3$. Substitute zxx for yy and its powers in the first equation, and it becomes $x^{10} \times \overline{z^5+1} = x^{10} \times \overline{z+1}|^{3} \times aaz$; $\therefore z^5 + 1 - \overline{z+1}|^{3} \times aaz = 0$; which, divided by $zz \times \overline{z+1}$ and reduced, gives $zz + \frac{1}{zz} - \overline{1+aa} \times \overline{z+\frac{1}{z}} + 1 - 2aa = 0$; or, putting $v = z + \frac{1}{z}$, $vv - \overline{1+aa} \times v - \overline{1+2aa} = 0$; $\therefore v = \sqrt{\frac{\overline{1+aa}|^{2}}{4} + 1 + 2aa} + \frac{1+aa}{2}$, which put $= 2c$, and then, $z + \frac{1}{z}$ being $= 2c$, z will be found $= \pm\sqrt{cc-1} + c$. Lastly, from the second equation is had $z^{\frac{5}{2}}x^{10} = x^8 \times \overline{z+1}|^{\frac{3}{2}} \times az^{\frac{1}{2}}$; $\therefore x = \overline{z+1}|^{\frac{3}{4}} \times \frac{\sqrt{a}}{z}$; which, as z is now known, will likewise be known, and from thence the triangle becomes known.

The solutions given by the Rev. Mr. *Crakelt* and Mr. *Gawith* (the proposer) are also very ingenious.

VIII. QUES-

VIII. QUESTION 604 *answered by Mr.* T. Moss.

CONSTRUCTION. Upon the given bisecting line AB describe a segment of a circle to contain the given angle of the required triangle, and in $BQ \perp AB$, take Bn = the given perpendicular. Bisect AB in m, and through the points m and n draw a right line meeting the arch of the said segment in I: Join I, A, and I, B, and draw $DnC \parallel AB$; in IB, produced, take $BH = BC$: Draw AH, and AHI will be the triangle required.

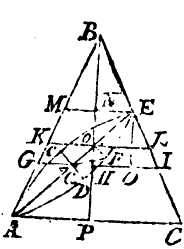


DEMONSTRATION. Draw HD, cutting AB in E: Then, because (by construc.) AB is bisected in m, and DC is $\parallel AB$, Dn is $= nC$; also (by construction) BH is $= BC$; whence HD is $\parallel Bn$, and consequently HE ($= \frac{1}{2} HD$) $=$ Bn, the given perpendicular, by construc. Moreover, because the given line AB is $\perp HD$ and bisects it at E, the $\angle EAD = EAH$; and AIH is $=$ the given angle, by construction. *Q. E. D.*

Elegant constructions to this question have also been received from the Rev. Mr. *Crakelt*, and Mess. *G. Coughron*, *H. Curtis*, *J. Dalby*, *P. George*, and *W. Wales*.——It is also answered by Mess. *T. Barker*, *J. Chipchase*, *Alex. Rowe*, and *W. Spicer*.

IX. QUESTION 605 *answered by the Rev. Mr.* Crakelt.

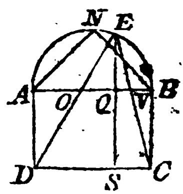
Let ABC be the representation of any upright cone; $ADEA$ that of any elliptic section made therein; $Ac = EC =$ the semi-transverse of such section; $De = ce$ its semi-conjugate; Fc any other semi-diameter; GHI, KoL, MNE the diameters of three circular sections made by planes passing through the points D, F, E, and EO a parallel to the axis of the cone, BP: Then, by similarity of $\triangle$s, $Lo : Ic :: Ec - co$ $(Eo) : Ec$, and $Ko : Gc :: Ec + co$ $(Ao) : Ec$ (Ac), and consequently $Lo \times Ko =$ (by prop. of the circle) $Fo^2 : Ic \times Gc$ or $Dc^2 ::$ Ec^2



$Ec^2 - co^2 : Ec^2$, the principal prop. of the ellipse: But Dc^2 or $Ic \times Gc =$ (by similarity of triangles and the nature of parallels) $Ic \times MN$ or $EN = Ic \times HO = \overline{HO + 2OI} \times HO = HO^2 + HO \times 2OI$, and $Ec^2 = OI^2\ (cH^2) + HO^2 + HO \times 2OI + EO^2$, and consequently $Ec^2 - Dc^2 = OI^2 + EO^2 = EI^2$, or $Dc^2 = Ec^2 - EI^2$: Wherefore $Ec^2 \times Fo^2 = Dc^2 \times Ec^2 - Dc^2 \times co^2 = Dc^2 \times Ec^2 - Ec^2 \times co^2 + EI^2 \times co^2$, or, by adding $Ec^2 \times co^2$ to both, $Ec^2 \times Fc^2 = Ec^2 \times Dc^2 + EI^2 \times co^2$: Whence, as a similar equality will obtain for all the other points of of the curve, DFE, and the greater co^2 becomes the greater will $Ec^2 \times Fc^2$ become likewise, the truth of the prob. is manifest.

Mess. *T. Barker* and *J. Dalby* have also demonstrated it.

X. QUESTION 606 *answered by* Clericus.



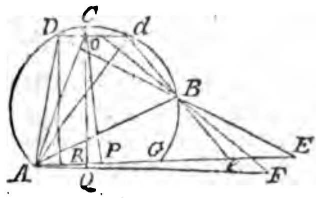
Join the points B, N; D, C; and draw $ES \perp DC$ intersecting AB in Q: Then, per sim. Δs, $DA : AO :: ES : DS = AQ$, and $BC\ (= DA) : BV :: ES : SC = BQ$, and consequently $DA \times DA : AO \times BV :: ES \times ES : AQ \times BQ = EQ^2$ (by property of the circle): Whence $2AO \times BV\ \left(= \frac{2DA^2 \times EQ^2}{ES^2}\right) = \frac{AB^2 \times EQ^2}{ES^2}$, because $2DA^2\ (= 2AN^2) = AB^2$ (47 Euc. 1.). But $OV^2 : DC\ (AB^2) :: EQ^2 : ES^2$ (per sim. Δs OEV, DEC), and consequently $OV^2\ \left(= \frac{AB^2 \times EQ^2}{ES^2}\right) = 2AO \times BV\ (= 2AO \times \overline{OB - OV}) = 2AO \times OB - 2AO \times OV$; $\because OV^2 + 2AO \times OV = 2AO \times OB$, and $AO^2 + OV^2 + 2AO \times OV + OB^2 = AO^2 + 2AO \times OB + OB^2$; i. e. $\overline{AO + OV}|^2 + OB^2 = \overline{AO + OB}|^2$, or $AV^2 + BO^2 = AB^2$

Demonstrations of this theorem have also been received from Mess. *T. Allen*, *T. Barker*, *J. Chipchase*, *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Dalby*, *P. George*, *R. Gibbons*, *M. Halls*, the Rev. Mr. *Lawson*, *Plus Minus*, *T. Robinson*, *W. Spicer*, the Rev. Mr. *Wildbore*, and *E. Williams*, Captain in the Royal Artillery.

XI. QUES-

XI. QUESTION 607 *answered by the Rev. Mr.* Lawson.

CONSTRUCTION. On AB the given line, describe a segment to contain the given angle. Bisect the circumference in D, and join A, D; D, B: Continue DB till BE equals BD; then, draw AE cutting the circumference continued in G. Bisect again the circumference ADG in C, and join A, C; C, B; and continue CB till BF equals CB; then join A, F, and I say ACF is the triangle required, whose perpendicular CQ is a maximum.

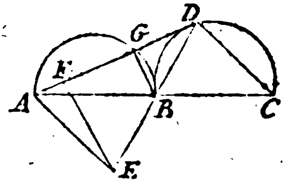


DEMONSTRATION. From C let fall $CP \perp AG$, and it will be the greatest perpendicular in the segment ACG. But CQ is still greater, because the locus of the remaining angle of the required triangle (viz. EFe) is a circumference similar to ADB, to which AF is a tangent.

NOTE, If the perpendicular be given in magnitude, the same construction may be used; and if Po be set off on PC equal thereto, and through o, Dod drawn $\parallel AG$, the points D and d will each give a solution to the problem.

CONSTRUCTION *to the same by the Rev. Mr.* Crakelt (*the Proposer*).

Let AB be supposed the given line drawn from the unknown angle at the base to the middle of its opposite side, and having produced it to C so that BC may be $= AB$, describe upon BC a segment of a circle capable of containing an angle $=$ the given one at the base, and draw the tangent thereto AD: Then, drawing DC, through A draw a parallel thereto to meet DB produced in E, and the $\triangle AED$, formed thereby, will be that required.



DEMON-

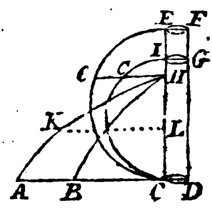
DEMONSTRATION. The line AE being ∥ DC, the angle AEB will be $=$ the $\angle CDB =$ the given angle at the base (by construction), and the $\triangle$s ABE, CBD similar; whence $AB : BC :: BE : BD$; but $AB = BC$ (by construction); wherefore $BE = BD$. And, that the perpend. EF will in this case be a maximum, is very plain, since in all other (possible) cases the line AD will cut the segment BDC in two points, and fall nearer the point B, and consequently not give the $\perp BG$ or its double, EF, a maximum.

SCHOLIUM. If it was required that EB should be to BD in a given ratio, or that the perpendicular instead of a maximum should be of a given magnitude, the construction would be the same very nearly.

This question is also solved in a curious manner by the Rev. Mr. *Wildbore*; and the constructions by Messrs. *G. Coughron* and *T. Moss* are not essentially different from the preceding, except in their strictly demonstrating the perpendicular to be a maximum.

XII. QUESTION 608 *answered by the Rev. Mr.* Wildbore.

If $HL = EH$, KL will $= 2EH$, and (per conics) $KL^2 : HL$, or $4EH^2 : EH :: AC^2 : HC$; $\because HE = \frac{AC^2}{4HC} = 25$ inches; and in like manner is found $IH = 9$ inches. Also (per Newton: Princip. edit. opt. p. 329, 330, 331) the times in which EH and IH may be emptied will be $= \frac{100EF^2\sqrt{2EH}}{\sqrt{193}}$ and $\frac{100EF^2\sqrt{2IH}}{\sqrt{193}}$ respectively; the difference of which is evidently $= 900''$ (the time in which EI is run out at H) per question; whence $EF = 6{\cdot}64872$, and the content of the cylinder $2117{\cdot}8266$ cubic inches.



Answer to the same by Mr. T. Allen.

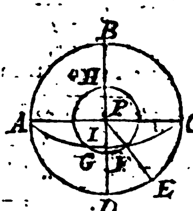
Let $CEFD$ represent the cylinder, H the hole, and HA, HB the spouting water, when the surface occupies EF and IG respectively. Upon the diameters EC, IC describe the semicircles EeC, IcC, and draw $ecH \perp EC$: Then, it is well known that the horizontal distances AC, BC, of the

the fpouting fluid, are refpectively $= 2 \times eH$ and $2 \times cH$. Therefore, fince eH and cH are given $= 2{\cdot}5$ and $1{\cdot}5$ feet refpectively, EH and IH are eafily found $= 2{\cdot}0833$ &c. and $0{\cdot}75$ feet: Therefore GE, the height of the cylinder, is $= 5{\cdot}0833$ &c. feet.——Put now, $EH = b$, $F = {\cdot}0000545415$ the area of the orifice in feet, $s = 16\frac{1}{12}$, $t =$ time of emptying $EFGI$ with the firft or greateft velocity, $IH = c$, $EI = 1{\cdot}33$ &c. $= d$, the given time $900'' = a$, $p = {\cdot}785397$, and EF (the diam. of the cylinder) $= x$. Then, per Emerf. Mech. 1 edit. pa. 173, $tF\sqrt{2bs} = pdxx$; $\therefore t = \frac{pdxx}{F\sqrt{2bs}}$: And, per Emerf. Flux. 2 edit. p. 139, $\frac{2pdxx}{F\sqrt{2bs}} - \frac{2pdxx\sqrt{c}}{F\sqrt{2bbs}} = a$; whence, &c.

Ingenious anfwers to this queftion have alfo been received from Meff. *J. Addifon*, *T. Barker*, *J. Chipchafe* (the propofer), *W. Cole*, *G. Coughron*, and the Rev. Mr. *Crakelt*.

XIII. QUESTION 609 *anfwered by Mr.* W. Wales *(the Propofer)*.

PROJECTION. Defcribe the primitive $ABCD$ for the equator, and draw BD for the meridian; from D, fet off $DE = \frac{1}{2}$ the arc of the equator paffed over in the given interval, and join P, E: Defcribe the horizon $AGEC$ interfecting PE in F; then, from P, the pole, as a center, through F, defcribe the parallel of declination FGH, which fhall be that of the ftar required.



DEMONSTRATION. It is a thing too well known to need demonftrating here, that every phænomenon whatever changes its azimuth the fafteft poffible when on the meridian, and with equal velocities at equal diftances therefrom; confequently, the change in azimuth will be the greateft poffible during the whole of any given interval, when the phænomenon is on the meridian precifely at the middle of that interval, or when the femi-diurnal arc DE, turned into time, is $= \frac{1}{2}$ the given interval.

SCHOLIUM. It is manifeft that when the given interval exceeds 12 hours, the declination will be of the fame name with the latitude, but of a contrary name when the interval is lefs.

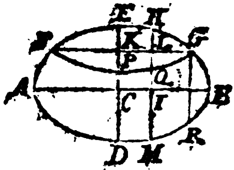
REMARK. It may at first sight seem to some, as if this method of projection would fail in particular cases; such as, if it was required to find the sun's declination when his change in azimuth was the greatest possible in any given time, reckoned from his rising, and less than the shortest day in the latitude given: But on more mature consideration this will be found a mistake; for in all those cases, the sun's declination must be the greatest possible, and of a different name with the latitude, because then the whole of the given interval is the nearest possible to the meridian, which is the very principle on which the preceding projection is founded.

I scarce need add that cos. FPI : rad. : : tang. IP (the given lat.) : tang. PF the co-declination required.

The Rev. Mr. *Wildbore* also gives a curious projection of this problem on the plane of the equinoctial; and the Rev. Mr. *Crakelt* and Mr. *G. Coughron* have given ingenious algebraic solutions to it.

XIV. QUESTION 610 *answered by Mr.* T. Allen.

Let $AEBD$ represent the given ellipsis, and $FEGP$ the spindle, generated by the revolution of the elliptic arc FEG about its axis FG.——Put $CB = a$, $CE = b$, $KC = LI = d$, $CI = KL = x$, $IH = y$, arc $EH = z$, and $p = 3.14159$; then, per conics, $y = \frac{b}{a}\sqrt{aa - xx}$, and therefore $HL = LQ = \frac{b}{a}\sqrt{aa - xx} - d$:



Whence $\dot{z}\ (= \sqrt{\dot{x}^2 + \dot{y}^2}) = \frac{\dot{x}}{a}\sqrt{\frac{a^4 - ccxx}{aa - xx}}$ (by putting $bb - aa = -cc$) and $\frac{2pb\dot{x}}{aa} \times \sqrt{a^4 - ccxx} - 2pd\dot{z} =$ the flux. of the spindle; whose fluent is $\frac{pb}{aa}\sqrt{a^4x^2 - c^2x^4} + \frac{pbaa}{c} \times$ circ. arc, whose rad. is 1 and sine $\frac{cx}{aa}$, $- 2pdz$, a general expression for the surface, either of the whole, or of any frustum of the spindle: But the two first terms thereof express the surface of the portion $EHMD$ of the spheroid $AEBD$; wherefore it is evident that the radius

of

of a circle = the surface $\left\{\begin{array}{c} EHQP \\ EGQ \\ HGQ \end{array}\right\}$ will be a mean proport. between the surf. $\left\{\begin{array}{c} EHMD \\ EGRD \\ HGRM \end{array}\right\} - 2p \times KC \times \left\{\begin{array}{c} EH \\ EG \\ HG \end{array}\right\}$ and $\frac{1}{p}$.

The Rev. Meff. *Crakelt* and *Wildbore* have likewife given ingenious and general folutions to this queftion.——Mr. *G. Gougbron* gives a general conftruction to it, and refers to pages 291 and 292 of Mr. Hutton's Menfuration for the demonftration.

XV. QUESTION 611 *anfwered by the Rev. Mr.* Wildbore.

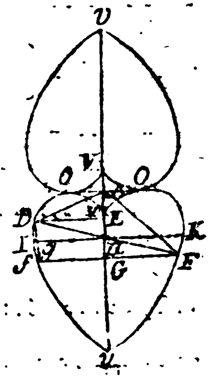
The terms of the propofed feries being multiplied feparately by x^5, x^9, x^{13}, &c. refpectively, and the fluxion taken, it becomes $\frac{x^4\dot{x}}{2.4} + \frac{1.3x^8\dot{x}}{2.4.6} + \frac{1.3.5x^{12}\dot{x}}{2.4.6.8} +$ &c. $= \frac{\dot{x}}{x^4} - \frac{\dot{x}}{2} - \frac{\dot{x}\sqrt{1-x^4}}{x^4} = \frac{\dot{x}}{x^4} - \frac{\dot{x}}{2} - \frac{\dot{x}\sqrt{1-x^4}}{x^4} - \frac{\frac{1}{3}\dot{x}}{\sqrt{1-x^4}} + \frac{\frac{2}{3}\dot{x}}{\sqrt{1-x^4}}$; the fluent of which, when $x = 1$, is $\frac{e + \sqrt{ee - 2f}}{3} - \frac{5}{6} = \cdot 040685$, the fum required; where $e = \frac{1}{4}$ the periphery of the ellipfis whofe femi-axes are $\sqrt{2}$ and 1, and $f = \frac{1}{4}$ of that of the circle, whofe rad. is 1.

Mr. *Allen* (the propofer) and the Rev. Mr. *Crakelt* alfo bring out the fame conclufion exactly, by means of theo. 1. pa. 146, of Mr. Landen's Lucubrations.

The PRIZE QUESTION *anfwered by Mr.* Tho. Allen, *of* Spalding.

Let DF ($= 2a$) reprefent any indefinite pofition of the ruler, and DAF ($= s$) the correfponding pofition of the thread. Put AH, the diftance of the tack A from the middle of the ruler, $= z$, $HG = HE = x$, and $p = 3.14159$ &c. then will $FG^2 = DE^2 = aa - xx$

(47 Euc. 1), $AE = z - x$, and $AG = z + x$, and consequently $\sqrt{aa - xx + \overline{z+x}^2} + \sqrt{aa - xx + \overline{z-x}^2} = s$; $\therefore z = \sqrt{\frac{s^4 - 4ssaa}{4ss - 16xx}}$ (generally) $= \frac{1}{2\sqrt{1-xx}}$ in the present case, where $a = \frac{\sqrt{3}}{2}$, and $s = 2$ (per quest.).——Further, when the extremity of the ruler passes the point O, the fluxion of EA, or $\dot{x} - \dot{z}$ will evidently be $= 0$; therefore $-\frac{1}{2}x\dot{x} \times \overline{1 - xx}|^{-\frac{3}{2}} + \dot{x} = 0$, or $4 \times \overline{1 - xx}|^{\frac{3}{2}} - xx = 0$; in which equation one root, or value, of x (which is that here required) is evidently $= \sqrt{\frac{1}{2}}$.——Now, $p \times : aa\dot{x} - x^2\dot{x} + \frac{1}{4}x^3\dot{x} - \frac{1}{4}aax\dot{x} \times \overline{1 - xx}|^{-\frac{1}{2}}$, $p \times : x^2\dot{x} - aa\dot{x} + \frac{1}{4}aax\dot{x} - \frac{1}{4}x^3\dot{x} \times \overline{1 - xx}|^{-\frac{1}{2}}$ and $p \times : aa\dot{x} - x^2\dot{x} + \frac{1}{4}aax\dot{x} - \frac{1}{4}x^3\dot{x} \times \overline{1 - xx}|^{-\frac{1}{2}}$ being the fluxions of the solids whereof OVO, $IOAOK$, and IvK are the respective sections (IK being the greatest ordinate), their fluents, by proper correction and substitution, will be $p \times : \frac{2}{3\sqrt{\frac{1}{2}}} - \frac{2+\sqrt{3}}{4}$, $p \times : \frac{2}{3\sqrt{\frac{1}{2}}} - \frac{5}{8}$, and $p \times : \frac{5}{8} + \frac{\sqrt{3}-2}{4}$; the sum of the two last of which minus the first gives $p \times \frac{\sqrt{3}}{2}$ = the content of the solid required; the ratio of which to that of its circumscribing cylinder will be as $1 : \frac{\sqrt{3}}{2} + \frac{3}{4}$, as is evident from what has been determined above.



Ingenious *Plus Minus*, you see how I prove
Your *heart's true content* from what's given above:
And since I'm so happy this truth to discover,
I hope you'll esteem me, a happy true lover.

The

The same answered by the Rev. Mr. Crakelt, *of* North-fleet, *in* Kent.

Suppose the lines drawn as in the preceding diagram, and the ruler to be moved from its first position Vv, into the position DF, and put $FA + AD$ (the length of the thread) $= 2a$, $FA - AD = 2x$, DF (the length of the ruler) $= 2b$, $DE = FG = y$, and $p = 3.14159$ &c. Then, by theo. 10, 11, book 2d, of Simpson's Geometry, 2d edit. and the nature of the problem, will be found $Gv = a + b - a\sqrt{\frac{aa-bb}{aa-bb+yy}} - \sqrt{bb-yy}$, $EV = b - a + a\sqrt{\frac{aa-bb}{aa-bb+yy}} - \sqrt{bb-yy}$, and, supposing the solid to be divided into two parts by a plane coinciding with its greatest ordinate, the sum of the fluxion of those parts $= \frac{2py^3\dot{y}}{\sqrt{bb-yy}}$; the correct flu. whereof ($-\frac{2p}{3}yy\sqrt{bb-yy} - \frac{4pb^2}{3}\sqrt{bb-yy} + \frac{4pb^3}{3}$) gives (when $y = b$) the solidity required $= \frac{4pb^3}{3} = \frac{8pb^3}{6}$, the solidity of a sphere, whose diameter is $=$ the length of the ruler.——Again, $\overline{aa-bb+xx} - ax \times \overline{aa-bb+xx}^{-\frac{1}{2}}$ representing the value of AE, its flux. made $= 0$, and properly reduced, gives $x^3 + \overline{aa-bb} \times x = \overline{aa-bb} \times a$, or (since $bb = \frac{3aa}{4}$, by the data) $x^3 + \frac{a^2}{4}x = \frac{a^3}{4}$; and the root $\frac{a}{2}$, by substitution, shews that, in the present case, the points E, A coincide, or that the altitude of the circumscribing cylinder is $= vA$, and consequently that its solidity is to that of the generated solid as $1 + \frac{2}{\sqrt{3}}$ to $\frac{4}{3}$.——The several curious corollaries deduced from this solution our narrow limits oblige us to omit; and Mr. *G. Coughron*'s answer is so nearly the same that it would be needless to repeat it.

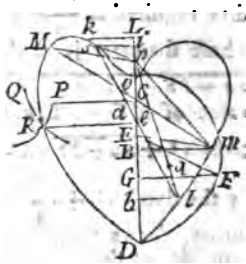
Mr. *James Terry* produces FG to meet the curve again in f; then, retaining the last substitution, and joining D, f and drawing $HG \perp$ thereto, $\sqrt{bb-yy} = EH = HG$ (per 47 Euc. 1.) and $4py\dot{y}\sqrt{bb-yy} =$ the fluxion of the solid

generated by the revolution of the space $VDgfv$ about the axis Vv; the correct fluent whereof $(\frac{4p}{3} \times b^3 - \overline{bb-yy}|^{\frac{3}{2}})$ when $y=b$, gives $\frac{4p}{3} \times b^3 = \frac{p}{6} \times 8b^3$, the content of the whole solid required, let the length of the thread be what it will, &c.

The ingenious proposer *Plus Minus*, after his curious solution, makes the following remark, viz. 'I have given 'the length of the thread in proportion to that of the ruler 'as $2 : \sqrt{3}$, that the conjugate hearts may osculate, as in O 'and O; had the thread been shorter, they would have 'intersected; had it been longer, they would not have 'touched; and had it been much longer, they would not 'have had this form of an heart, but would have been two 'ovals. Yet in all cases the solid generated by the rotation 'of one of the curves is equal to a sphere, whose diameter 'is equal to the length of the ruler.'

General Answer to the same by the Rev. Mr. Wildbore.

The curve being generated by the motion of the point m from D to O, let the semicircle DFO, whose rad. is ED $(=\frac{1}{2}OD = \frac{1}{2}Mm = Cm)$ be supposed to be generated by that of the point F. Through the center E draw $EF \parallel Cm$, and then, the $\angle$s mCD, FED being equal, m and F must be co-temporary positions of the two points. Moreover, $p.B\dot{D}.Bm^2 = p.B\dot{G}.Bm^2 + p.G\dot{D}.Bm^2$ is evidently $=$ the flux. of the solid generated by the revolution of BmD about $BD = p.B\dot{G}.GF^2 + p.G\dot{D}.GF^2$ (because $Bm = GF$): But $p.G\dot{D}.GF^2 =$ the flux. of the spheric segment generated by the revolution of GFD about GD; whence the corrected fluent of $p.B\dot{G}.GF^2 +$ the said segment $=$ the said solid; but, let the relation of BG to GF be what it will, when each of them is $\mp$ nothing at the same time, the corrected fluent of $p.B\dot{G}.GF^2$ must necessarily be $=0$: In which case therefore, the segment will be $=$ the solid. Now m and F evidently arrive at O in the



the ſame time, making GF and BG each equal to nothing; and, by conſequence, the ſphere generated by $OFD =$ the ſolid generated by OmD.——Which property is not at all affected by $AM + Am$, but it is univerſal to the whole family of curves, when Cm is a conſtant quantity. Further, Mm is evidently equal to the greateſt diameter of the ſolid, or to that of its circumſcribing cylinder, and $LD =$ its height; ∴ the ratio of the ſolidities will be $\frac{2}{3}OD : \frac{1}{2}LD$.

Now, to determine the ſolid anſwering to any particular data; from the center i with the rad. $iQ = \frac{1}{2}.\overline{AM + Am}$ deſcribe a circle, and from A as a center, with the power $Ad^2 = iQ\sqrt{iQ^2 - Cm^2}$, deſcribe the equilateral hyperbola PR, and where it cuts the circle at R let fall the perpendicular Re; at i erect the $\perp\, is$, and through e draw sl, which will be $= Mm$; for (per trigonom.) $iQ : AE :: ie : \frac{1}{2}.\overline{Al - As} = ql$ (taking $Aq = iQ$) and $\overline{ql + Aq}^2 = Al^2 = \overline{Ae + eb}^2 + es^2 - ie^2 = Ae^2 + 2Ae.ei + es^2$; ∵ $Ae^2 . Aq^2 - ie^2 . Ae^2 = Aq^2 \times \overline{Aq^2 - es^2} = Ad^4$: But $Aq^2 (= iQ^2) - ie^2 = Re^2$ (47 Euc. 1.); therefore $Re.Ae = Ad^2$, a known property of the hyperbola, whoſe aſymptote is AD and power Ad^2. It is moreover evident that, when Ai is the greateſt poſſible, or when i coincides with L, the hyperbola will not cut but touch the circle, AL being univerſally the height of the circumſcribing cylinder above AD; and, in the particular caſe propoſed, becauſe $\sqrt{iQ^2 - Cm^2} = \frac{1}{2}iQ$, $Qd = Re = Ad$, $iQ^2 - Ad^2 = Ad^2$, $\frac{1}{2}iQ^2 = Ad^2$, ie^2, $Ai = 0$, $AL = 0$, AD is the height of the circumſcribing cylinder, and the ratio above will become $\frac{2}{3}OD : \frac{1}{2}AD$ or $1 : \frac{3 + 2\sqrt{3}}{4}$ :: the ſolid content of the heart to that of its circumſcribing cylinder.

COR. 1. The fluxion of the area $BmD = \dot{BD}.Bm = BG.\dot{GF} + GD.\dot{GF} = BG.\dot{GF} +$ flux. of the circ. ſegment GFD; whence, when BG and GF each $= 0$, the area $BmD =$ circular ſegment GFD; wherefore the area of the whole circle $=$ that of the curve.

COR. 2. From what is ſhewn above, the ſolidity of any ſegment of the heart, cut perpendicular to the axis OD, is eaſily derived, being $=$ the ſpheric ſegment above $+$ the correct fluent of $p.BG.GF^2 = \frac{1}{6}p.GD \times \overline{3FG^2 + GD^2} + \frac{1}{4}p \times \overline{\frac{AE^4}{AC} + AE^2 . AC - AE^2}$ $=$ the ſolid generated by revolution of BmD about BD.

COR.

COR. 3. In the same manner it will be found that the fluent of $BG.GF$ is $= \frac{1}{4}AE\sqrt{AE^2 - AC^2} - \frac{1}{4}AE^2 \times$ hyp. log. of $\frac{AE-\sqrt{AE^2-AC^2}}{AE+\sqrt{AE^2-AC^2}}$; which + the circ. segment GFD is = the area of the curve BmD.

COR. 4. If Cm is not $= ED$, but in a given ratio $n \cdot Cm = ED$, then will $\frac{1}{nn} \times$ globe = the solid, as is evident from what is demonstrated above; and the same universal property may be still farther extended.

SCHOLIUM. The above universal properties may be applied to the finding a number of fluents of very difficult forms; but the subject is much too copious for this place.

Can I mistake the problem, miss the prize!
On whom my Lucy's choicest blessings rise;
For whose content she labours and she lives:
Friendship, truth conjugate she daily gives
Entwin'd with love unspotted;——then may I
E'er guide her on to bliss;——to endless joy.

NOTE, The initials answer the *Prize Enigma*.

Ingenious answers to this question have also been received from Mess. *T. Barker*, *J. Chipchase*, *P. George*, *Wm. Hardy*, and *Wm. Sewell*; but the first prize of 12 Diaries for the solution of it are fallen to the lot of the Rev. Mr. *Wildbore*, and that of 8 to *Plus Minus* the proposer.

The Eclipses calculated for 1770.

There will happen only two eclipses this year; both of the sun, and invisible in Europe.

The first, on Friday the 25th day of May, at two in the morning, visible in America, towards the tropic of Cancer.

The second, on Saturday the 17th day of November, at 10 in the morning, visible in the southern parts of Africa and America. J. KEACH.

On the 28th of August, Jupiter will be occulted by the moon, according to the following computation for Greenwich Observatory, viz. immersion 11 h. 59′ 20″, and emer. 12 h. 57′ 47″ apparent time. REUBEN ROBINS.

New

New Questions.

I. QUESTION 612, *by Mr.* Tho. Sadler.

In Whitchurch now a maid doth dwell,
Her neighbours stile her bonny Nell:
She likes to live a maiden's life,
And won't consent to be a wife,
Tho' Harry, Richard, Ben, and Joe
To Nelly oft a courting go.
Above a score sue for the maid,
Fop, clown, and leering ganymede;
But to them all 'tis Nelly's song,
To marry she is yet too young.
Her age and fortune you will find
From the * equations here subjoin'd;
Come artists say, from what is said,
Is Nelly old enough to wed?
For should the maiden stay too long,
She may forget her fav'rite song.

* Given $\left\{\begin{array}{l} x^2+y^2=13900+x^4, \text{ and} \\ x^3+y^3=654508000000+y^4 \end{array}\right\}$: To find x her age in years, and y her fortune in pounds.

II. QUESTION 613, *by Mr* Paul Sharp.

Given the difference of the transverse and conjugate diameters of an ellipsis $=8$, and the diff. of the length of the greatest inscribed parallelogram and radius of curvature at the end of the conjugate diameter $=3\cdot28426$, to find the said diameters and rad. of curvature at the said point.

III. QUESTION 614; *by Mr.* Wm. Gawith.

Given $\overline{x^5+y^5+z^5}|^2+\overline{x^3+y^3}|^2=a$, $\overline{x^5+y^5+z^5}|^3\times\overline{x^3+y^3+z^3}|^3=b$, and $\overline{x^3+y^3}|^2+\overline{x^5+y^3+z^3}|^3=c$; to find x, y, and z by means of quadratics.

IV. QUESTION 615, *by Mr.* J. Chipchase.

In the midst of the Market-place at Stockton is erected a fine column of the Doric order, on the top of which stands an urn 5 feet in height, that appears the greatest possible to a person

a person (the height of whose eye is $5\frac{1}{2}$ feet) standing at the distance of 32 feet from the base of the column, on level ground: From whence the column's height is required.

V. QUESTION 616, *by Mr.* Wm. Spicer.

Required the area of the expponential curve, whose equation is $\overline{x}|^{x^2} = b^{y^{\frac{1}{x}}}$.

VI. QUESTION 617, *by Mr.* Samuel Vince.

If the subtang. of a curve be expressed by $\dfrac{\overline{dx - x^4}|^2 \times y\dot{y}}{\overline{a - bx^3}|^2 \times \dot{x}}$; required the value of the semi-ordinate y thereof, when the abscissa x is = a given quantity s.

VII. QUESTION 618, *by Mr.* J. Addison.

Suppose a triangular prism, whose length is 12 feet, and base a right-angled isosceles triangle, the longest side of which is also 12 feet, to be placed perpendicularly in a stream of water 12 feet deep, with its right angle directly facing the stream; required the velocity of the stream per second, when its force against the prism is = 187072 lb. avo.

VIII. QUESTION 619, *by Mr.* Steph. Ogle.

Given an angle A in magnitude, and a point P in position; to draw a right line PLS in such a direction, that $PLS + M$ may be $= AS + AL$, M being a given right line. [*See the fig. to the solution.*]

IX. QUESTION 620, *by Mr.* Edw. Williams, *Captain in the Royal Artillery.*

The same construction remaining as in Monf. Fermat's theorem (see the last year's Diary); it is required to demonstrate, geometrically, when OV will be a maximum.

X. QUESTION 621, *by the Rev. Mr.* Wildbore.

Given the right line DC and the angle D, it is proposed geometrically to determine the point B so, that BA, DA being,

being in a given ratio, the sum of BA and BC may be a given quantity. [*See the fig. to the solution.*]

IX. QUESTION 622, *by the Rev. Mr.* Crakelt.

Given the right line bisecting the vertical angle of any plane triangle and terminating in its base, and the rectangle under each side and its adjacent segment of the base made by the said bisecting line; to construct the triangle.

XII. QUESTION 623, *by Mr.* T. Moss.

If from the extremities of the diameter of a circle ever so many chords are drawn, two and two, intersecting each other in an ordinate perpendicular to that diameter, the chords joining the extremities of every corresponding two of them, being produced, will all intersect the said diameter produced in one point; query the demonstration?

XIII. QUESTION 624, *by Mr.* J. Powle.

Given $1^3.1^2 + 2^3.3^2 + 3^3.5^2 + 4^3.7^2$ &c. .... to x terms $= 845815702$, to find x;

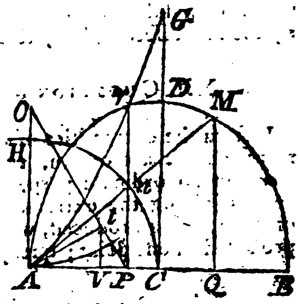
Where x is the age of my partner for life:
A fond mother, true friend, and good wife.

XIV. QUESTION 625, *by* Plus Minus.

Just after reading in the book of Genesis of the tower of Babel, whose top was to reach up to heaven, I fell asleep, and dreamed, that I was got upon the top of this tower. When St. Paul's clock, in London, began to strike 12, I saw my own shadow fall on that church, and some time after saw it quit the earth's surface, just at the spot where a man seemed to be very hard at work. I called out (as loud as I could bawl) surely, friend, tis time to leave off; I think so too, says he, for our clock wants but a quarter of 7. This adventure happened on the first of May, as I knew by the garlands on the May-polls all about the country, and so the sun's declination was 15° 19′. Hence I would know the height of the tower, the place where it stood, and the habitation of my friend aforesaid.

XV. QUESTION 626, *by Mr.* Tho. Allen.

Let ADB be a given semi-ellipse, CnH a circular arc described on the center A with the rad. $AC =$ the semi-conjugate, and suppose the right line AnM cutting the circle in n and the ellipse in M, to revolve about the point A, until it coincides with AB. Let, also, QM, Pn be drawn perpendicular to AB in the points Q and P, and on Pn (produced when necessary) let Pr be taken always $= AQ$; then, whilst the point M describes the semi-elliptic arc ADB, the point r will describe the curve line ArG. It is required to determine the content of the solid generated by the revolution of the said curve about its axis AC.



The PRIZE QUESTION, *by the Rev. Mr.* Wildbore.

A lady sent her servant to a well famous for its excellent water, who filled the bucket, and drew it up very uniformly and steadily; but when the bottom arrived at land, found that the last drop of water was running out of it; for some unlucky boys had bored an hole therein 1·066105 inch diameter. Hence the time it was in drawing up is required, the diameter and depth of the bucket being each of them one, and the depth of the well to the surface of the water 29 feet.

1771.

Questions answered.

I. QUESTION 612 *answered by Mr.* Wm. Wilkin.

PUT $a = 13900$ and $b = 654508006000$; then, from the 1st equation, $y = \overline{a + x^3 - x^2}|^{\frac{1}{2}}$, which value substituted in the 2d gives $x^8 + \overline{a + x^3 - x^2}|^{\frac{3}{2}} = b + \overline{a + x^3 - x^2}|^2$: From whence x is found $= 30$, her age in years, and thence $y = 200$, her fortune in pounds.

In this manner, nearly, it is also answered by Mess. *J. Bartlet*, *G. Coughron*, *Curiosus*, *R. Dening*, *J. Hellings*, *J. Hooper*, jun. *Pamphagus*, *E. Parnel*, *E. Reed*, *Wm. Reynolds*, *Alex. Rowe*, *T. Sadler* (the proposer), *J. Shadgett*, *T. Smith*, *Wm. Spicer*, *Mich. Taylor*, and *J. Young*, who are all greatly surprised that Nelly should think herself too young to marry!

II. QUESTION 613 *answered by Mr.* Wm. Spicer.

Let $x =$ the semi-transverse; then will $x\sqrt{2} =$ the length of the greatest inscribed parallelogram, and (by art. 71 of Simpson's Flux.) $\frac{xx}{x-4} =$ rad. of curvature at the extremity of the conjugate diam. whence (putting $a = 3{\cdot}28426$) $\frac{xx}{x-4} = x\sqrt{2} - a$, per quest. and $x = 20$: Consequently the transverse diameter $= 40$, the conjugate $= 32$, and the rad. of curvature $= 25$.

Much after this manner it is likewise answered by Mess. *J. Addison*, *J. Chipchase*, *G. Coughron*, *J. Hellings*, *Pamphagus*, *E. Parnel*, *E. Reed*, *Tho. Robinson*, *Alex. Rowe*, *P. Sharp* (the proposer), *W. Wilkin*, and *Ja. Young*.

III. QUESTION 614 *answered by Mr.* Geo. Coughron.

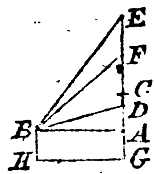
Put $\overline{x^{1.5}+y^{1.5}+z^{1.5}}|^3 = u$, $\overline{x^3+y^3}|^2 = v$, and $\overline{x^3+y^3+z^3}|^3 = w$; then the given equations will become $u+v=a$, $uw=b$, and $v+w=c$: Take the last equation from the first, and $u-w=a-c$, from which and the 2d equation u is found $=\frac{1}{2}\times\sqrt{\overline{a-c}|^2+4b}+a-c$, $w=\frac{1}{2}\times\sqrt{\overline{a-c}|^2+4b}-a+c$, and from thence v ($=a-u$) $=\frac{1}{2}\times\overline{a+c-\sqrt{\overline{a-c}|^2+4b}}$; whence $x^{1.5}+y^{1.5}+z^{1.5}$ ($=u^{\frac{1}{3}}$), x^3+y^3 ($=v^{\frac{1}{2}}$) and $x^3+y^3+z^3$ ($=w^{\frac{1}{3}}$) all become known, and from the 2d and 3d of these equations $z=\overline{w^{\frac{1}{3}}-v^{\frac{1}{2}}}|^{\frac{1}{3}}$; whence $x^{1.5}+y^{1.5}$ ($=u^{\frac{1}{3}}-z^{1.5}$) also becomes known, which put $=r$ and x^3+y^3 ($=v^{\frac{1}{2}}$) $=2s$, and then (by quest. 102 p. 63 of Simp. Select Exerc.) x^3-y^3 will easily be found $=2\times\overline{\overline{\frac{1}{10}rs^{-1}+\frac{4}{3}s^4}|^{\frac{1}{2}}-s^2}|^{\frac{1}{2}}$ (supposing x to be greater than y) which put $=2d$, and then x and $y=\overline{s+d}|^{\frac{1}{3}}$ and $\overline{s-d}|^{\frac{1}{3}}$ respectively.

In this manner, exceeding nearly, it is also answered by the Rev. Mr. *Crakelt*, Mr. *Gawith* (the proposer), Mr. *Tho. Robinson*, of *Biddick*, and Mr. *Wm. Spicer*.

Mr. *Mich. Taylor* has also sent a solution to it.

IV. QUESTION 615 *answered by the Rev. Mr.* Crakelt.

CONSTRUC. Take $AB=32$ feet (the dist. of the observer from the column), and in an indefinite perpendicular thereto, at the point A, the dist. $AG=5$ feet (the length of the urn): Bisect AC in D, and take $DE=DB$; also take $EF=AC$ and $AG=5\frac{1}{2}$ feet, the height of the observer; then GF will be the required height of the column.



DEMONST. Since $BA^2=BD^2-AD^2=\overline{BD+AD}\times\overline{BD-AD}=\overline{ED+DA}\times\overline{ED-DC}$ (by construc.) $=EA\times EC=EA\times AF$, a circle

circle described through the three points F, E, B will have the line AB for a tangent, and by Simp. Algebra, p. 358, 2d edit. the line FE will appear under the greatest angle to an eye at B, and consequently $FA + AG$ or FG will be the perpendicular height of the column.*

Upon this principle it is also answered by Mess. *J. Chipchase* (the proposer), *Gemini*, *J. Hellings*, *E. Reed*, *Wm. Reynolds*, *Tho. Robinson*, *Alex. Rowe*, *J. Spencer* (Discip. *T. Allen*), *W. Spicer*, and the Rev. Mr. *Wildbore*.

Mr. *Geo. Coughron* gives an ingenious construction to it, founded on art. 55 of Rowe's Flux. 3d edit. and from thence calculates the height of the column required = 35·09751 &c. feet.

V. QUESTION 616 *answered by Mr.* J. Spencer, *Discip.* T. Allen.

Put z and a for the hyp. logs. of x and b respectively; and then $z\dot{x}^2 = \frac{a\dot{y}^{\frac{1}{2}}}{x}$, and $\dot{y} = a^{-2}z^2x^6$; whence $y\dot{x} = a^{-2}z^2x^6\dot{x}$, and the fluent (by art. 341 of Simp. Flux.) is $\frac{1}{7}a^{-2}x^7 \times z^2 - \frac{2}{7}z + \frac{2}{49}$ = the area of the curve required.

And in this manner the answer is also given by Mess. *J. Addison*, *G. Coughron*, the Rev. Mr. *Crakelt*, Mess. *E. Parnel*, *Tho. Robinson*, *Alex. Rowe*, *Paul Sharp*, *Wm. Spicer* (the proposer), *Wm. Wilkin*, and the Rev. Mr. *Wildbore*.

VI. QUESTION 617 *answered by Mr.* Geo. Coughron.

Per quest. $\frac{\overline{dx - x^4}|^2 \times y\dot{y}}{\overline{a - bx^2}|^2 \times \dot{x}} = \frac{y\dot{x}}{\dot{y}}$; $\because \dot{y} = \frac{a\dot{x} - bx^2\dot{x}}{dx - x^4} = \frac{a\dot{x}}{dx} + \frac{a}{d} \times \frac{x^2\dot{x}}{d - x^3} - \frac{bx\dot{x}}{d - x^3}$, and, taking the correct fluent, $y =$

* See Mr. Crakelt's remark on this question, at the bottom of p. 37, Diary 1774.

$y = \frac{a}{d} \times$ hyp. log. of $x + \frac{a}{3d} \times$ hyp. log. of $\frac{d}{d - x^3} + \frac{b}{3r}$ $\times \overline{N\sqrt{3} + M - M'}$ (where $N =$ the circular arc whose sine is $\frac{x\sqrt{\frac{3}{4}}}{\sqrt{rr + rx + xx}}$ to rad. 1, $r = d^{\frac{1}{3}}$ and $M, M' =$ the hyp. logs. of $\frac{r - x}{r}$ and $\frac{\sqrt{rr + rx + xx}}{r}$ respectively, see arts. 329 and 330 of Simpson's Flux. 2d edit.); whence the value of y may be determined let that of x be what it will.

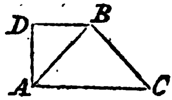
The Rev. Mr. *Wildbore* solves it thus, viz. $\dot{y} = \dot{x} \cdot \frac{a - bx^3}{dx - x^4}$ (per question) $= \frac{a\dot{x}}{dx - x^4} - \frac{bx\dot{x}}{d - x^3} = \frac{a\dot{x}}{dx - x^4} - b\dot{x} \times :$ $\frac{-\frac{1}{3}x + \frac{1}{3}d^{\frac{1}{3}}}{d^{\frac{1}{3}}x^2 + d^{\frac{2}{3}}x + d} + \frac{\frac{1}{3}}{d^{\frac{1}{3}}x - d^{\frac{2}{3}}} = \frac{a}{d}\,\frac{\dot{x} - 4d^{-1}x^3\dot{x}}{x - x^4 d^{-1}} +$ $\frac{a}{d} \times \frac{4d^{-1}x^2\dot{x}}{1 - x^3 d^{-1}} + \frac{b\dot{x}}{3d^{\frac{1}{3}} \cdot \overline{d^{\frac{1}{3}} - x}} + \frac{b}{3d^{\frac{1}{3}}} \times \frac{t\dot{t}}{cc + tt} - \frac{1}{2}b$ $\times \frac{\dot{t}}{cc + tt}$ (putting $cc = \frac{3}{4}d^{\frac{2}{3}}$ and $t = x + \frac{1}{2}d^{\frac{1}{3}}$), the correct fluent of which is $y = \frac{a}{d} \times$ h. l. $\overline{x - \frac{x^4}{d}} - \frac{4}{3} \times$ h. l. $\overline{1 - \frac{x^3}{d}}$ $- \frac{b}{3d^{\frac{1}{3}}} \times$ h. l. $\overline{d^{\frac{1}{3}} - x} + \frac{b}{6d^{\frac{1}{3}}} \times$ h. l. $\overline{cc + tt} - \frac{b}{2c} \times$ circular arc whose tang. is $\frac{t}{c}$ to rad. $1 + \frac{b}{2c} \times$ circular arc tang. $\frac{1}{\sqrt{3}}$ and rad. 1; which, placing s for x, gives what is required.

The Rev. Mr. *Crakelt* and Mess. *Alex. Rowe* and *S. Vince* (the proposer) have likewise answered it.

VII. QUES-

VII. QUESTION 618 *answered by Mr.* J. Addison (*the Proposer*).

Let *ABC* represent a section of the prism parallel to its ends, and *BD* and *AD* be parallel and perpendicular to *AC* respectively; then, the number of forcing particles of the fluid acting upon the side *AB*, being as *BD*, and their force as the square of the sine of the angle of incidence *BAD* (45°), by mechanics, the whole force upon the



two sides thereof, *AB*, *BC*, will manifestly be = half that upon its base *AC* (supposing *AC*, directly, to face the stream): But, putting v = the velocity of the water (per second) reckoned in feet, and $s = 16\frac{1}{12}$ feet, the force against *AC* is $= \frac{144vv}{2s} \times 62\cdot5$ lb. Av. (per prop. 107 of Emerson's Mechanics, 1st edit. a cubic foot of water weighing $62\frac{1}{2}$ lb. Av.); therefore $\frac{144vv}{4s} \times 62\cdot5 =$ the force against *ABC* = 387072 lb. Av. per quest. Whence $v = 52\cdot6008$ &c. feet, the velocity of the stream per second, which was required.

Upon the same principle, and in the very same manner nearly, the answer is also given by the Rev. Mr. *Crakelt* and Mess. *J. Bartlett*, *J. Chipchase*, *G. Coughron*, *J. Spencer* (Discip. *T. Allen*), and *S. Vince*.

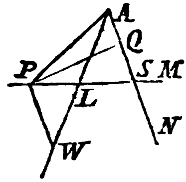
The Rev. Mr. *Wildbore* has also favoured us with a curious solution to this quest. founded on the principle, that the effect which the pressure of the water has to move the prism in the direction of the stream shall be equal to that of a weight of 387072 lb. Av. from which, and the consideration of the action of the back water, he proves that the effect of the pressure against low-arched bridges, in time of floods, such as those made by Mr. *Brindley* for conveying his aqueducts over rivers, is not so great as is commonly imagined, not varying according to the depth of the water above, but nearly as the square of the difference of the depth above and below the bridge: Also, that the arch ought in this case to be a parabola, that so its width may be every where proportional to the quantity of water which is to be discharged through it; for the water runs with the greatest velocity at the bottom, and when the upper part of the arch is choaked up with hay, ice, &c. if

 it

it be thus formed it will discharge the most water when and where there is the greatest necessity for it, &c.——but we cannot spare room for the solution at present.

VIII. QUEST. 619 *answered by the Rev. Mr.* Wildbore.

If the conchoid ruler revolve about the pole P upon the given line ASN with the given line or dist. $SM = M$, till $PM = AL + AS$, it will give the position, or positions, of PM required, with much more ease than any solid construction derived from the following



ANALYSIS. Join A, P, and draw PQ perpendicular and PW parallel to AS. Put $AP = a$, $AW = b$, $AQ = g$, $PW = d$, $As = x$, and $AL = y$; then, per sim. triangles, $b - y$ $(AW - AL)$: d (PW) :: y (AL) : x (AS), whence $x = \frac{dy}{b-y}$. Also, by Euc. 2. 13, and the conditions of the question, $x + y - M = \sqrt{aa + xx - 2gx}$; reduced, $y^3 - \overline{2d + 2M + b}\,.y^2 + \overline{2Md + 2Mb + M^2 - a^2 - 2gd}\,.y + \overline{a^2 - M^2}\,.b = 0$: Which equation not being of the kind that admits of reduction by any of the methods of divisors, except in particular cases, it should seem as if the problem could not generally be reduced to a plane one.—— But the roots of the equation may be found with great facility after the manner of Newton and Leibnitz by means of the above mechanism.

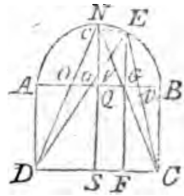
Mr. *G. Coughron*'s solution is not essentially different from this.——He also shews, by way of corollary, that where the point P coincides with L, the cubic reduces to a simple equation.

It is likewise answered by the Rev. Mr. *Crakelt* and Mess. *J. Hellings* and *Alex. Rowe*.

IX. QUES

IX. QUESTION 620 *answered by the Rev. Mr.* Crakelt.

CONSTRUC. From the vertices, D, C, of the given perpendiculars, AD, BC, to the middle N of the semicircle ANB draw the right lines DN, CN, and the part, OV, which they intercept on the diameter AB will be the maximum required.



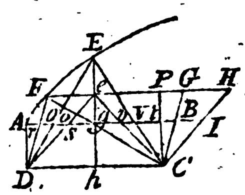
DEMONSTRA. For to any other point, E, of the said semicircle draw the right lines DoE, CvE, and from N and E let fall the perpendiculars NQS and EGF upon DC, and draw $Ec \parallel BA$; then, by similarity of triangles, $NS : NQ :: DC : OV$, and $EG : EF :: ov : DC$, or, by taking the rectangles of the corresponding terms, $NS \times EG = \overline{Nc + cS} \times cQ$: $NQ \times EF = \overline{Nc + cQ} \times cS :: ov : OV$: But $\overline{Nc + cQ} \times cS$ is greater than $\overline{Nc + cS} \times cQ$; wherefore OV will be greater than ov.

SCHOLIUM. Whatever the segment ANB, and the lengths of the perpendiculars AD, BC, are, the construction and demonstration will hold good.

The demonstration by Capt. *Edw. Williams* (the proposer) is also very curious, and purely geometrical.

The Rev. Mr. Wildbore's *Demonstration is thus, viz.*

At any given dist. whatever, draw $AB \parallel$ to DC and $Eh \perp$ to the same; then 'tis self-evident that the higher the point e or E is taken above h, the greater will ov or OV be; through e draw a right line parallel to DC, in which take any point F; join FD, FC, cutting AB in r and s: Compleat the parallelograms DG, DH; then, Fe being $= GH$, and PC ($\perp$ to FeH) common (to the Δs CGH, CFe, DFe) BI will be $= sv = ro$, and adding os (common), rs will evidently be $= ov$; therefore 'tis manifest that OV will be the greatest at the vertex, and the same at eqral heights above DC, let gh and the curve AFE be what they will.

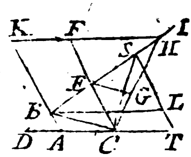


Mess.

Mess. *J. Chipchase*, *G. Coughron*, *J. Hellings*, *E. Parnel*, *T. Robinson*, *Alex. Rowe*, *J. Spencer* (Discip. *T. Allen*), *W. Spicer*, *S. Vince*, and *W. Wilkin* have likewise demonstrated it.

X. QUESTION 621 *answered by Mr.* Steph. Ogle.

CONSTRUC. Make the $\angle CDI =$ the given $\angle D$, and draw CE to terminate in DI so, as to be to DC in the given ratio of BA to DA; produce it till $CF =$ the given quantity, and through F draw $KH \parallel$ to DC meeting DI in H; join C, H, and draw EG to terminate therein so as to be $= EF$; then, drawing CB, $BA \parallel$ to EG, CE respectively, the thing will be done.

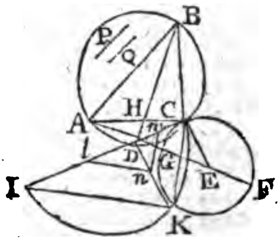


DEMONTST. Produce AB to meet KH in K; then, per sim. triangles, $EF : BK (:: HE : HB) :: EG : BC$, where the antecedents EF, EG being equal (by construction) the consequents BK, BC must be equal also: Consequently $AB + BC (= AB + BK) = CF$, the given quantity by construction.——The Rev. Mr. *Crakelt*'s construction is not materially different.

Elegant constructions to this problem have also been received from Mess. *G. Coughron*, *Plus Minus*, *T. Moss*, and the Rev. Mr. Wildbore (the proposer); and Mess. *T. Robinson*, *Alex. Rowe*, *W. Wilkin*, and *Ja. Young* answer it algebraically.

XI. QUESTION 622 *answered by the Rev. Mr.* Crakelt (*the Proposer*).

CONSTRUC. Let P and Q represent the sides of two squares equal in magnitude to the two given rectangles, and find a fourth proportional, DC, to the given bisecting line, P and Q; perpendicular to which erect $CE = \frac{1}{2}$ the said bisecting line, and therewith as rad. describing a circle, draw $DGEF$: In CD produced take $DI = DF$, and on CI as diam. describe the semicircle CKI; erect the per-



pendicular

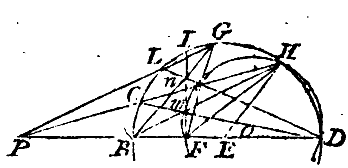
pendicular DK, and having taken thereon $Dn = Q$, draw nl, nm ∥ to KI, KC respectively; then with DC, DG, and Dm constitute the $\triangle\, DHC$, and in DH produced take $HB = GF$, and join the points B, C: Circumscribe the $\triangle\, BDC$ with a circle, and produce CH to meet it in A; then, joining the points A, B, ABC will be the required triangle.

Demonstra. By construc. $BD \times DH = DF \times DG = DC^2$; whence the $\triangle$s BDC and HDC are similar, and consequently $BD : DC :: BC : CH$: But, by reason of parallels, we also have $BD\ (ID) : DC :: ID : CH\ (Dm)$; whence $BC = Dl$, and $BC \times CH = Dl \times Dm =$ (since the $\angle\, lnm$ is right) $Dn^2 = Q^2$, by construction. The $\angle\, ABD = \angle\, ACD = \angle\, DBC$; whence BH ($= GF = 2CE =$ the given bisecting line) bisects the $\angle\, ABC$: Therefore $CB : HB :: DC\ (AD) : AH$, and $CH : HB :: DC : AB$, and consequently $CB \times CH : HB^2 :: DC^2 : AH \times AB$: But, by construction, $HB^2 : P^2 :: Q^2 : CD^2$; wherefore, *ex æquo perturbate*, $CB \times CH : P^2 :: Q^2 : AH \times AB$; but, from what has been already proved, $CB \times CH = Q^2$, and of consequence $AH \times AB = P^2$.

Ingenious constructions to this probem have likewise been received from Mr. *G. Coughron*, *Gemini*, and the Rev. Mr. *Wildbore*; and Mess. *Wm. Reynolds*, *T. Robinson*, *Alex. Rowe*, *Wm. Spicer*, *Mich. Taylor*, *Wm. Wilkin*, and *Ja. Young* have given neat algebraic solutions to it.

XII. Question 623 *answered by Mr.* T. Moss (*the Proposer*).

Demonstra. Let any two (unequal) chords BH, DC be drawn, intersecting each other in the point m; thro' which point draw the perpendicular (or semi-chord) FI; draw the radius EH, and also FH and DH: Then it is evident that the $\angle\, FHB$ is equal to $\angle\, BHP$, each being $=$



$\angle\, FDm$ ($= \angle\, BDC$). Moreover, the $\angle\, EBH + \angle\, FHB = \angle\, EFH$; but $\angle\, EBH$ is $= \angle\, EHB$: Therefore $\angle\, EHB + \angle\, BHP\ (FHB) = \angle\, EHP = \angle\, EFH$; whence the $\angle\, E$ being

being common, the $\triangle$s EHP and EHF are similar, and so $HE : EF :: PE : HE$, or $EB\ (HE) : EF :: PE : EB\ (HE)$, or (dividedly) $EB : EB - EF\ (FB) :: PE : PE - BE\ (BP)$, or, alternately, $EB : PE :: FB : BP$: Again, by division of ratios, $EB : PE - EB :: FB : BP - FB$, i. e. $EB : BP :: FB : BP - FB$; in which circumstance it is demonstrated (see theo. 15. p. 74. Simp. Geom. 1st edit.) that if any two right lines be drawn from the points F, P meeting any where in the periphery of the circle described with the rad. EB, they will be in the constant ratio of FB to BP, and consequently a right line (BG or BH) drawn from B to the point where the two said lines meet in the periphery will always bisect the angle formed by those lines. ——Therefore, through any other point n, in the perpend. FI, let now two chords be drawn from B and D to meet the periphery in G and L; then, having drawn GF and GL, it is evident from the former part of this demonstration, that the $\angle FGB\ (= FDn) = \angle BGL$, and consequently that (by the above-mentioned theorem) GL produced must necessarily meet DB produced, in the very same point P where HC produced met it.

The Rev. Mr. Wildbore's *Demonstration is as follows, viz.*

In the given ordinate FI take any point m, and draw DmC, BmH; and HCP meeting DB produced in P. Bisect Dm in O, and with OD rad. describe the circ. FmD, which, per Euc. 3. 31, will circumscribe the quadrilateral $FmHD$; then, per 1. 32. the $\angle HBE = \angle BPH + \angle BHP$, the $\angle HED = \angle EHP + \angle EPH$, and, per 3. 21, the $\angle CHB = \angle CDB\ (mDF) = mHF$; therefore the $\angle PHE = \angle EBH\ (BHE) + \angle PHB\ (CDB)$, the $\angle HPD = \angle HBE - \angle CDB$, and the $\angle EFH = \angle EBH + \angle mHF\ (mDB)$: Consequently the $\angle EFH = \angle PHE$, and the $\angle E$ being common, the $\triangle$s FEH, HEP (per 1. 32.) are similar, and (per 6. 4.) $FE : EH\ (= BE) :: EH : PE$; where 'tis manifest that FE and EH will continue the same wherever in the given ordinate FI the point m is taken, and consequently that their third proportional PE must continue the same likewise.

COR. 1. If, in like manner, a circle be described about the quadrilateral $BCmF$, it will appear that the $\angle FCm$ is $= \angle mBF = \angle HCm$; $\because CD$ bisects the $\angle FCH$: and it is shewn above that BH bisects the $\angle CHF$; $\because IF$ bisects the $\angle CFH$, and m is the cent. of the circle inscribed in the $\triangle CFH$. It is evident likewise that PI is a tangent at I.

COR.

COR. 2. Because $FE : BE = EH :: FH : PH :: BF : BP$, ∵ by conversion and division, $BP : BP - BF :: BE : BE - FE = BF$. Wherefore if, from any two given points P and F, two right lines in a given ratio are to be drawn, divide PF in that ratio in B, and take $BE : BF :: BP : BP - BF$; then two lines meeting any where in the circumference of the circle described with the rad. BE, will be in the required ratio, which is the property laid down at *P.* 336 of Simp. Alg. 3d edit. and applied to the solution of many difficult problems by that ingenious author.——And with the same ease may other curious and useful proportions be derived.

We are also favoured with demonstrations of this curious property by Mr. *J. Chipchase*, the Rev. Mr. *Crakelt*, Mr. *G. Coughron*, *Plus Minus*, and Mr. *Ja. Young*.

XIII. QUESTION 624 *answered by Mr.* Cha. Hutton.

Put s for the given sum of x terms of the series; then the $x+1$ term or $\dot{s}$ is $= \overline{x+1}|^3 . \overline{2x+1}|^2 =$ (putting $z = x+1$) $z^3 . \overline{2z-1}|^2 = 4z^5 - 4z^4 + z^3 = 4zz_1z_2z_3z_4 - \overline{40\dot{z} + 4} . zz_1z_2z_3 + \overline{100\dot{z}^2 + 24\dot{z} + 1} . zz_1z_2 - \overline{60\dot{z}^3 + 28\dot{z}^2 + 3\dot{z}} . zz_1 + \overline{4\dot{z}^4 + 4\dot{z}^3 + \dot{z}^2} . z$; and, by taking the integrals, &c. we then obtain $s = \frac{1}{60} \times : 40x^6 + 72x^5 - 5x^4 - 50x^3 - 5x^2 + 8x$, the sum of the series proposed ($\dot{z}$ being $= 1$ and $\dot{x} = z - 1$); from which the remaining part of the quest. might be easily determined, &c.

And by other different but ingenious methods the sum of the series is likewise determined by Mr. *G. Coughron*, the Rev. Mr. *Crakelt*, Mess. *J. Hellings*, *E. Reed*, *Alex. Rowe*, *W. Spicer*, *S. Vince*, and *W. Wilkin*: But the ingenious proposer having, by some mischance, given us a wrong number (845815702), none of them could find the right age, that sent by the proposer along with the quest. in the year 1767 being 48.

Answer to the same by the Rev. Mr. Wildbore.

The proposed series $1^3 . 1^2 + 2^3 . 3^2 + 3^3 . 5^2 + 4^3 . 7^2$ .... to x terms is evidently $= 1^3 . \overline{2.1-1}|^2 + 2^3 . \overline{2.2-1}|^2$

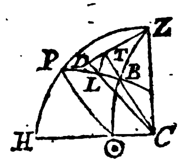
+

$+ 3^3 \cdot \overline{2 \cdot 3 - 1}|^2 + 4^3 \cdot \overline{2 \cdot 4 - 1}|^2 \ldots\ldots$ to x terms $= 4 \times \overline{1^5 + 2^5 + 3^5 \ldots\ldots x^5} - 4 \times \overline{1^4 + 2^4 + 3^4 \ldots\ldots x^4} + 1^3 + 2^3 + 3^3 \ldots\ldots x^3 =$ (by case 3d on p. 206 of Simp. Alg. 2d edit. &c.) $\frac{2}{3}x^6 + \frac{6}{5}x^5 - \frac{1}{12}x^4 - \frac{5}{6}x^3 - \frac{1}{12}x^2 + \frac{2}{15}x$, which suppose $= 8458957024$,* and then $x = 48$, the truly valuable lady's age required by (the proposer) Mr. *Powle*.

* NOTE, This truly ingenious gentleman having discovered this number so as to give the value of x a whole number, it happens luckily to be the very number that gives $x =$ the age meant by the proposer, &c.

XIV. QUESTION 625 *answered by Mr.* W. Wales.

From the data of this quest. it appears that the tower stood to the south of London, and under the same meridian, and that the sun set at 6 h. 45′ by the time at the labourer's meridian, which let the arc ZPH represent, where Z is the zenith, or geographical situation, P the pole, and ☉ the sun in the horizon HC. Draw P☉, the sun's polar distance, the vertical Z☉ and PLB for the meridian of London,



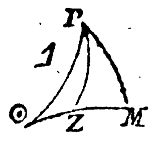
and the point B where these two last intersect will be the situation of the tower, and L that of London. Now, in the right-angled spherical $\triangle PH$☉, having given P☉ and the $\angle P$, HP, the labourer's latitude, is readily found $= 35°\ 27'\ 46''$ north; and if ZT be a tangent to the arc ZB in Z, and meeting CB produced in T, TB will be the height of the tower. Join T, L, and from T, on CL produced, let fall the perpendicular TD, and the $\angle DLT$ will be $=$ the ☉'s zenith dist. at London on the given day, whose sine and cosine denote by r and p, the sine and cosine of ZP by s and c, those of LP by a and b, the cosine of $\angle PZB$ ($= H$☉) by v, and DL by x. Then $1 + x = DC$, and $\frac{rx}{p} = TD$; also (Euc. 47. 1.) $\sqrt{DC^2 + DT^2} = TC$: Moreover $TC : 1 :: TD :$ sine $\angle DCT =$ sine LB, whose cosine also is thence known; and (Simp. Trig. p. 55) cosine $PL \times$ cos. $LB -$ sine $PL \times$ sine $LB =$ cos. PB. Again, since TC is the secant of ZB; its cosine $= \frac{1}{TC}$, from whence its sine also is determinable, and (per Anderson's theo.) sine $ZB \times$ sine $ZP \times$ cos. $\angle PZB +$ cos. $ZB \times$ cos. $ZP =$

cos.

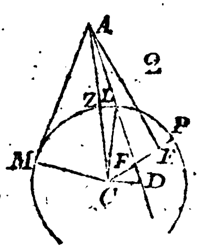
cos. PB. These two values for the cos. PB expressed in symbols, according to the preceding notation, and equated, give $b+\frac{x}{p}\times\overline{pb-ar}=sv\sqrt{2x+\frac{xx}{pp}+c}$; but $pb-ar$ and sv are each = the sine of the ☉'s declination, for which putting d, and $b-c=m$, the equation will be $\frac{m}{d}+\frac{x}{p}=\sqrt{2x+\frac{xx}{pp}}$; whence $x=\frac{pm^2}{2d\times dp-m}=5{\cdot}9433341$. Hence TC and the $\angle TCD$ are readily found $=8{\cdot}1928755$, and $32^\circ\ 4'$; and thence the tower's height $=7{\cdot}1928755$ radii of the earth, and its latitude $19^\circ\ 27'$ north; and lastly, the labourer's longitude $=95^\circ\ 19'$ east.

Answer to the same by the ingenious Proposer, Plus Minus.

In fig. 1. P represents the north pole, ☉ that place of the earth to which the sun was vertical when the shadow quitted its surface, Z the place of the tower, and M that of the man. It is evident that at the time aforesaid the sun must be in the horizon of M, and therefore ☉$M=90^\circ$; ☉P is the comp. of declin. of the sun, and the $\angle$☉$PM=101^\circ\ 15'$: Hence the man's lat. will be found $=35^\circ\ 17'\ 46\frac{1}{2}''$ north.



Find the $\angle P$☉M and call its cos. q, and let the sine and cos. of ☉P be s and d respectively. Then, to find expressions for the sine and cosine ☉Z and the cosine of ZP, consider fig. 2d, where A represents the top of the tower, Z its base, L London, P the north pole, C the earth's center, and M the man's place (which is here put in the meridian of L to prevent confusion in the diagram): Draw AL cutting the axis CP in F, and let fall CD, $AE\perp$ to AL and CP, and join CL, CM, which call r, and AL, z; moreover, let the sine and cosine of the sun's merid. alt. at London be m and n ($=LD$ and CD). Then $AC=\sqrt{zz+2mz+rr}$, and the sine of the $\angle CAM$ (= ☉Z) $=rr\div\sqrt{zz+2mz+rr}$, and its cosine $=r\sqrt{zz+2mz}\div\sqrt{zz+2mz+rr}$. Now the $\angle$s FCD, FAE being equal



to the sun's declination, $s : d :: n\,(CD) : \frac{dn}{s}\,(FD)\; s : r :: n : \frac{rn}{s} = CF,\; r : d :: z + m - \frac{dn}{s}\,(AF) : \overline{dsz + dsm - ddn} \div rs\,(FE)$, and $FE + FC$ (or CE) $= \overline{dz + dm + sn} \div r$ (putting ss for $rr - dd$): Hence the cosine of $ZP = \overline{dz + dm + sn} \div \sqrt{zz + 2mz + rr}$, and (by means of an excellent theor. in the Lady's Diary 1732, pa. 8.) $qrrs \div \sqrt{zz + 2mz + rr} + drr\sqrt{zz + 2mz} \div \sqrt{zz + 2mz + rr} = rr \times \overline{dz + dm + sn} \div \sqrt{zz + 2mz + rr}$; whence $x = \overline{dm + sn - sq}|^2 \div 2ds \times \overline{q - n}$.——Hence the rest will be found as follows, viz. The height of the tower $AZ = 7{\cdot}370827$ radii of the earth $= 29364{\cdot}296$ miles, its latitude $= 19^\circ\ 22'$ N. and the man's longitude from London $= 95^\circ\ 26'\ 30''$ E. nearly.

This question is also answered ingeniously by the Rev. Mess. *Crakelt* and *Wildbore*.

XV. QUESTION 626 *answered by Mr.* Geo. Coughron.

Put $AP = x$, $Pr\,(AQ) = y$, $AC = a$, $CD = b$, and $3{\cdot}1416 = p$ (vid. last year's fig.); then will $Pn^2 = \overline{aa - xx}$, and, per prop. of the ellipsis, $QM^2 = b^2a^{-2} \times \overline{2ay - yy}$: Whence, by means of the sim. Δs APn, AQM, will be found $y = 2abbxx \div \overline{a^4 + ccxx}$ the equation of the curve, cc being $= bb - aa$; consequently $pyy\dot{x}$ (the flux. of the required solidity) $= 4paab^4x^4\dot{x} \div \overline{a^4 + ccxx}|^2$, (and taking the flu.) the solidity itself, when $x = a$, will be either $2pa^3b^4c^{-5} \times \overline{\overline{3aac + 2c^3} \times \overline{aa + cc}|^{-1} - 3a \times \text{cir. arc whose tan. is } \frac{c}{a} \text{ and rad. } 1}$, or $2pa^3b^4c^{-5} \times \overline{\overline{3aac - 2c^3} \times \overline{aa - cc}|^{-1} - \frac{3}{2}a \times \text{h. log. of } \frac{a+c}{a-c}}$, according as b or a is the semi-transverse, &c.

And in this manner, nearly, the answer is also given by Mr. *J. Addison*, the Rev. Mr. *Crakelt*, Mess. *J. Hellings*, *C. Hutton*, *E. Parnel*, and *S. Vince*.

Answer

Answer to the same by the Rev. Mr. Wildbore.

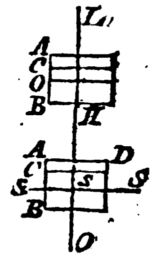
The figure being drawn as directed in the quest. erect the $\perp AO = AC^2 + \sqrt{\overline{DC^2 - AC^2}}$, with which, as rad. and cent. O, describe the arc of a circle, and join O, P cutting it in s; let fall At, $tV \perp$ upon OP and AC respectively, then will AV (by sim. triangles) $= AP \cdot AO^2 \cdot OP^{-2}$, which, because $AO^2 + AP^2 = OP^2$, is evidently the fluent of $\dot{AP} \cdot AO^2 \cdot \overline{AO^2 + AP^2 - 2AP^2} \cdot OP^{-4}$. Take any where a right line $M = 2AC \cdot DC^2 \cdot \overline{DC^2 - AC^2}|^{-1} = 2AO^2 \cdot DC^2 \cdot AC^{-3}$; then, by conics, &c. $AM^2 = 2CD^2 \cdot AQ \cdot AC^{-1} - AQ^2 \cdot AC^2 \cdot AO^{-2}$, and, by means of similar triangles, $M : AQ :: AO^2 + AP^2 : AP^2$; $\therefore AQ = Pr = M \cdot AP^2 \cdot OP^{-2}$, $Pr^2 (= M^2 \cdot AP^4 \cdot OP^{-4}) = M^2 \times \overline{\overline{AO^2 + AP^2}|^2 - \frac{1}{2}AO^2 \cdot \overline{AO^2 + AP^2} + \frac{1}{2}AO^2 \cdot \overline{AO^2 + AP^2 - 2AP^2}} \times \overline{AO^2 + AP^2}|^{-2}$ and the solidity generated round AP, or the fluent of $p \cdot rP^2 \cdot \dot{AP}$, is $p \cdot M^2 \times AP - \frac{1}{2}$ the arc $As + \frac{1}{2}AV$, which was required.

COROLLARY. $Pr (= M \cdot AP^2 \cdot OP^{-2}$ (being $= M \times \overline{1 - AO^2 \cdot OP^{-2}}$, the flu. of $Pr \cdot \dot{AP}$ is $M \cdot AP -$ the arc As $=$ the area of the curve ArP.

Plus Minus puts $AC = c$, $CD = t$, $AP = z$, and $Pr = v$, and then finds the equation of the curve ArG to be $v = 2cttzz \times \overline{c^4 \pm mmzz}|^{-1}$ (see fig. in the last year's Diary), where $mm = tt - cc$, and is affirmative or negative according as CD is the longer or shorter semi-axis. In the former case, the curve ArG is the conchoidal hyperbolism of an ellipsis with a diameter without a center, and that diameter is a line perpendicular to AC in the point A, and its asymptote is parallel to AC; and, in the latter case, the curve is the hyperbolism of an hyperbola with a diameter, its two parallel asymptotes being perpendicular to AC and equidistant from A, and the 3d parallel to AC, which is the tangent at the vertex of the axis, viz. A.—If $tt = cc$, or $m = 0$, i.e. if the semi-ellipsis AMD becomes a semi-circle, the curve degenerates into a common parabola, and AC is still a tang. at A the vertex of the axis. The fluxion of the solid required is $4pc^2t^4z^4\dot{z} \times \overline{c^4 \pm m^2z^2}|^{-2}$, and the fluent, or solid itself, is easily found from Emerson's forms, &c.

The PRIZE QUEST. *answered by the Rev. Mr.* Wildbore (*the Proposer*).

Suppose SS to be the surface of the water, L the land, O the bottom of the bucket when filled with water, B the bottom of it when the part AS is drawn out of the water, C the surface of the water in the bucket in this position, $LS = 348$ inches $= n$, $AB = SO = AD = 12 = a$, $1{\cdot}066105 = c$, $193 = d$, $s =$ the velocity per second with which the bucket is drawn up, $AS = BO = r$, and $AC = x$. Then $\sqrt{d} : 2d :: \sqrt{\frac{1}{2}CS} : d^{\frac{1}{2}}\sqrt{2r-2x} =$ the velocity per second of descent of a column of water whose height is CB and immersed part SB, owing to its own pressure, which increased by s gives $s + d^{\frac{1}{2}}\sqrt{2r-2x} =$ the real velocity per second with which the water runs out of the orifice: Wherefore $c^2a^{-2} \times \overline{s + d^{\frac{1}{2}}\sqrt{2r-2x}} =$ the relative velocity per second with which the surface of the water descends in the bucket at C, and $s - c^2a^{-2} \times \overline{s + d^{\frac{1}{2}}\sqrt{2r-2x}} =$ the absolute velocity per second, with which the surface of the water ascends in the well. Whence $\frac{aa \cdot \overline{\dot{r} - \dot{x}}}{\overline{aa - cc.s} - c^2 d^{\frac{1}{2}}\sqrt{2r-2x}} =$ the flux. of the time of ascent from S to C or from O to B; the fluent of which corrected by taking r and x each $= 0$, will, when $AS = BO = r = a$ (putting $\gamma = \frac{aa - cc.s}{cc\sqrt{d}}$), give $\frac{aa}{cc\sqrt{d}} \times \gamma \times$ hyp. log. of $\frac{\gamma}{\gamma - \sqrt{2a-2x}} - \sqrt{2a-2x} = \frac{n}{s}$, the time in which the bucket ascends out of the water.



——When it has ascended to H, suppose O the surface of the water therein, $b = a - x = CB$, and $y = CO$; then, in the same manner as before, will $\frac{cc}{aa} \times \overline{s + d^{\frac{1}{2}}\sqrt{2b-2y}} =$ the relative velocity per second with which the surface of the water descends in the bucket at O, and the fluxion of the

time of ascent from S to H = that of descent in the bucket between those two positions $= \frac{aay}{cc \times s + d^{\frac{1}{2}} \sqrt{2b - 2y}}$; the fluent of which, when $y = b = a - x$, or when H arrives at L (putting $\frac{s}{\sqrt{d}} = e$), is $\frac{aa}{cc\sqrt{d}} \times :$ $\sqrt{2a - 2x} + e$. hyp. log. of $\frac{e}{e + \sqrt{2a - 2x}} = \frac{n}{s}$ = the time of ascent of the bottom of the bucket from the surface of the water till it arrived at land: From which equation and that above x is found $= .4557$, $s = 12$, and $\frac{n + a}{s} =$ 30″ the time required.

COROLLARY. $\frac{AD^2 \sqrt{2AB}}{.088842|^2 \sqrt{16.0833}} = 44.67768''$ is the time in which the bucket would be emptied at rest.

SCHOLIUM. From the same principles may one part of the theory of the curious wheel for raising water upon the river Limat at Zurich in Switzerland be determined.*

Ingenious.

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* This is the solution in which was proposed that new principle of effluent water, which occasioned that long altercation on the subject, in our Miscellany, by two learned and ingenious gentlemen. This dispute was carried on, by both parties, with that true gentlemen-like candour, openness, and modesty, which has done them much honour. And of their several ingenious arguments, every reader is left to avail himself, and adopt that principle which to him seems to be the true one.

The above solution is given on the principle of the velocity of the effluent stream being increased by that of the vessel itself. And we shall here subjoin the solution on the contrary principle; that every person may take which he pleases.

Let a, c, d, n, and s denote the same quantities as in the above original solution; also x = any variable alt. of the surface of the water in the bucket above that in the well, while the bucket is ascending out of the water; and t the corresponding time. Then $\sqrt{2dx}$ = the velocity of the issuing stream, and $\because c^2 a^{-2} \sqrt{2dx}$ = the velocity of the descending surface in the bucket; consequently $s - c^2 a^{-2} \sqrt{2dx}$ = velocity per sec. with which the surface of the

Ingenious answers to this question have also been received from Mess. *J. Addison*, *T. Allen*, *T. Barker*, *G. Coughron*, the Rev. Mr. *Crakelt*, *C. Hutton*, and *Sam. Vince*; but the prize of 12 Diaries for the solution thereof is fallen to the lot of the Rev. Mr. *Wildbore* (the proposer) and that of 8 to Mr. *Hutton*.

The Eclipses calculated for 1771.

There will be four eclipses this year, two of the sun, and two of the moon, according to the following order.

The first is a lunar eclipse, on Monday the 29th day of April; Beginning 0 h. 58 m. Middle 2 h. 4½ m. End 3 h. 11 m. and digits eclipsed 4° 18'.

The

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the water in the bucket ascends above that in the well; ∵ $s - c^2 a^{-2}\sqrt{2dx} : 1^2 :: \dot{x} : \dot{x} \div \overline{s - c^2 a^{-2}\sqrt{2dx}} = \dot{t}$; hence t is $= \frac{-a^2}{dc^2}\sqrt{2dx} + \frac{a^4 s}{c^4 d} \times$ hyp. log. $\frac{s}{s - c^2 a^{-2}\sqrt{2dx}}$ which by the question will be $= \frac{a}{s}$ when the bottom of the bucket arrives at the surface of the water in the well, or when the bucket is just totally emerged from the water in the well.

Now the remaining part of the time, or that in which x depth of water will issue from the bucket, is $\frac{a^2}{dc^2}\sqrt{2dx}$ by prob. 2 art. 2 Miscel. and this must be $= \frac{n}{s}$ by the question. Adding now this to the former time, we have $\frac{a^4 s}{c^4 d} \times$ h. l. $\frac{s}{s - c^2 a^{-2}\sqrt{2dx}} = \frac{a+n}{s}$ the whole time of emptying. In this equation write $\frac{c^2 dn}{a^2 s}$ for $\sqrt{2dx}$ (its value by the equation $\frac{a^2}{c^2 d}\sqrt{2dx} = \frac{n}{s}$), &c. and we have $\frac{a+n}{a^4} \times c^4 d = s^2 \times$ hyp. log. $\frac{s^2}{s^2 - c^4 dna^{-4}}$; or, in numbers, $1{\cdot}879796 = s^2 \times$ com. log. $\frac{s^2}{s^2 - 4{\cdot}1841096}$. From hence we find $s = 7{\cdot}9667$. And ∵ $\frac{a+n}{s} = 45{\cdot}188094$ the whole time required, on this principle.

The second is of the sun, on Tuesday the 14th day of May, about 8 m. after three o'clock in the afternoon. It is invisible in Europe, but may be seen in the Atlantic ocean.

The third is of the moon, on Wednesday the 23d day of October; part visible at London, the moon rising at 5 h. 5 m. in the afternoon. Eclipsed 3° 26′, and ends 5 h. 55¼ m.

The fourth and last is another invisible eclipse of the sun, on Wednesday the 6th day of November, about seven o'clock in the evening; but may be seen in North America.

Isaac Tarratt.

New Questions.

I. Question 627, *by Mr.* Tho. Sadler.

Young Hodge, a homely country swain,
Long courted Susan of the plain,
But never could a method find
To bring the fair one to be kind.
Her pride is center'd in herself;
She calls him clown and country elf,
And mimics fashion with an air:
For dressing few with her compare.
This charmer's name with ease you'll find
From what is underneath subjoin'd.*
O! tell the way to make her kind.

* Given $\left\{\begin{array}{ll} x + y^2 + z & = 201, \\ x^2 + yz & = 37, \\ z^2 + x^2 y & = 130; \end{array}\right\}$ To find x, y, and z; their values expressing the places of the letters in the alphabet that compose her name.

II. Question 628, *by Mr* Wm. Gawith.

Given $\overline{x^5+y^5+z^5}\big|^5 + \overline{u^5+z^5}\big|^5 = a$, $x^5+y^5+u^5+2z^5 = b$, $\overline{u+z}\big|^2 \times \overline{x^5+y^5+z^5} = c$, and $\overline{u^5+z^5} \times \overline{x+y}\big|^5 = d$, to find x, u, y, and z by quadratics.

III. Question 629, *by Mr.* G. Cetii.

If any plane triangle be circumscribed by a circle, and have another circle inscribed therein, and if from any one of the angular points a straight line be drawn to the center of the inscribed circle, and produced to meet the periphery

of

of the circumscribing one: Then will the sum of the sides which include the angle from whence the straight line is drawn, be to the said straight line, so produced, as the the third side of the triangle is to that part of the straight line which is produced beyond the center of the inscribed circle. It is required to demonstrate this geometrically.

IV. QUESTION 630, *by Mr.* R. Mayo.

To determine the locus of all the places on the earth which have the same altitude of the sun with any given place, at a given instant of time.

V. QUESTION 631, *by Mr.* Wm. Spicer.

A shell was discharged from a mortar, which in its flight just touched the top of a steeple, and in four seconds of time after fell at the distance of $3262\frac{6}{7}$ feet from the bottom of the steeple; From whence the report of its fall was heard at the mortar just 12 seconds after the explosion: Required the steeple's height.

VI. QUESTION 632, *by Mr* Paul Sharp.

Suppose a tub in the form of the frustum of a common parabolic conoid, whose top diameter $= 4$, bottom diameter $= 1\cdot52982$, and depth $= 3$ feet, to be filled with water: Required the time of its emptying through a circular hole of 1 inch diameter in its bottom.

VII. QUESTION 633, *by Mr.* T. Moss.

Having given the hypothenuse of a right-angled plane triangle, and the part of the leg intercepted between the right angle and a point where a perpendicular on the middle of the hypothenuse meets the said leg, to determine the triangle.

VIII. QUESTION 634, *by Mr.* S. Clark.

Required to draw a right line CB from a given point C between two right lines DE, DF given in position to terminate in one of them (DE) so, that drawing another right line BA from the point of termination (B), parallel to another right line DG given, likewise, in position to terminate in the other of them (DF), the difference of the lines, CB, BA, so drawn may be equal to a given right line (MN).

IX. QUES-

IX. Question 635, *by the Rev. Mr.* Crakelt.

Two of the sides of a plane triangle, together with the line drawn from the angle they include to the center of the inscribed circle being given, to determine the triangle by construction.

X. Question 636, *by Mr.* T. Moss.

To draw a right line through the given point E in the side BF of the given plane triangle BFC meeting BC in G and CF produced in H so, that the sum of the perpendiculars Gm, Hn drawn from the points of intersection G, H to the said side BF may be equal to a given right line PQ.

XI. Question 637, *by Mr.* J. Chipchase.

A gentleman has a direct south-reclining dial in his garden. Its style is a streight pin erected perpendicular to the plane of the dial, at the distance of 18 inches from its center, the shadow of which returns at a particular time of the day, for several days in summer. On the 21st of June the arch described by the returning shade contained $4^\circ\ 24'\ 40''$, and the height of the sun above the plane of the dial at 3 h. $27'\ 57''$ that afternoon, was = to his altitude above the horizon: Hence is required the latitude of the place and height of the pin that the top of its shade may shew the true hour of the day.

XII. Question 638, *by the Rev. Mr.* Wildbore.

What right-angled plane triangle is that, the numerical value of whose double area being subtracted from that of each of its sides severally shall leave three square numbers?

XIII. Question 639, *by* Plus Minus.

If any semidiameter of any conic ellipsis be lengthened in the ratio of the side of a square to its diagonal, and from the end of that line two tangents be drawn to the ellipsis, the points of contact will be the vertices of two conjugate diameters: And moreover, the line joining those vertices will be to the diameter which is parallel to it, in the same ratio of the side of a square to its diagonal; the demonstration is required.

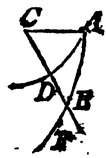
XIV. Ques-

XIV. Question 640, *by Mr.* Wm. Wales.

Having given the representation of any lesser circle of the sphere stereographically projected on a given plane, it is required to describe the representation of a great circle which touches the lesser one, and has a given arc intercepted between the point of contact and its intersection with some other given great circle.

XV. Question 641, *by Mr.* Tho. Allen.

Suppose ABF to be a certain spiral line described by a body in motion, beginning at A, and having the following property, viz. $x = \frac{N^{2z}+1}{2N^z}$, where $AC = CD = 1$, CB, any distance from the center C, $= x$, circ. arc $AD = z$, and N = the number whose hyp. log. $= 1$: Required the length of the said spiral, or the space described by the revolving body, when $x = 1000$.



The Prize Question, *by the Rev. Mr.* Wildbore.

In the latitude of London what is the declination of that star, whose azimuth increaseth the least possible in two hours after rising; — and on what day of the year does the sun's azimuth increase the most possible in two hours after rising?

1772.

Questions answered.

I. Question 627 *answered by Mr.* Hen. Clark.

It is evident from the 1st equation that y cannot be greater than 14 nor less than 12 (because x, y, and z are to be whole positive numbers by the nature of the quest. not exceeding 24); therefore supposing $y = 14$, xx from the 2d equation becomes $= 37 - 14z$, which value substituted in the

the 3d equation it becomes $zz - 196z = -338$; whence, by compleating the square, &c. $z = 2$, a whole number; consequently $x = 3$, $y = 14$, and $z = 2$, answering to the letters in the alphabet *COB*, the lady's name required.

It is also answered by Mess. *J. Bartlett*, *G. Cetii*, *J. Chipchase*, *S. Clarke*, *G. Coughron*, *R. Dening*, *Gemini*, *J. Haycock*, *J. Hellings*, *L. Ker*, *R. Mallock*, *T. Nicholson*, *Pamphagus*, *E. Reed*, *W. Reynolds*, *T. Robinson*, *A. Rowe*, *T. Sadler* (the proposer), *J. Shadgett*, *P. Sharp*, *W. Spicer*, and *W. Wells*.

Mr. *Thomas Adcock* concludes his solution with the following consolatory advice:

Friend Hodge, forbear to sigh or sob
For such a one as Susan COB;
Address some other charming she,
That is more complaisant and free:
This method take; perhaps you'll find
The haughty fair will grow more kind.

II. QUESTION 628 *answered by Mr.* G. Coughron.

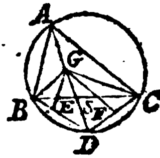
The two first given equations being the sum and the sum of the 5th powers of $x^5 + y^5 + z^5$ and $u^5 + z^5$, it will be found by quest. 48 pa. 105 of Simp. Alg. 2d edit. that $x^5 + y^5 + z^5$ and $u^5 + z^5 = \frac{1}{2}b \pm \sqrt{\sqrt{\frac{1}{5}ab^{-1} + \frac{1}{20}b^4} - \frac{1}{4}bb}$, the upper of the double signs giving the value of the one, and the lower the other of them; these values being put $=$ m and n respectively, from the 3d and 4th equations, by division, &c. will be had $u + z = c^{\frac{1}{2}} m^{-\frac{1}{2}}$, which call $2e$, and $x + y = d^{\frac{1}{2}} n^{-\frac{1}{2}}$ which call $2r$; and from thence, as above, will be found u and $z = e \pm \sqrt{\sqrt{\frac{1}{10}ne^{-1} + \frac{4}{5}e^4} - e^2}$; consequently $x^5 + y^5 = m - z^5$ becomes known, which call s, and then from it and $x + y = 2r$, x and y come out $= r \pm \sqrt{\sqrt{\frac{1}{10}sr^{-1} + \frac{4}{5}r^4} - r^2}$, the affirmative sign giving the value of the greater, and the negative sign that of the lesser of them.

Ingenious answers to this question have also been received from Mess. *G. Cetii*, *J. Chipchase*, *Hen. Clark*, *S. Clarke*, the Rev. Mr. *Crakelt*, *Gemini*, Mess. *J. Hellings*, *L. Ker*, *E. Reed*, *W. Reynolds*, *Tho. Robinson*, *P. Sharp*, and *W. Spicer*.

III. QUES-

QUESTION 629 *answered by Mr.* Isaac Dalby.

DEMONSTRA. The line bisecting the $\angle A$ of the proposed $\triangle BAC$ being produced to meet BC in S and the circumf. of its circumscribing circ. in D, let the points B, D and C, D be joined, and right lines drawn from B, C to G the cent. of the inscribed circle: Likewise draw GE, $GF \parallel AB$, AC meeting BD, CD in E and F; then (E. 3. 21.) the $\angle DBC$ being $= \angle DAC = \angle BAD$ and also $= \angle EGD$ (by construction), the $\angle SBG = \angle GBA$ (Euc. 4. 4.) and the $\angle DGB =$ the $\angle$s GAB, GBA (Euc. 1. 32.) BD will be $= DC$ (Euc. 1. 6.) $= DG$, and the $\triangle$s BSD, DEG sim. and equal; $\because GE = BS$: And in the same manner GF is proved to be $= CS$; and consequently, the trapeziums $BACD$, $EGFD$ being sim. and alike situated, $AB + AC : AD :: EG + GF$ $(BS + SC = BC) : DG$.——Mr. *R. Mallock's* demonstration is the same as this exceeding nearly.



It is also demonstrated in a curious manner by Mess. *G. Cetii* (the proposer), *J. Chipchase*, *H. Clarke*, *S. Clarke*, *G. Coughron*, the Rev. Mr. *Crakelt*, *C. Hutton*, *Tho. Nicholson*, *Tho. Robinson*, *J. Turner*, and the Rev. Mr. *Wildbore*.

IV. QUESTION 630 *answered by Mr.* G. Cetii.

As the ☉'s alt. is const. and given, its complement, or the ☉'s zenith dist. will be const. and given also. Now, nothing can be plainer than that the ☉'s zenith dist. is always equal to the dist. between the given place and that to which the sun is vertical at the given time; whence this dist. is given and constant, and therefore if the point to which the ☉ is vertical at the given time be found, and round it, as a pole, a circle be described through the given place, it will be the locus required.

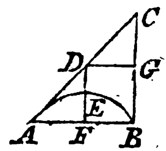
COROLLARY. Hence, if it was required to find the locus of all the places where the sun rises or sets at the same time on a giving day, it is manifest it will be a great circle, whose pole is the point to which the sun is then vertical.

Much after the same manner this question is also answered by Messrs. *J. Chipchase*, *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Dalby*, *J. Haycock*, *C. Hutton*, *R. Mayo* (the proposer), *Alex. Rowe* and the Rev. *Mr. Wildbore*.

V. QUEST.

V. QUESTION 631 *answered by Mr.* G. Coughron.

Let AEB represent the path of the shell, AC its direction at the commencement of motion, EF the the steeple; and let DG be parallel and BC perpendicular to the horizontal line AB. Put FB $(=DG) = 3262\frac{6}{7}$ feet $=a$, the time of its description $= 4'' = b$, the velocity of sound per second $= 1142 = c$, and the time of the shell's flight, or the time of its describing AEB or $AC = x$; then will $b : a :: x : ax \div b = AB$, and



consequently $x + ax \div bc = 12'' = t$; whence $x = \frac{bct}{a+bc}$ $= 7''$, $AB = ax \div b = 5710$ feet, $BC = 7^2 \times 16\frac{1}{12} = 788\frac{1}{12}$ feet, $DE = 3^2 \times 16\frac{1}{12} = 144\frac{3}{4}$ feet, $AF = \frac{3}{7} \times 5710 = 2447\frac{1}{7}$, FD $(= AF \times BC \div AB) = 337\frac{3}{4}$, and the steeple's height FE $(= FD - DE) = 193$ feet.

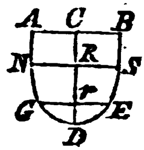
The same answered independent of Algebra, by Mr. Cha. Hutton.

The time in which sound moves from B to F is $= FB \div 1142 = 3262\frac{6}{7} \div 1142 = 2\frac{6}{7}''$; consequently the horizontal velocity of the shell is to that of sound as $2\frac{6}{7}$ to 4. But, as $4 + 2\frac{6}{7} : 4 :: 12 - 4 - 2\frac{6}{7} : 5\frac{1}{7} \times 4 \div 6\frac{6}{7} = 3''$, the time of the shell's flight from A to E or F, so that the whole time of its flight is $7''$. Also, $AF = \frac{3}{7} AB$, $FD = \frac{3}{7} BC$; and, by the law of falling bodies, $1^2 : 7^2 :: 16\frac{1}{12} : 16\frac{1}{12} \times 49 = BC$, and $1^2 : 3^2 :: 16\frac{1}{12} : 16\frac{1}{12} \times 9 = ED$; whence FE $(= FD - ED = \frac{3}{7} BC - ED = 16\frac{1}{12} \times 21 - 16\frac{1}{12} \times 9 = 16\frac{1}{12} \times 12) = 193$ feet, the height of the steeple required. ——And in this manner almost exactly the answer is also given by the Rev. Mr. *Wildbore*, without algebra.

It is also ingeniously answered by Mr. *G. Cetii*, the Rev. Mr. *Crakelt*, Mess. *Hen. Clark*, *S. Clark*, *J. Bartlet*, *J. Dalby*, *J. Haycock*, *J. Hellings*, *W. Reynolds*, *Alex. Rowe*, *P. Sharp*, *W. Spicer* (the proposer), *J. Turner*, and *W. Wells*.

VI. QUESTION 632 *answered by Mr.* Isaac Dalby.

Let *AGEB* represent the tub, and suppose the parabolic conoid to be compleated. Put the top diameter ($=4$) $= d$, the bottom diameter ($= 2\cdot52982$) $= b$, the depth ($= 3$ feet) $= h$, the part of the axis $Dr = n$, $\cdot7854 = a$, $t =$ the time in which the tub's circumscribing cylinder would empty itself thro' the same orifice with the first or greatest velocity uniformly continued, x ($= rR$) any variable alt. of the running water's surface above the bottom of the tub, and $y =$ the time sought.



Then, by the nature of the curve, $NS^2 = d^2 \times \overline{n+h}|^{-1} \times \overline{n+x}$, and by reasoning as in examples 4, 5 on pa. 139, 140, &c. of Emerson's Fluxions, 2d edit. $\dot y = -\overline{n+h \times \sqrt{h}}|^{-1} \times \overline{tnx^{-\frac{1}{2}}\dot x + tx^{\frac{1}{2}}\dot x}$; the correct fluent whereof, when $x = 0$, is $y = 2t \times \frac{3n+h}{3n+3h} = 14'\ 4\cdot3476''$ the time required.

COROLLARY. The times of emptying the tub and its circumscribing cylinder are as $\frac{3}{5}$ to 1; and if the tub was inverted, the time through an equal orifice would be $2t \times \frac{3n+2h}{3n+3h} = 18'\ 45\cdot769''$.

The Rev. Mr. *Wildbore* answers it thus, viz. ' From the ' data are easily found $p = \frac{4}{5}$ the parameter of the genera' t ng parabola, and the depth wanting to compleat it at ' bottom $= 2$ feet $= a$. Let c ($= 3$ feet) $=$ the depth of the ' vessel, $bb = \frac{1}{4 \times 144} = \square$ of rad. of the orifice, $x =$ any ' variable height of the water above it, $y =$ the ord. cor' responding, $d = 16\frac{1}{12}$ feet, and $t =$ the time required; ' then, by the laws of hydrostatics, $\frac{bb}{yy}\sqrt{2dx} = \frac{bb\sqrt{2d}}{p} \times$ ' $\frac{\sqrt{x}}{a+x} =$ the velocity of descent along the axis of the ' vessel; by which dividing $-\dot x$, it gives $\dot t$, whose corrected ' fluent, when $x = 0$, is $t = \frac{p\sqrt{2c}}{bb\sqrt{d}} \times \overline{a + \frac{1}{3}c} = 844\cdot3476$ ' $= 14'\cdot07246'$, the time required.'

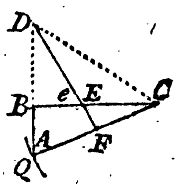
For

For the solution of this quest. Mr. *Hutton* refers to art. 1 of his New Mathematical Miscellany, where the subject of such exhaustions is fully treated on.

Mess. *G. Cetii*, *J. Chipchase*, *Hen. Clark*, *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Hellings*, *J. Hitchcock*, *T. Robinson*, *Alex. Rowe*, *P. Sharp* (the proposer), and *W. Spicer* have likewise answered it.

VII. QUESTION 633 *answered by Mr.* T. Moss (*the Proposer*).

CONSTRUCTION. On BE the given intercepted part of the leg produced, take EC (by prob. 18. pa. 107 of Simp. Geom. 2d edit.) such that $BC \times EC$ may be $=$ the rectangle under the given hypothenuse and its half, and about C, as a center, with the given hypothenuse as rad. describe an arch of a circle cutting $BQ \perp CB$ in A; then, join the points A, C and the thing is done.



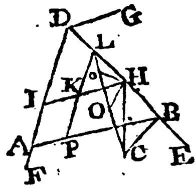
DEMONSTRATION. Upon the middle of AC erect the perpendicular Fe meeting BC in e; then, by sim. triangles, $AC : BC :: eC : FC$, and consequently (by Euc. 16. 6.) $AC \times FC = BC \times eC = BC \times EC$ (per construction); whence $eC = EC$, and the points e, E coinciding, $Be = BE$ (the given intercepted part, per construction), and ABC is a right angle, per construction. Q. E. D.

COROLLARY. If AB, FE be produced to meet in D, and CD be drawn, it evidently appears that DE bisects the $\angle BDC$, and that the bisecting line ED is to the hypothenuse CD in the given ratio of BE to FC ($\frac{1}{2} AC$); whence it manifestly follows that the preceding construction is, in effect, the very same as that of a right-angled triangle having the hypothenuse and a line bisecting one of its acute angles, and terminating in the opposite leg, given.

We are also favoured with ingenious constructions to this problem from Mess. *G. Cetii*, *J. Chipchase*, *Hen. Clark*, *S. Clark*, *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Dalby*, *Gemini*, *J. Haycock*, *J. Hellings*, *C. Hutton*, *L. Ker*, *R. Mallock*, *Tho. Nicholson*, *E. Reed*, *Wm. Reynolds*, *Tho. Robinson*, *Alex. Rowe*, *P. Sharp*, *W. Spicer*, and *J. Turner*.

VIII. QUESTION 634 *answered by Mr.* G. Coughron.

CONSTRUCTION. Draw CH and $HI \parallel DF$ and DG. On HI take $IK =$ the given diff. MN, and through K draw $LKP \parallel DF$; join C, L, and with the rad. HK and cent. H, interfect CL in o, O, and $\parallel HO$ and DG draw CB and BA, which will be the lines required.

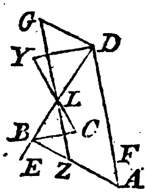


DEMONSTRA. By sim. $\triangle$s, $HK : BP (:: LH : LB) :: HO (= HK) : BC$; consequently $BC = BP = AB - AP = AB - IK = AB - MN$, and therefore $AB - BC = MN$, as it ought. Q. E. D.

COROLLARY. The rad. HK cutting CL in two points shews that the prob. admits of two answers.——When MN is $\sqsubset HI$ the construction is still the same, only HI must be continued beyond H till it is $= MN$; and when CB is to be greatest, MN must be set from I to the left hand, on HI produced, and then the rest of the construction will be the same as above.

CONSTRUCTION *to the same by Mr.* Cha. Hutton.

Take $DG = MN$, and draw $GZ \parallel DF$ cutting DE in L. Draw CLY so that $DY = DG$: then, drawing CB, $BA \parallel DY$ and DG, meeting DE and DF in B and A respectively, and the thing will be done.——For by sim. $\triangle$s, $DL : LB :: DG : BZ :: DY : BC$; but $DG = DY$; $\because BZ = BC$, and $ZA = DG$.

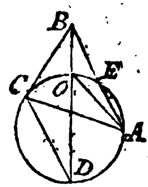


This problem is also constructed in an ingenious and very elegant manner by Mess. *G. Cetii*, *S. Clark* (the proposer), the Rev. Mr. *Crakelt*, *J. Dalby*, *J. Haycock*, *J. Hellings*, *R. Mallock*, *E. Reed*, *Tho. Robinson*, *Alex. Rowe*, *P. Sharp*, *J. Turner*, and the Rev. Mr. *Wildbore*.

IX. QUES-

IX. QUEST. 635 *answered by the Rev. Mr.* Wildbore.

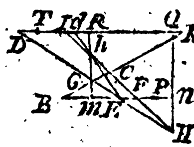
CONSTRUCTION. Upon BO the given bisecting line, produced, take BD a fourth proportional to BO and the two given sides of the triangle, and on the diameter DO describe a circle; to which from B set off the two given sides BC, BA (the latter of them intersecting the circumf. suppose in E); join A, C, and ABC is the required triangle——For (per Euc. 3. 37.) DB is a fourth proportional to OB, AB, and EB, and (by construc.) to OB, AB, and BC; therefore $EB = BC$, and $\angle CBO = \angle EBO$, and the sides about the equal angles being (by construc.) proportional, the Δs AOB, DCB (per Euc. 6. 6.) are similar, and (per 3. 21.) $\angle CDO (= BAO) = \angle CAO$; ∵ (per 4. 4.) O is the center of the inscribed circle, and the triangle is that required.—— And hence by way of corollary is easily deduced an elegant demonstration to quest. 3d.



The construction by Mess. *Isaac Dalby* and *Cha. Hutton* agree with the above in almost every particular; and it is also constructed in an ingenious and curious manner by Mess. *G. Cetii*, *Hen. Clark*, *S. Clark*, *G. Coughron*, the Rev. Mr. *Crakelt* (the proposer), *J. Haycock*, *J. Hellings*, *R. Mallock*, *E. Reed*, *Tho. Robinson*, *Alex. Rowe*, and *J. Turner*.

X. QUESTION 636 *answered by Mr.* G. Cetii.

CONSTRUCTION. At a distance from $BF = PQ$ and parallel thereto draw DK, and produce BE and FC to meet it in K and I. Make $IT = 2FE$, and take TD so, that $TD \times DK = BE \times EF$; then draw DE, meeting CF produced, in H, and the thing is done.



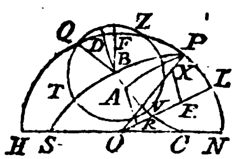
For $DT : EF :: BE : DK$ (by construction), and (Simp Geom. 20. 4.) $Gm : PQ - Gm$ (GR) $:: BE : DK$; also $PQ + nH : nH :: DT + 2EF$ $(DI) : EF$, or by division, $PQ - Hn : Hn :: DT : EF$; ∵ by equality, $PQ - Hn : Hn :: Gm : PQ - Gm$, or by compos. $PQ : Hn :: PQ : PQ - Gm$. Now the antecedents being here equal, the consequents will be so also, and ∵ $PQ = Gm + Hn$.

But if FC be produced towards I, take Id so, that $Id \times Kd = BE \times EF$, and draw dE for the line required; the demonstration of which being much more simple than that of the above case, needs not pointing out.

Very ingenious and curious constructions to this question have also been received from Mess. *S. Clark*, *G. Coughron*, the Rev. Mr. *Crakelt*, Mess. *J. Dalby*, *J. Haycock*, *J. Hellings*, *Cha. Hutton*, *T. Moss* (the proposer), *E. Reed*, *Tho. Robinson*, *Alex. Rowe*, *P. Sharp*, and the Rev. Mr. *Wildbore*.

XI. QUESTION 637 *answered by Mr.* W. Wales.

PROJECTION. Let $HZPN$ be the meridian, P the elevated pole, and PO the semi-axis of the sphere. Describe DC, the parallel of the ☉'s declin. for the given day, and the great circle QAR to touch it, making the $\angle Q$, which it forms with the meridian, equal to half the arc the returning shade describes, and Q will be the pole of the dial-plane, OL. Draw the hour-circle PBS, answering to 3h. 17m. 57s. and cutting DC in B, the place of the sun at that time. Round B, as a pole, describe, through Q, the less circle TQZ, cutting the meridian again in Z, which will be the zenith of the place required; and ∵ ZP is the complement of its latitude. Lastly, make $OE = 18$ inches, and draw $EX \perp OL$, meeting PO in X, and EX will be the height of the pin, and the $\triangle OEX$ the proper gnomon of the dial.



DEMONSTRATION. It is plain that the motion of the shade will be direct until the sun comes to A, i. e. into the vertical to the dial-plane, QAR, which touches the parallel of declination; after which it can go no farther westward, but will return; and when the sun gets on the meridian at D, will have described an arc = the measure of the $\angle AQP$, and which is therefore = half the arc described by the returning shade. Again, Q and Z being the poles of the planes OL, HN, and $BQ = BZ$ by construction, it is plain that the altitudes of the point B above those planes are equal. Lastly, OL being the plane of the dial, PO the semi-axis of the earth, and $OE = 18$ inches, EX must be the height of the pin; for its top must be in the right line drawn through the pole and the center of the dial.

Answer

Answer to the same by Mr. J. Chipchase (*the Proposer*).

It is evident that the plane of this dial is parallel to the horizon of some place within the tropics, and that the time of the shadow's returning will be when the sun's azimuth from the north is the greatest possible at that place. Let LO be the horizon of the place, which is parallel to the plane of the dial (see fig. preceding), Q the zenith thereof, P the north pole, DC the parallel of declination on the 21st of June, A the place of the sun when his azimuth from the north is the greatest possible, and the $\angle QAP$ will therefore be a right one. Put the sine of $PQ = x$, sine of $AP = s$, its cosine $= c$, and then the sine of the $\angle AQP$ or arch RL, will be $= \frac{s}{x}$, and the sine of the amplitude $OV = \frac{c}{x}$, and its cosine, or the sine of the arch VL, $= \sqrt{1 - \frac{cc}{xx}}$. The arch RV or $RL - VL$ described by the returning shade, being $= 4^\circ\ 24'\ 40''$, call its sine b, and then (per Emers. Trig. prop. 6. b. 1.) $\frac{sc}{xx} - \sqrt{1 - \frac{cc}{xx} \times 1 - \frac{ss}{xx}} = b$; which reduced gives $x = \sqrt{\frac{1 - 2bsc}{1 - bb}} = \cdot 9743693$ the nat. sine of 77°; therefore the height of the pole above the dial-plane is 13°; and rad. : 18 inches :: tang. 13° : 4·156 inches, the height of the pin required——Describe a merid. to make an angle of 3h. 17′ 57″ or $49^\circ\ 29'\ 15''$ with HQP, cutting the parallel of declin. in B, and describing $BF \perp QP$, make $FZ = FQ$, and Z will be the zenith of the place where the dial is fixed: For, by the quest. at 3h. 17′ 57″ the height of the ☉ above the plane of the dial was equal to his alt. above the horizon of the place where it stands; B is the ☉'s place, and QB, ZB are the complements of his alt. above the plane of the dial and horizon of the place where it is fixed, at that time, which are equal by construction.—Lastly, cotang. BP : rad. :: cos. $\angle FPB$ ($49^\circ\ 29'\ 15''$) : tang. $FP = 56^\circ\ 13'\ 30''$; whence $ZP = 35^\circ\ 27'$, the comp. of the required latitude.

The Rev. Mr. *Crakelt*, Mess. *Isaac Dalby*, *Cha. Hutton*, and the Rev. Mr. *Wildbore* have also favoured us with curious projections of this prob. and calculations to the same, deduced therefrom; and it is also answered in a very ingenious manner, without algebra, by Mr. *Geo. Coughron*.

XII. QUES-

XII. QUESTION 638 *answered by the Rev. Mr.* Crakelt.

Let the legs of the required triangle be represented by x and nx; then are $x - nxx$, $nx - nxx$ and $\sqrt{xx + nnxx} - nxx$ to be square numbers (per quest.): Suppose $x - nxx = xx$; then will $nx = 1 - x$, by which means two of the conditions of the quest. will be fulfilled, and therefore what remains is to make $\sqrt{xx + nnxx} - nxx = \sqrt{1 - 2x + 2xx} + xx - x$ a square number. Now, to effect this, imagine $1 - 2x + 2xx = \overline{mx - 1}|^2$, and x will be $= \frac{2m - 2}{m^2 - 2}$, $\sqrt{1 - 2x + 2xx} + xx - x = \frac{m^4 - 4m^3 + 6m^2 - 4}{\overline{m^2 - 2}|^2}$, and m greater than 2: but, since $m^4 - 4m^3 + 6m^2 - 4$ is not a square number, feign it $= \overline{r - 1 - m^2 + 2m}|^2$, and m will be $= \frac{2r - 2 + \sqrt{2r^3 + 2r + 4}}{2r}$, and r greater than $\frac{3}{2}$; but $2r^3 + 2r + 4$ not proving a square number, suppose it $= \overline{2 + 2vr}|^2$, and r will be $= vv + \sqrt{v^4 + 4v - 1}$, and v greater than 1; let $4v - 1 = z$, and $v^4 + 4v - 1$ will be $= \frac{z^4 + 4z^3 + 6z^2 + 260z + 1}{256}$, which being supposed $= \overline{\frac{1 + 130z - zz}{16}}|^2$ we shall get $z = \frac{4223}{66}$, and thence $\frac{18465217}{20590417}$, $\frac{18325825}{20590417}$, and $\frac{2264592}{20590417}$ for the sides of the triangle required.

Answer to the same by the Rev. Mr. Wildbore, (*the Proposer*).

Let x and $1 - x$ = the legs of the triangle; then will $\sqrt{1 - 2x + 2xx}$ = the hypothenuse, and $x - xx$ = the double area, which subtracted from these three values of the sides, leaves xx, $1 - 2x + xx$, and $\sqrt{1 - 2x + 2xx} - x + xx$ for the three square numbers required, the two first being necessarily squares; and in order to make the last a square, put $v \cdot \overline{1 - x} + x = \sqrt{1 - 2x + 2xx}$, and the said square will become $\overline{1 + 4v^3 - v^4} \times \overline{1 + 2v - vv}|^{-2}$; therefore, if $1 + 4v^3 - v^4$ be a square number less than unity

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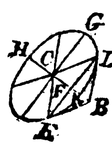
the conditions of the queſtion will be anſwered. Make $v = ba^{-1}$, then muſt $a^4 + 4ab^3 - b^4 =$ a ſquare, or, putting $a = b + c$, $c^4 + 4bc^3 + 6bbcc + 8b^3c + 4b^4 =$ a ſquare; and in order to take off the two firſt terms and the laſt by tranſpoſition, make it = a ſquare whoſe ſide is $cc + 2bc - 2bb$, and we find by reduction, $3c = -8b$, or in the leaſt integers, $c = 8$ and $b = -3$: but b muſt not be a negative quantity; ∵ let $b = d - 3$ and $a = b + c = d + 5$, and ſubſtituting theſe values for a and b we obtain $4d^4 + 16d^3 + 24d^2 + 1040d + 4 =$ a ſquare; ſuppoſe its ſide $= 2dd - 260d - 2$ in order to deſtroy the firſt and two laſt terms by tranſpoſition, and we find $d = \frac{4223}{66}$, $b = \frac{4025}{66}$, $a = \frac{4553}{66}$, $v = \frac{4025}{4553}$, and the ſides of the triangle required = three fractions whoſe numerators are 2264592, 18325825, and 18465217, and com. denom. 20590417; which is the anſwer (though ſomewhat differently deduced) given by Ozanam at pa. 604 of his Nouveaux Elemens d'Algebre (which has been handed to me ſince I ſent the queſtion laſt year); and this being the only anſwer that the proceſs thus far admits of, has been ſaid to be the only one that could be given to the queſt.: But by artifices ſimilar to the preceding, v will next be found $= \frac{491050}{555466}$, $\frac{75846408}{1232216833}$, &c. and other values of the ſides of the triangle correſponding equal to three fractions whoſe numerators are 8426546834, 68190394825, 68709072245, and com. denom. 76616941657, &c. &c.

Solutions to this queſtion have likewiſe been received from Meſſ. *G. Coughron*, *J. Dalby*, and *C. Hutton*.

XIII. QUESTION 639 *anſwered by the Rev. Mr.* Crakelt.

DEMONSTRATION. Produce any ſemi-diam. CA of the annexed ellipſis, till $CB : CA ::$ the diagonal of a ſquare : its ſide, and having drawn the tangents BD, BE, join the points C, D; D, E, and E, C; then, ſince by conics, DE will be biſected by CB in F, by the known prop. of tangents, $CF : CA :: CA : CB ::$ (by Euc. 1. 6.) $CA \times CB : CB^2 = 2CA^2$ (per queſt.) $:: CB : 2CA$, and conſequently $CB = 2CF$, or $CF = FB$; whence the △s CFE, BFD being (by Euc. 4. 1.) equal in all reſpects, CE will



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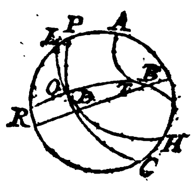
(by Euc. 27. 1.) be ∥ BD and $CD \parallel BE$, and thence CE and CD conjugate semi-diameters, which is the first thing to be proved.——Next, draw $CG \parallel FD$; then, by the common prop. of the ellipse, $FD^2 : CG^2 :: CA^2 - CF^2 = CB \times CF - CF^2 = CF^2 : CA^2 = CF \times CB :: CF : CB :: CF \times CB = CA^2 : CB^2$, or $FD : CG$ $(:: 2FD : 2CG)$ $:: CA : CB$. Q.E.D.

The Rev. Mr. *Wildbore* observes by way of corollary to his demonstration (which is both very concise and curious) that from pa. 201 of Simp. Geom. best edit. it appears that $ADHE$ will be the greatest ellipsis that can be inscribed in the rhomboides formed by the intersections of the tangents BD, BE produced, and two others drawn parallel to them, respectively.

This property is also curiously demonstrated by Mess. *G. Cetii, S. Clark, G. Coughron, J. Dalby, C. Hutton, Plus Minus* (the proposer), *T. Robinson*, and *Alex. Rowe.*

XIV. QUEST. 640 *answered by the Rev. Mr.* Wildbore.

PROJECTION. Describe $ACRP$ the primitive, ATB the given lesser circ. POC the given great one, and the right circ. BDR touching ATB, suppose in B: From B set off BD = the given intercepted arc, and through D describe $OD \parallel$ to ATB, cutting the given circ. POC in O; then, through O describe the great circ. OT, touching ATB in T, and it will be that required;—because the two arches OT, BD, touching the lesser circ. ATB, and being bounded by the same parallel OD, are equal.



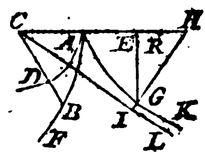
Mess. *J. Chipchase, Geo. Coughron, J. Dalby, C. Hutton,* and *W. Wales* (the proposer) have likewise favoured us with curious projections of this problem.

XV. QUESTION 641 *answered by the Rev. Mr.* Crakelt.

From the given equation ($x = \dfrac{N^{2z}+1}{2N^z}$) by multiplying, compleating the square, and taking the hyp. log. is had

had $z =$ h. l. of $\overline{x + \sqrt{xx - 1}}$. But 1 ($= CD$, see the last year's fig.) : x (CB) :: $\dot{x} + \sqrt{xx - 1}$ (the angular velocity of a body at D) : $x\dot{x} \div \sqrt{xx - 1} =$ the angular velocity of the same body at B; whence the fluxion of the curve's length will be $= \dot{x} \times \sqrt{2xx - 1} \div \sqrt{xx - 1}$, and the the length itself = the arch of that equilat. hyperbola, which hath two for each of its diameters and $x - 1$ for its abscissa = (when $x = 1000$) to 1414·2132088, &c.

Answer to the same by the Rev. Mr. Wildbore.



DESCRIPTION. With the center C and vertex A describe the equilat. hyperbola AGK; draw the asymp. CL and ord. EG. Take $EH = EG$, $AR =$ the hyp. log. of CH, and the circ. arc $AD = AR$; then, drawing a right line from C through D, take upon it $CB = CE$, and the point B will be in the spiral; and thus may points be determined therein *ad libitum*.

DEMONSTRATION. By conics, $CE = \dfrac{CH^2 + CA^2\,(1)}{2CH}$, $CH = CE + \sqrt{CE^2 - 1}$, and by construc. $AR = AD =$ the hyp. log. of CH; $\because CH = N$ to the AD power, and consequently $CE = CB = \dfrac{N^{2z} + 1}{2N^z}$, as per question.—— Moreover, the flux. of the spiral AB is well known to be $= \sqrt{\dot{CE}^2 + CE^2 . \dot{AD}^2}$, and $\dot{AG} = \sqrt{\dot{CE}^2 + \dot{EG}^2}$; Likewise, $EG^2 = CE^2 - 1$, $EG . \dot{EG} = CE . \dot{CE}$, $\dfrac{CE . \dot{CE}}{EG}$ ($\dot{EG}$) $+ \dot{CE} = \dot{CH}$ (because $EG = EH$), $EG . \dot{CH} = CH . \dot{CE}$, $\dfrac{\dot{CH}}{CH} = \dot{AR} = \dot{AD} = \dfrac{\dot{CE}}{EG} = \dfrac{\dot{EG}}{CE}$; $\because CE^2 . \dot{AD}^2 = \dot{EG}^2$, and $\dot{AB} = \dot{AG}$, and because they both begin together, $AB = AG$.

COMPUTATION. Because when $CE = 1000$, both it and EG are very great in comparison of CA, the common series and approximations for AG are useless; but here the series given at p. 511 of Simp. Flux. may be applied to very great advantage:

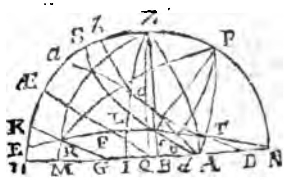
advantage: For, the diff. betwixt the part of the asymp. beyond I and that of the curve beyond G being exceedingly small, or quite inconsiderable in comparison of the diff. betwixt CI and AG, $CI - AG =$ the sum of that series very nearly $= \cdot 32111$; ∵ this being subtracted from $CI = CH \times \frac{1}{\sqrt{2}}$ (HGI being $\perp CL$) gives $AG = 1413 \cdot 8921 = AB$, the length of the curve required.

COROLLARY. The hyp. log. of CH (1999·9995) being $= 7 \cdot 6009024 = AD$, this divided by the whole circumference of the circle, shews that the body will have made 1·209725 revolutions round the center when its distance therefrom is 1000.

Mess. *T. Allen* (the proposer), *G. Cetii*, *Hen. Clark*, *G. Coughron*, *J. Hellings*, and *Cha. Hutton* have likewise answered this question according to the first of the preceding methods, very nearly.

The PRIZE QUESTION *answered by Mr.* Wm. Wales.

Let $HZPN$ be the meridian, HQN the horizon, $ÆQ$ the equator, Z the zenith, and P the pole. Make QF = the measure of the given interval, and, having described the circ. of perpetual apparition, Nob, describe, through F, the great circle EFD to touch it. Then by prop. 13. b. 2. of Theodosius's Spherics, the arcs of all circles ∥ Nob, or $ÆQ$, intercepted between the circles HQN, EFD are similar and described in the same time with QF. Suppose, now, the circle KRG to be the parallel of that star whose change in azim. GM, is a max. and SCA the parallel of that star whose change in azim. AB is a min. in the given interval of time, ZRM and ZCB being verticals passing through R and C, the places of the two stars when situated in the circle EFD, and ZLI the vertical whose pole is D. By the above prop. of Theod. $QG \doteq FR$; ∵ MG, the change of azim. in the given time, is the diff. between QM and FR, and so $QM - FR$ is to be a max.; but when $QM - FR$ is a max. $IM - LR$ is a max. because QI and FL are constant quantities. Now IM is the measure of the $\angle RZL$; therefore when the change in azim.



azim. is a max. the diff. between the $\angle RZL$ and its opposite side, RL, is a max. and consequently its sine is so too: But the $\angle L$ is right, and the side ZL given; also, by spherics, rad. : sin. ZL :: tang. Z : tang. LR, and by composition, division, and equality of ratios, rad. + sin. ZL : rad. − sin. ZL :: tang. Z + tang. LR : tang. Z − tang. LR :: sin. $\overline{Z+LR}$: sin. $\overline{Z-LR}$ (by prop. 4. p. 58. Simp. Trig.). Now, the two first terms of this analogy being constant, the last will manifestly be a max. when the third, or sin. $\overline{Z+LR}$, is so, *i.e.* when $Z+LR=90°$; ∵ the sine of their diff. $\overline{Z-LR}$, when a max. $=\frac{\text{rad.}-\text{sin.}\,ZL}{\text{rad.}+\text{sin.}\,ZL}$; and hence, having the sum and diff. of Z and LR, those arcs themselves are known, and of course every thing else in the $\triangle RZL$; and, as the $\angle LZP$ is known by the construc. the $\angle RZP$ will be known, and we shall have in the triangle RZP, the sides RZ and ZP with the included angle, to find RP, the parallel's dist. from the elevated pole, = (in the case given) 109° $27\frac{3}{4}'$, or its declin. 19° $27\frac{3}{4}'$ south; and this parallel the sun occupies Nov. 18th and Jan. 22d.

Again, because $FC=♎A$, the change in azim. $BA=FC-♎B$, which is therefore to be a min.; or $LC-IB$ is to be a min.; or, which amounts to the same thing, $IB-LC$ must be a max. and hence the point C will be determined exactly in the same manner that the point R was, and we shall have in the $\triangle CZP$, the sides CZ and ZP with the included angle to find CP, = 58° $55\frac{1}{6}'$, and so the declin. required is 31° $4\frac{5}{6}'$, north.

SCHOLIUM I. The declin. of the parallel SCA will, it is plain, be of the same name with the latitude of the place until the circ. EFD comes into the position aod, that is, to touch the circ. of perpetual apparition in the prime vertical, $Z♎$: For then, the intersecants Nd, do being equal, $od+d♎=90°$, which has been proved to be the case when the change in azim. is a max. or min. and consequently the circle Nob is the parallel required, and the star rises due north or south; after which the declin. of the parallel SCA becomes of a different name from the lat. of the place till the time given is 18h. when it becomes again of the same name therewith.

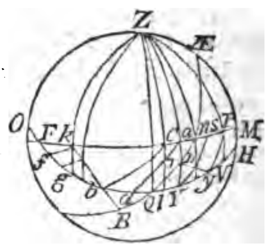
II. In like manner, the declin. of the parallel KRG will be of a diff. name until the given time be 6h. after which it becomes of the same name with the lat. until the circle EFG touches the circle of perpetual apparition in the western prime vertical, and then it will be again of a diff. name from

the latitude. And all these determ. hold equally true of a star moving from the horizon at setting, only the declin. here said to be of the same name with the lat. will there be of a contrary one, and *vice versa*; for it is only considering them as stars rising to the Antipodes.

III. In all these conclusions I have supposed the given lat. was not less than 45°; for if it be, the given time may be such as to cause the circ. *EFD* to cut the merid. between the zenith and elevated pole: in which case the question may admit of more answers than one; but they cannot always be exhibited by this, or, I believe, any other general method. It may, however, be easily done by others, adapted to the particular cases where this fails; but your limits are much too narrow for so copious a subject.——I must, before I take leave, return thanks to the ingenious proposer for bringing this question again under consideration, as I had passed it over in too hasty a manner before; the reasons for which you are well acquainted with.

The same answered by the Rev. Mr. Wildbore, *(the Proposer).*

The annexed scheme is an oblique representation of the concavity of the eastern hemisphere, bounded by the merid. *OZÆH* of the place, *HO* the horizon, *ÆQ* the equinoctial, and the given hour arch is set off from *Q* to *n*; at *n* the $\angle anT$ is made $= TQ\Upsilon$, and thus is the great circ. *FnM* described. Then, by Gregory's Astron. b. 2. prop. 40. the arches of the equinoctial and its parallels intercepted between this circle and horizon are similar, and $aT = T\Upsilon$, $an = Q\Upsilon$, $QT = Tn$, $an = dQ$, $ac = db$, $ak = df$, &c. Now, suppose the circle *FanM* to be generated by the motion of a point both ways from *a*; then, when the sun or star rises at *d*, its alteration of azim. in the time *Qn* will be $d\Upsilon$, and when at *b* it will be bl, and because the $\angle$s at Υ, l, and a are right, Υl will be $\sqsubset ac$; but ac is $= bd$: ∵ bl is $\sqsupset d\Upsilon$, or the alteration of azim. when the phœnom. rises at *b*, will be $\sqsupset$ when it rises at *d*, the diff. being $l\Upsilon - ca$: But in the right-angled isosceles spheric $\triangle F\Upsilon a$, $l\Upsilon$ is the decrem. of $F\Upsilon$, and ca of Fa; and if *k* be taken for



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any other position of the moving point, if in the right-angled spherical $\angle Fgk$, the decrem. of Fk is $\sqsupset$ that of Fg, the said alteration of azim. may still decrease by taking the point k nearer F, for the same reason as at a: But if the decrem. of Fk be equal to that of Fg, it can decrease no further; for at the next point, the decrem. of Fk being greatest, the said quantity will be upon the increase; therefore, when the required increase of azim. is a min. the decrem. of Fk is $=$ that of Fg.——And in the very same manner, it appears that when the point moves the other way from a (M being placed at the other intersection of the circle FaM with the horizon) the alteration of azim. as the declin. varies, will increase till the decrements of PM and VM are equal, and then it will be a max.——And hence, per spherics and pa. 280 of Simp. Flux. is easily found $\angle kFg = 18° 32'$, $kg = 13° 10'$, $\angle Fkg = 76° 50'$, $Fg = ka = 44° 14'$, $Fk = g\Upsilon = 45° 46'$, $an = 11° 51'$, $kn = 56° 5'$, $kB = 31° 5'$ the star's declin N. required; $gd = 22° 4'$, and thence $fg = 22° 10'$, the min. azim. in 2 hours from rising corresponding, $sP = 20° 32'$, $yV = 25° 14'$ the max. increase (or alteration) in 2 hours, and $Dy = 19° 27\frac{2}{3}'$, S. the corresponding declination answering to Nov. 19 or Jan. 22.

Answer to the same by Mr. Cha. Hutton.

Let P be the north pole (see fig. 1st preceding), Z the zenith, HN the horizon, AC a great circle passing through the object at its rising and 2 hours after; describe the perpendicular circ. PU, and the circles PA, PC, ZA, and ZCB: Put $a =$ sine of 15° or $\angle APU$, $b =$ its tang. $c =$ sine of lat. or cos. of ZP, $d =$ its cos. or sine of ZP, $x =$ s. declin. or cos. of AP or PC, and $z =$ cos. of declin. or s. of AP. Then, in the right-angled spherical $\triangle PUA$, $1 : a :: z : az =$ s. AU or $\frac{1}{2}AC$, and hence $2az\sqrt{1-aazz} \div \overline{1-2aazz} =$ tang. AC; also, $1 : b :: x : bx =$ cotang. PAU; and hence $\overline{1+bbxx}|^{-\frac{1}{2}} =$ sine and $bx \times \overline{1+bbxx}|^{-\frac{1}{2}} =$ cos. PAU. In the quadrantal $\triangle AZP$, $z : 1 :: c : \frac{c}{z} =$ cosine and $\frac{1}{z}\sqrt{zz-cc} =$ sine PAZ: Whence $\overline{c-bx\sqrt{zz-cc}} \div z\sqrt{1+bbxx} =$ s. ZAC or cos. CAB.——And in the right-angled spherical $\triangle ABC$, $1 :$ tang. $AC ::$ cos. $CAB :$ tang. $AB = 2a\sqrt{1-aazz} \times \overline{c-bx\sqrt{zz-cc}} \div \overline{1-2aazz} \times \sqrt{1+bbxx} =$ (since $\frac{a}{b}$

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$= \sqrt{1-aa}$, and putting $e = \frac{1-2aa}{2aa}) \frac{1}{b} \times \frac{c-bx\sqrt{dd-xx}}{e+xx}$ = a max. or min.——This in flux. &c. gives $2ex\sqrt{dd-xx} = b \times \overline{dd+2e.xx-dde}$, or $x^4 - \text{·}3780667xx = \text{·}0296788$; and hence $x = \sqrt{\text{·}2668459 \text{ or } \text{·}1112207} = \text{·}5165713$ or ·3334977, answering to 31° 6′ 15″ and 19° 28′ 52″, the two declinations required.

This question is also ingeniously answered by Mess. *J. Chipchase*, *Hen. Clark*, *G. Coughron*, *Sam. A Day*, and *J. Haycook*; but the prize of 12 Diaries for the solution thereof is fallen to the lot of Mr. *Wm. Wales*, and that of 8 to Mr. *Geo. Coughron*.

The Eclipses calculated for 1772.

This year will produce four eclipses of the sun, and two of the moon, as under:

1st eclipse, of the ☉, will be on Friday the 3d day of April, about 5 in the morning; therefore invisible in England; but will be seen in the northern parts of Asia.

2d will be a total eclipse of the ☽, on Friday the 17th day of April: Beginning 2 h. 16 m. 18 s. Middle 4 h. 3 m. 1 s. End 5 h. 49 m. 44 s. Dig. 15 deg. 20 m. apparent time at London, afternoon.

Note, This eclipse will be over before the ☽ rises at London.

3d will be an eclipse of the ☉, on Saturday the 2d day of May; invisible at London; but visible in North America, at near our 10 at night.

4th will be an eclipse of the ☉, on Sunday the 27th day of September; invisible at London; but visible near the south pole, about our 1 o'clock in the morning.

5th is a visible eclipse of the ☽, on Sunday the 11th day of October: Beginning 3 h. 28 m. 40 s. Middle 5 h. 26 m. 2 s. End 7 h. 23 m. 24 s. Digits eclipsed 13 deg. 52 m. apparent time at London.

Note, The moon will rise totally eclipsed at 5 h. 24 m. and continue so till about 16 m. after 6 o'clock; when the vertical limb of the ☽ will begin to emerge out of the shadow.

6th

6th is a small visible eclipse of the ☉, on Monday the 26th day of October: Begin. 8 h. 22 m. Middle 8 h. 30 m. 12 s. End 8 h. 40 m. Dig. 0 deg. 6½ m. apparent time, in the morning, at London.——By the late ingenious Gael Morris's Tables.

Isaac Tarratt.

New Questions.

I. Question 622, *by Mr.* John Shadgett.

In friendship two sisters together reside,
With virtue replete; each a stranger to pride:
Maria for beauty with Venus may vie,
And Cloe for wisdom Minerva defy:
Maria is prudent in ev'ry degree,
Whilst Cloe is court'ous, good-natur'd, and free.
From what's under-written * their ages I ask:
Resolve it, dear ladies, nor think't a hard task.

* Given $\begin{cases} x^2 + xy + y^2 = 1087, \\ x^4 + x^3y^3 + y^4 = 45777295; \end{cases}$ to find the value of x the age of Maria, and that of y the age of Cloe.

II. Question 643, *by Mr.* Wm. Spicer.

A piece of timber in the form of the frustum of a cone, whose girths at the ends are 10 and 3 feet, was measured by the customary way of taking ¼ of the girth in the middle for the side of the square: Now, if the said piece of timber be cut into two pieces at the distance of 14 feet from the small end, and both the pieces measured according to the same method, they will amount to 7 solid feet more than the whole piece: Required the length of the said piece of timber?

III. Question 644, *by Mr* Paul Sharp.

Given the sum of the transverse and conjugate diameters of an ellipsis = 100, and the difference of its area and that of its greatest inscribed parabola = 845·73: Required the area of the ellipsis, and the axis of the said parabola?

IV. Question 645, *by Mr.* Steph. Hodges, *at the Right Hon. the Earl* Spencer's, *at* Althorp.

Having given a common parabola; it is required to draw a tangent thereto so, that a right line drawn from the point

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of contact to a given point within the same, may be the shortest possible?

V. Question 646, *by the Rev. Mr.* Wildbore.

The difference betwixt the perpendicular and base, the difference of the segments of the base, and the difference of the angles at the base being given, to construct the triangle?

VI. Question 647, *by Mr.* T. Moss.

From the given point E in the side DR produced, of the given rectangle $BDRF$, to draw a right line EA cutting the sides RF, DB, thereof in a and C, and the side FB produced in A so, that the trapezium $CDRa$ may be to the $\triangle ABC$ so formed, in the given ratio of m to n.

VII. Question 648, *by Mr.* Isaac Dalby.

In the right-angled plane triangle, there is given one of the legs, also a line drawn parallel thereto and terminated by the other leg and hypothenuse; to determine geometrically, the triangle so, that the rectangle under the hypothenuse and a line drawn from the acute angle next the given leg, to the point where the said parallel line meets that other leg, may be of a given magnitude.

VIII. Question 649, *by Mr.* S. Ogle.

Two lines AB, AC being given in position, and a point P in one of them (AB); it is required to draw from thence a right line PH meeting the other given line AC in H so, that if another right line HI be drawn to meet AB and make a given angle therewith, the perimeter of the triangle PHI so formed may be the least possible.

IX. Question 650, *by Mr.* J. Chipchase.

On a certain day last summer, at a quarter past midnight, when the sun was just rising, a person set forward on a journey along a parallel of latitude, which he ended at sunset, having by his watch performed it in 24 hours; at noon the greatest shadow of his walking-stick (which is $3\frac{1}{2}$ feet long) exceeded its shadow when placed perpendicular to the horizon by 1·522267 feet; from whence is required the latitude he was in, the day of performing the journey, and distance travelled?

X. Ques-

X. QUESTION 651, *by Mr.* W. Chartreux.

The latitude of the place, and the poſition of two hour-circles, with reſpect to the meridian, being given; it is required to determine what the declination of a ſtar muſt be, ſo that in paſſing over the interval contained between thoſe hour-circles, the change in altitude may be the greateſt poſſible.——This queſtion has been propoſed before, but never anſwered.

XI. QUESTION 652, *by Mr.* Cha. Hutton.

If water runs through a pipe $1\frac{1}{4}$ inch diameter with a conſtant velocity of 6 feet per ſec. into an empty conical veſſel, having a hole of 1 inch diameter in its bottom: Required the time when the ſurface of the water in the veſſel will be juſt 1 foot above the bottom of it; alſo, what will be the greateſt height to which the water will riſe, and the time in which it will riſe to the ſaid greateſt height, the the diameter of the bottom of the veſſel being 3, the diameter of the top 5, and the altitude 6 feet?

XII. QUESTION 653, *by* Aſtronomicus.

To inveſtigate or demonſtrate the nature of the curve which a fixed ſtar by means of the abberration would appear to deſcribe, if the earth, inſtead of revolving in an ellipſis, was to move in a parabolic or an hyperbolic orbit.

Note, This curious queſtion has been propoſed before, but as it was thought to have been neither inveſtigated nor fully demonſtrated, we have been requeſted to re-propoſe it.

XIII. QUESTION 655, *by the Rev. Mr.* Wildbore.

Abſtracting from refraction, at what time on the 21ſt of June, at Salton in Norway, lat. 67° N. will the velocity of the ſhadow of the ſummit of an erect object be the greateſt poſſible on an horizontal plane?

The PRIZE QUESTION, *by* Peter Puzzlem.

Suppoſe a body to be faſtened to one end of a ſtring, the other end whereof is faſtened to a fixed point *c*, and ſuppoſe it to be impelled from a certain given point *b* with a given velocity in a direction at right-angles to *bc*: To find the law and direction of the force which muſt continually act on ſuch body ſo, that its velocity ſhall vary according to any propoſed law, and itſelf be always found in a circle (whoſe center is *c*) revolving with any propoſed velocity about the given fixed diameter *bcd*.

Queſtions

1773.

Questions answered.

I. QUESTION 642 *answered.*

PUT $xx + yy = v$, $xy = w$, $1087 = a$, and $45777295 = b$, and then the given equations become $v + w = a$ and $vv - 2w^2 + w^3 = b$, from the former of which $v = a - w$, and that value substituted in the latter of them, &c. gives $w^3 - w^2 - 2aw = b - aa$; whence $w = 357$, and consequently from the first given equation, $xx + 2xy + yy = a + 357$ and $xx - 2xy + yy = a - 3 \times 357$; and thence x and $y = \frac{1}{2}\sqrt{a + 357} \pm \frac{1}{2}\sqrt{a - 3 \times 357} = 21$ and 17, or 17 and 21, the two ages required.——According to this method the answer is given by Mess. *Joseph Cowley*, *William Dent*, *Pamphagus*, *William Reynolds*, *Alex. Rowe*, and *Michael Taylor*: but as x and y are concerned exactly alike in both the given equations, Mess. *John Aspland* and *William Smart* put $s = \frac{1}{2}$ the sum and $d = \frac{1}{2}$ the diff. of them, and then find, by an easy process, $64d^6 - 52128d^4 + 4074476d^2 - 55467952 = 0$; whence $d = 2$, and the ages required come out the same as above.

Mr. *Geo. Coughron*, by transposing xy in the 1st equation, and squaring and taking it from the 2d, deduces the cubic equation $\overline{xy}|^3 - \overline{xy}|^2 - 2axy = b - aa$; from which $xy = 357$, and $x + y$ $(= \sqrt{xx + 2xy + yy}) = \sqrt{a + xy} = 38$; and thence x and y are found the same as before.

Mess. *Tho. Adcock*, *J. Bartlett*, *C. Burton*, *J. Dalby*, *R. Dening*, *Mark Elstob*, *Edw. Fidler*, *Wm. King*, *Bewley Mellor*, *J. Parker*, *Wm. Richardson*, *Abr. Robertson*, *Tho. Robinson*, *J. Shadgett* (the proposer), *Tho. Smith*, *Wm. Spicer*, *Elean. Suggett*, and *Wm. Wilkin* have likewise answered it; and Mr. *Wm. Wells*'s answer is as follows, viz.

Friend Shadgett, I thought fit to try
If I could find your x and y:
Your x then wants three of a score,
And y, I find, is just one more.*

* Viz. 17 and 21.

II. QUES-

II. QUESTION 643 *answered by Mess.* J. Aspland, Mark Elstob, J. Turner, Wm. Smart, *and* Wm. Wilkin.

Put $x =$ the required length; then by the nature of the cone, $x : 7\ (= 10 - 3) :: 14 : \frac{7 \times 14}{x} = \frac{98}{x}$; and $\frac{1}{2} \times \overline{\frac{98}{x} + 6} = \frac{98 + 6x}{2x}$ and $\frac{1}{2} \times \overline{\frac{98}{x} + 13} = \frac{98 + 13x}{2x}$ are the girths of the two places respectively; whence $\overline{\frac{98 + 6x}{8x}}\Big|^2 \times 14 + \overline{\frac{98 + 13x}{8x}}\Big|^2 \times \overline{x - 14} = \frac{13}{8}\Big|^2 \times x + 7$, by the question; reduced, $238x = 9604$, and $x = 40\frac{6}{17}$ feet, the length required.——Mr. *G. Coughron*, after his solution, which is very ingenious, refers to Mr. Hutton's Mensuration, whereby may be seen how to cut it so, as that the content of the two parts may amount to the greatest quantity possible.

It is also ingeniously answered by Mess. *J. Bartlett*, *J. Cowley*, the Rev. Mr. *Crakelt*, *J. Dalby*, *R. Dening*, *W. Dent*, *Edw. Fidler*, *W. King*, *Bewley Mellor*, *J. Milbourn*, *W. Reynolds*, *Abr. Robertson*, *Tho. Robinson*, *Alex. Rowe*, *W. Spicer* (the proposer), and *Mich. Taylor*.

III. QUESTION 644 *answered by Mr.* Isaac Dalby.

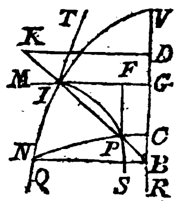
It is easily shewn that the axis of the greatest parab. that can be inscribed in any ellipsis is $\frac{3}{4}$ of its conj. diam. the vertex being at the extremity thereof. Put $100 = 2a$, $845{\cdot}73 = b$, $\cdot 7854 = v$, and let $a + x$ and $a - x$ represent the transverse and conjugate diameters of the ellipse; then $\frac{1}{4} \times \overline{3a - 3x}$ is $=$ the axis of the parab. and $\frac{1}{4} \times \overline{a - x}$ the dist. between the center of the ellipse and the parabola's greatest ord. and by the prop. of the ellipse, $a - x : a + x :: \sqrt{\overline{\tfrac{1}{2} \times \overline{a - x}}\big|^2 - \tfrac{1}{4} \times \overline{a - x}\big|^2} : \overline{a + x}\sqrt{\tfrac{3}{16}} =$ the parabola's greatest ord. and $\overline{aa - xx}\sqrt{\tfrac{3}{16}} =$ its area; whence per quest. $\overline{aa - xx}\sqrt{\tfrac{3}{16}} = \overline{aa - xx} \times v - b$; and by reduction, $xx = aa - b \div \overline{v - \sqrt{\tfrac{3}{16}}} = 100$; consequently $x = 10$, and the diameters of the ellipsis are 60 and 40, and the axis of the parab. 30.

COROL.

COROLLARY. The area of every ellipsis is to that of its greatest inscribed parab. as ·7854 is to $\sqrt{\frac{3}{16}}$, or as 3·1416 to $\sqrt{3}$.

The Rev. Mr. *Crakelt*'s solution is almost exactly the same with this; and it is likewise answered by Mess. *J. Aspland*, *J. Bartlett*, *G. Goughron*, *J. Cowley*, *Wm. Dent*, *Wm. Reynolds*, *Tho. Robinson*, *Alex. Rowe*, *Wm. Smart*, *P. Sharp* (the proposer), *Wm. Spicer*, *Mich. Taylor*, *J. Turner*, and *Wm. Wilkin*.

IV. QUESTION 645 *answered by Mr.* Geo. Coughron.



Let VQR be the given parabola, P the given point, I the required point of contact, and let IG and PC be $\perp$ to VR, and $PF \parallel$ to the same. Then, since PI is to be a minimum, it manifestly must be $\perp$ to the curve at I, and therefore, per sim. Δs, $BG : GI :: PF\ (VC - VG) : FI = GI - CP$; whence $BG \times \overline{GI - PC} = GI \times \overline{VC - VG}$; which equat. is general let the nature of the curve VIQ be what it will, and from which and the relation between VG and GI, the lines VG, GI, &c. may be determined; for both VC and CP are given, and BG is determinable from the equation of the curve.

When the curve is a parab. as given in the quest. let $VC = a$, $CP = b$, the param. $= p$, $VG = x$, and $GI = y$; then, px being $= yy$ and $\frac{1}{2}p = BG$, the above equation becomes $\frac{1}{2}p \times \overline{y - b} = y \times \overline{a - x}$, or, by exterminating x, $y^3 + \overline{\frac{1}{2}p - a} \times py = \frac{1}{2}bpp$; from whence y may be determined.

Answer to the same by Mr. Burrow.

From P, the given point, draw $PC \perp$ to the axis VR, and take CD therein $= \frac{1}{2}$ the parameter, and draw $DK \parallel$ to CP: with DK and DB as asymptotes, describe an hyperb. MS through P, cutting the parab. in I; draw the right line PI, and it will be the min. required.

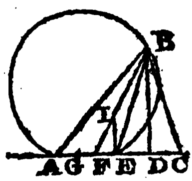
DEMONSTRA. Draw $IG \parallel$ to PC, and continue IP to meet VR and DK in B and K: then $IK = BP$ (by prop. 72. b. 2. of Emerson's Conics), whence $DG = CB$; Consequently $BG = CD =$ the semi-parameter, and therefore (by prop. 11. b. 3. of Emer. Conics aforesaid) BI is the shortest

shortest line that can be drawn from B.——Also, PI is the shortest that can be drawn from P to the curve: for if not, let PN be it, and join B, N; then BN is ⊏ BI (by what has just been shewn), and $BP + PN$ (⊏ than BN, by 20 Euc. 1.) is therefore much greater than BI; take away BP which is common, and there remains PN ⊏ PI; but it was supposed ⊐ PI; therefore PI is the line required; perpendicular to which draw IT, the tangent required.

Ingenious solutions to this question have likewise been received from Mess. *J. Aspland*, the Rev. Mr. *Crakelt*, *J. Dalby*, *S. Hodges* (the proposer), *Tho. Robinson*, *Alex. Rowe*, *Wm. Smart*, *Michael Taylor*, *J. Turner*, and *Wm. Wilkin*.

V. QUESTION 646 *answered by Mr.* J. Chipchase.

CONSTRUCTION. On AE the given diff. of the segments of the base, describe the segment of a circle capable of containing an angle = the given diff. of the angles at the base, and take AG = the given diff. of the base and perpendicular, and bisecting GE in F, erect the perpendicular EI = EG, and draw the right line FIB cutting the circumference in B; from whence upon AE produced, let fall the perpendic. BD, and having taken $DC = DE$, let the points A, B and B, C be joined, and ABC will be the triangle required.

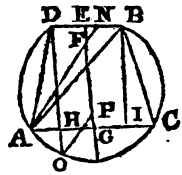


DEMONSTRATION. Join the points E, B; then the angles BEC, BCE are equal, and the $\angle ABE$ is the diff. between the angles at the base BAC, BCA (BEC) by 32 Euc. 1. which is of the given quantity by construction. Also, be-because $DC = DE$ by construc. AE is the diff. of the segments of the base, which by hypoth. is of the given mag. likewise. Lastly, EI being $= EG = 2EF$ (by construc.) DB will be $= 2DF = 2DE + 2EF = CE + EG = CG$; whence the diff. of the base AC and perpendic. DB is $= AG$, the given diff. by construc.——In this solution the base is supposed to be greater than the perpendicular; but if the perpendicular be greatest, the construction will be the same, only AG must be set off the contrary way on the base produced.

In this manner exceedingly near it is also constructed by Mess. *Geo. Coughron*, *H. Curtis*, *J. Turner*, *W. Wales*, and the Rev. Mr. *Wildbore* (the proposer); and Mess. *J. Aspland*, *Wm.*

Wm. Reynolds, *Tho. Robinson*, *Alex. Rowe*, *Mich. Taylor*, and *Wm. Wilkin* have anſwered it algebraically.

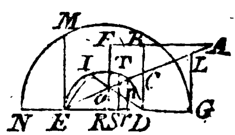
Mr. *Iſaac Dalby*'s conſtruction is conſiderably different, and truly ingenious; and the Rev. Mr. *Crakelt*'s conſtruction is as follows, viz.



Upon DB, equal to the given diff. of the ſegments of the baſe, deſcribe the ſegment of a circle capable of containing the given diff. of the angles at the baſe; and having taken thereon and on an indefinite perpendicular biſecting it, EN and EF reſpectively equal to the half and the whole of the diff. betwixt the perpendicular and the baſe, through N and F draw a right line to meet the circular arc in A: then draw $AGC \parallel$ to BD, and join the points A, B; C, B, and ABC will be the triangle required.

DEMONSTRATION. On AC let fall the perpendiculars DH, BI, and draw DA: then ſince, by 29 Euc. 1. and 29 of 3d, $AG = GC$, and HG (by 34 Euc. 1.) $= DE = EB$ (by conſtruction) $= GI$, therefore will $AH = IC$, and $AI - IC = HI = DB$. Moreover, by ſim. triangles, $FE : EN :: FG : GA$; but by conſtruction FE is $=$ twice EN; wherefore FG will be $=$ twice $AG = AC$; and conſequently $BI - AC = EG - AC = EF$; and the diff. of the angles BCA, BAG is manifeſtly $=$ the $\angle BAD$.

VI. QUESTION 647 *anſwered by Mr.* T. Moſs (*the Propoſer*).



CONSTRUCTION. Biſect DR in r, and take $rG = Er$; alſo, in GE produced, take EN a fourth proportional to m, n and DR, and upon GN deſcribe a ſemicircle, and erect the perpendicular EM meeting its circumference in M: then in FB produced, take $BA = EM$; join E, A and the thing is done.

DEMONSTRA. Draw $GL \perp GE$ meeting AE in L: then, by the prop. of the circle and conſtruc. the rectangle $NEG = EM^2 = AB^2$, and therefore $EG : AB :: AB : EN$; but

but by sim. $\triangle$s, $EG : AB :: GL : BC$; whence, by equality, $AB : GL :: EN : BC$; but by construction, $Er = rG$, $Rr = rD$, and $GL \parallel rn$; wherefore $GL = 2rn$, and consequently the rectangles contained by AB, BC, and EN, $2rn$, and also their halves, are equal: whence m being to n ($:: DR : EN$ by construc.) $:: DR \times rn$ (= trapez. $CDRa$) : $EN \times rn$ ($= \frac{1}{2}AB \times BC = \triangle ABC$).

Mr. G. Coughron

Constructs it by taking BA so, that m may be to $n :: ED^2 - ER^2 : BA^2$; then drawing EA, the thing required will be done.——For, by similarity of $\triangle$s, the $\triangle EDC : \triangle ERa :: ED^2 : ER^2$, and dividedly, the trapez. $CDRa : ED^2 - ER^2 :: \triangle EDC : ED^2 :: \triangle ABC : AB^2$, and alternately, the trapez. $CDRa : \triangle ABC$ $(:: ED^2 - ER^2 : AB^2) :: m : n$, per construction.

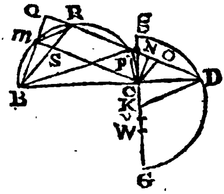
Answer to the same by the Rev. Mr. Crakelt.

CONSTRUCTION. Upon ED describe a semicircle, and inscribe therein $EI = ER$, and divide ED in S in the given ratio of m to n; then take BA (in FB produced) = a fourth proportional to ES, ST ($\perp$ to ED and meeting the circumference in T) and DI, and draw $ACaE$ for the line required.

DEMONSTRATION. By Euc. 6. 19, the $\triangle EDC : \triangle ERa :: ED^2 : ER^2$; $\because$ by division, permutation, &c. the trapezium $CDRa : ED^2 - ER^2 :: \triangle EDC : ED^2 :: \triangle ABC : BA^2$; whence by permutation, &c. the trapez. $CDRa : \triangle ABC :: ED^2 - ER^2$ or $ED^2 - EI^2 =$ (Euc. 1. 47.) $DI^2 : BA^2 ::$ (by construc.) $ES^2 : ST^2 =$ (by prop. of the circle) $ES \times SD ::$ (Euc. 6. 1.) $ES : SD :: m : n$, by construction.

We have likewise been favoured with ingenious and elegant constructions to this question from Mess. *R. Burrow*, *J. Chipchase*, *J. Dalby*, *J. Turner*, and the Rev. Mr. *Wildbore*; and algebraic solutions from Mess. *J. Aspland*, *Wm. Reynolds*, *Tho. Robinson*, *Alex. Rowe*, *Mich. Taylor*, and *Wm. Wilkin*.

VII. QUESTION 648 *anſwered by Mr.* Iſaac Dalby.



CONSTRUCTION. At right angles to the given baſe Cv take CB ſo, that their rectangle may be $=$ the given magnitude. Upon BC let a ſemicircle be deſcribed, and produce BC ſo, that BD may be a fourth proportional to the given parallel line, the baſe Cv and BC; alſo, produce Cv to make vW a fourth proportional to BD, CD, and Cv: Biſect CW in K, and draw the line DK; with which as rad. and cent. K, deſcribe another ſemicircle meeting CW produced both ways in G and g; from D to the former ſemicirc. on BC, apply $DP = CG$, producing it to meet the circumference again in R, and on DR produced let fall the perpendic. BQ, cutting the ſaid ſemicircle in m; join B, R, and BQR will be the triangle required.

DEMONSTRATION. Join Rm, mC, CP, BP, and draw CN, $CO \parallel BQ$, BR reſpectively. By Euc. 3. 35, $gC : CD :: CD : CG$ ($= DP$, by conſtruc.), and by ſim. $\triangle$s, $DO : DC :: DC : DP$; $\because$ (5. 9) $DO = gC$; but, becauſe Kg, KG are equal, and CW is biſected in K (by conſtruc.), WG is $= Cg$; $\because OP = CW$. Moreover, the $\angle$s BmC, BQP being right ones, Cm is $\parallel PQ$; conſequently the arcs mR, CP, and their chords mR, CP are equal to each other reſpectively, and thence $QR = PN$: but, by ſim. triangles, $BD : CD :: BQ : CN :: QR\ (PN) : NO$, and by conſtruc. $BD : CD :: Cv : vW$; whence by equality, $PN : NO :: Cv : vW$, and, compounding, $OP : CW :: NO : vW$; but we have already proved that $OP = CW$; $\because$ (5. 9.) $NO = vW$, and conſequently $PN\ (= QR) = Cv$, the baſe. Again, by ſimilar triangles, $Bm : BQ$ ($:: mS : QR$) $:: BC : BD$; $\because mS$ is $=$ the given parallel line, by conſtruction. Laſtly, by ſim. triangles, $BC : CP\ (mR) :: BR : QR$; whence $mR \times BR = BC \times QR\ (Cv)$, the given magnitude, by conſtruction.

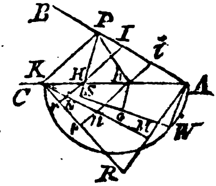
This conſtruction, though leſs conciſe than ſome others, may perhaps be found acceptable to ſome of our readers, as it is derived from firſt principles only, without either tranſforming it to another prob. or calling in the aſſiſtance of any other prob. already known in order to its ſolution, &c. whereby it appears to be an original, independent problem.

Meſſ.

Meff. *G. Coughron*, the Rev. Mr. *Crakelt*, *S. Ogle*, *J. Turner*, *W. Wales*, and the Rev. Mr. *Wildbore* have conftructed this problem in a neat, elegant, and concife manner; and Meff. *J. Afpland*, *T. Robinfon*, *Alex. Rowe*, *Mich. Taylor*, and *Wm. Wilkin* have given algebraic folutions to it.

VIII. QUESTION 649 *anfwered by Mr.* Burrow.

CONSTRUCTION. Draw PK making with the given line AB, the given angle APK, and defcribing on AK a femicirc. take AW = the diff. between AP and PK, and apply it therein from A, and join K, W: then draw $PS \perp$ to KW, cutting AK in H, the point required; for drawing $HI \parallel$ to PK, PHI will be the triangle whofe perimeter is a minimum.



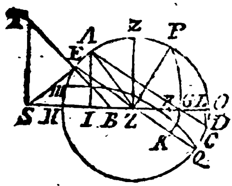
DEMONSTRATION. Draw $AR \parallel$ to PK and $= AP$, and join K, R, and draw $KM \parallel AP$, meeting AR in M, and continue IH to meet KM in N and KR in r: then I fay the perimeter of the $\triangle PHI$ is always $= PH + HS + IN$; for, $KH : KA :: PI : PA :: Hr : AR$; but $PA = AR$ (by conftruc.); $\because PI = Hr$, and HS is always $= Nr$: for fince HS and AW are each $\perp$ to KW, $AW : HS :: AK : HK :: AP : IP :: MK : NK :: MR : Nr$; but $AW = MR$, each being by conftruc. $= AP - PK$: whence $HS = Nr$; therefore $HS + HN = PI$, and $PH + HS + IN =$ the perimeter of the $\triangle PHI$, as was affirmed. Now, the fame holds true wherever the point H is taken betwixt A and K; but of the three lines which conftitute the perim. IN is always the fame, and therefore the perim. will be a min. when the fum of the two variable lines PH and HS is fo, and that will evidently be when they make one ftraight line $PHS \perp$ to KW.

The conftructions by Meff. *G. Coughron*, the Rev. Mr. *Crakelt*, *J. Dalby*. *S. Ogle* (the propofer), *J. Turner*, *W. Wales*, and the Rev. Mr. *Wildbore*, are likewife exceedingly elegant and ingenious; from which, by way of corollary and remark, they have, in a curious manner, pointed out various limitations, &c. to it, but our great want of room at prefent obliges us to omit them.

Meff. *J. Afpland*, *Tho. Robinfon*, *Alex. Rowe*, *Wm. Smart*. *Mich. Taylor*, and *Wm. Wilkin* have folved it algebraically,

IX. QUESTION 650 *answered by the Rev. Mr.* Crakelt.

It will readily appear that the staff will cast the greatest shad. at the given time, when the sun's rays fall upon it in a perpendicular direction; therefore, if TSB and SAZ be supposed two right-angled triangles, having ST (the given staff $= 3\frac{1}{2}$ feet) perpendic. to the horiz. SO, and $= SA$ so inclined to it as to make its shad. SZ a max. as likewise $TB \parallel$ to AZ, they will be sim. and equal, and of course $SZ - SB = BZ = 1{\cdot}522267$ feet $= TB - SB$, and thence the $\angle TBS$ or $AZS = 47^\circ\ 0'\ 42''$ = the sun's merid. altitude.——Now, with ZA rad. describe the circumference of a circle intersecting SZ in H, and when produced in O, and having let fall $AI \perp$ on HZ, drawn $zZ \parallel$ thereto, and made OF = the versed sine of $3^\circ\ 45'$ the given time of the ☉'s rising after midnight, perpendicularly to HO apply to the circumf. of the said circ. LD a fourth proportional to HF, FO, and AI; then draw AD, cutting HO in G, and perpendic. thereto ZP, and PO will be = the latitude required.



For, if PG be the representation of a merid. arc passing through G and meeting EQ drawn through the center $Z \parallel$ to AD, in the point K, by the nature of the orthograph. project. sim. triangles, and construc. it will be $EK : KQ :: AG : GD :: AI : LD :: HF : FO$, or, by composition, $EQ : KQ :: HO : FO$, and consequently, KQ (corresponding to the semi-nocturnal arc GD) $= FO$, answering to $3^\circ\ 45'$ as it ought.

The calculation will from hence be very easy; for, since ZK and KQ are respectively = the cosine and versed sine of $3^\circ\ 45'$, and AI = sine of $47^\circ\ 0'\ 42''$ the ☉'s merid. alt. by supposing ZA or $ZE = 1$, will be had $DL (= KQ \times AI \div EK) = {\cdot}0007839$, the nat. sine of $0^\circ\ 2'\ 42''$ the ☉'s depress. at midnight; therefore by the nat. of the sphere, the comp. of the required lat. HE or $zP (= \frac{1}{2} \times \overline{HA + OD})$ $= 23^\circ\ 31'\ 42''$, and the ☉'s declin. $= EA (= \frac{1}{2} \times \overline{HA - OD})$ $= 23^\circ\ 29'$, giving June 21st for the day on which the traveller performed his journey.——Lastly, if a degree on the equator be supposed to measure $69\frac{1}{2}$ miles, we shall, by sim. sectors, have 1 : sine of $23^\circ\ 31'\ 42'' :: 69\frac{1}{2} \times 7\frac{1}{2} : 208{\cdot}0843$, &c. miles, the length of the required journey.

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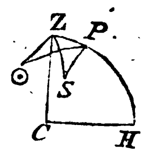
In this manner nearly the solution is also given by Mess. *J. Chipchase* (the proposer), *G. Coughron*, *J. Dalby*, *J. Turner*, *W. Wales*, and the Rev. Mr. *Wildbore*. And Mess. *Edw. Fidler*, *Tho. Robinson*, *Alex. Rowe*, *Elea. Suggett*, *Mich. Taylor*, and *W. Wilkin* have solved it algebraically.

X. QUESTION 651 *answered by the Rev. Mr.* Crakelt.



When the change in alt. or $ZS - Z\odot$, in the given time is a max. the $\angle$s $Z\odot P$, ZSP will, by art. 293 of my translation of Mauduit's Trigonometry, be equal; therefore, if h and h' be put for the hour angles $ZP\odot$ and ZPS, L for the lat. or comp. of ZP, and D for the required declin. or comp. of $P\odot = PS$, we shall have, by art. 214, sin. $h \div \overline{\text{cos. } D \times \text{tang. } L - \text{sin. } D \times \text{cos. } h} =$ tang. $Z\odot P =$ tan. $ZSP =$ sin. $h' \div \overline{\text{cos. } D \times \text{tan. } L \mp \text{sin. } D \times \text{cos. } h'}$, according as the $\angle ZPS$ is acute or obtuse; or, by multiplying, dividing by cos. D, substituting tang. for $\frac{\text{sin.}}{\text{cos.}}$, and transposition, sin. $h' \times$ cos. $h \times$ tang. $D \mp$ cos. $h' \times$ sin. $h \times$ tang. $D =$ (by art. 85.) sin. $\overline{h' \mp h} \times$ tang. $D = \overline{\text{sin. } h' - \text{sin. } h} \times$ tang. L; and therefore, by art. 87 and 92, cos. $\frac{1}{2} \times \overline{h' - h}$: cos. $\frac{1}{2} \times \overline{h' + h}$ :: tang. L : tang. D, and sin. $\frac{1}{2} \times \overline{h' + h}$: sin. $\frac{1}{2} \times h' - h$ :: tang. L : tang. D; which are the two theorems given on p. 189 of the above-quoted book.

We are also favoured with ingenious solutions to this question from Mess. *G. Coughron*, *J. Dalby*, *Wm. Smart*, *W. Wales*, and the Rev. Mr. *Wildbore*.

XI. QUESTION 652 *answered by Mr.* Hutton, (*the Proposer*).

Let x denote any variable height of the fluid in the vessel, and z the corresponding time, $m = 32\frac{1}{6}$, $p =$ the area of the section of the pipe, $h =$ that of the hole in the vessel, and $a =$ the velocity of the water in the pipe.——Then $h\sqrt{mx}$ = the quantity running out of the vessel per second (see p. 4 of my Math. Miscel.), and $ap =$ the quantity running in per second: wherefore $ap - h\sqrt{mx} =$ the rate of the vessel's filling per sec. and consequently $apz - hz\sqrt{mx}$ will be the increase in z time, and is therefore = the flux.

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of the quantity in the vessel. Now, the vessel being given, by the rules of mensuration, the content of the part whose height is x, will be found $= \frac{1}{27} \times \overline{xx + 27x + 243} \times \cdot 7854x$, whose flux. or $\frac{1}{9} \times \overline{xx + 18x + 81} \times \cdot 7854\dot{x}$, must therefore be $= ap\dot{z} - b\dot{z}\sqrt{mx}$: whence $\dot{z} = \frac{xx + 18x + 81}{2s - 2r\sqrt{x}} \times \dot{x}$, by putting the given numbers instead of b, m, a, and p, and making $r = \frac{\sqrt{32\frac{1}{6}}}{32}$, and $s = \frac{\cdot 75}{256}$; or $\dot{z} = \frac{v^5 + 18v^3 + 81v}{s - rv} \times \dot{v}$, by putting vv for x: the fluent of which is $z = -\frac{v^5}{5r} - \frac{sv^4}{4rr} - \frac{ss + 18rr}{3r^3} . v^3 - \frac{ss + 18rr}{2r^4} . svv - \frac{\overline{ss + 9rr}^2}{r^5} . v + \frac{\overline{ss + 9rr}^2}{r^6} . s \times$ hyp. log. of $\frac{s}{s - rv}$; which is a general expression of the time for any alt. vv (x).——Now, when $x = 1$ foot, v is $= 1$ also, and then the above expression brings out 4′ 47·555″ for the time when the water will be one foot high in the vessel.

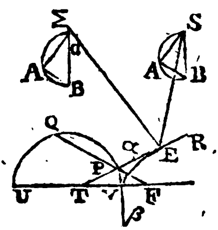
Also, since the greatest height is evidently when the water runs as fast out of the vessel as it runs in by the pipe, $s - rv$ must, in that case, be $= 0$; $\because v = \frac{s}{r} = 1 \cdot 652983$, and x ($= vv$) $= 2 \cdot 732352$ feet, the greatest height of the fluid. But when $s - rv$ vanishes, the expression $\frac{s}{s - rv}$ becomes infinite, and the value of z infinite also of consequence; so that the water will be an infinite time in rising to the said greatest height 2·732352, that height being the limit to which it continually approaches from nothing, but to which it can never attain in any given finite time.

The solution by the Rev. Mr. *Wildbore* is very ingenious, and founded on a new principle, which our narrow bounds cannot contain. Mr. *G. Coughron*, Mr. *J. Dalby*, and the Rev. Mr. *Crakelt* have also favoured us with ingenious answers to it.

XII. QUES-

XII. QUESTION 653 *answered by Mr.* W. Wales; *at at whose Request it was re-proposed.*

Let *VER* represent the conic-section in which the earth is supposed to revolve, *F* the focus, *V* the place of the perihelion, *E* the place of the earth at any assigned time, and *S* that of a star: Draw the tangent *ET*, which will give the direction of the earth's motion at *E*, take $E\alpha$ to *ES*, as the velocity of the earth when at *E*, is to the velocity of light, and $V\beta$, $\perp VF$, as the velocity when at *V*. From *S*, on a plane passing through the star parallel to that of the earth's orbit, draw *SB* and *SA*, = and $\parallel$ to $V\beta$ and $E\alpha$, and *A* and *B* will be the apparent places of the star in that plane, when the earth is at *E* and *V*, on account of aberration. Let fall on *ET* the perpendicular *FP*, which, it is well known, will be reciprocally as $E\alpha$, or its equal *SA*; moreover, *FV* will be reciprocally as *SB*. Now, if the trajectory *VER* be a parabola, the tang. which is drawn parallel to the axe will be at an infinite distance therefrom, and the perpend. thereon from *F* will be so likewise; its reciprocal therefore is = o, and so the path in question passes through the points *S*, *A*, and *B*. Draw *AB*. By a prop. of the parab. $FV : FP :: FP : FT$, and because $FV \times SB = FP \times SA$, $FV : FP :: SA : SB$, and by equality, $FP : FT :: SA : SB$; whence, the $\angle$s *F* and *S* being equal by construction, the Δs *FPT* and *SAB* are similar; and consequently, *P* being a right angle, *A* will be a right angle also, and that where-ever the point *A* may fall; the curve *SAB* (or path in this case, required) is therefore a circle.



But if the earth's orbit *VER* be an hyperbola, and Σ the true place of the star, describe on the transverse axe *UV* the circ. *UQPV*, &c. then it is well known that all perpendiculars from the focus *F* will meet the tangents to this orb on which they are demitted, in the circumf. of this circ. and if *FP* be produced to meet it again in *Q*, by the prop. of the circ. *FP* and *FV* will be reciprocally as *FQ* and *FU*; but they are also reciprocally as $A\Sigma$ and $B\Sigma$; therefore $FU : FQ :: B\Sigma : A\Sigma$; and as the $\angle$s *QFU* and $A\Sigma B$ are equal, and that wheresoever *A* and *Q* fall, it is evident

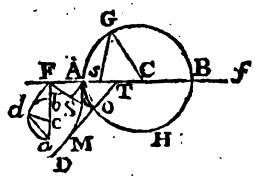
evident that the path of the point *A* will be similar to that of the point *Q*; but that of the point *Q* is known to be a circle; consequently that of the point *A* (or the curve apparently described by the star on account of the abberration, in this case) is also a circle.

SCHOL. I. Because the apparent path of the star will not be on the plane above specified, unless its true place be the pole of the ecliptic, but on one which is perpendicular to the line joining the eye and star, it follows from the nat. of the orthographic projection, that the apparent path will be in every other case an ellipsis, excepting that when the star is in the ecliptic, and then it will be a right line.

II. Because the asymptote becomes a tang. to the hyperb. when both are produced out to an infinite dist. it follows that the perpendic. from the focus on the asymp. will be greatest; therefore its reciprocal will be least, and the star can apparently approach no nearer to its true place than by that dist. Moreover, because the asymp. passes through the center of the circ. *UQPV*, the said perpendic. will be a tang. to it; therefore if from Σ, ∥ to the asymp. a line be drawn = to the said reciprocal, it will be a tang. to the path at its nearest approach to the star's true place; and therefore if the circ. was compleated, the true place of the star would be without it.

Answer to the same by the Rev. Mr. Wildbore.*

Let *AMD* represent the earth's orbit, *AB* its transverse axis when an hyperbola, *F*, *f* its foci, &c. *MT* a tang. to it at *M*, *FO* a perpendic. let fall from *F* upon *MT*, *aF* another upon *Ff*, and *AGBH* the circumference of a circ. described about the diameter *AB*; and suppose, when the earth is at *A*, that *F* and *a* are the real and apparent places of the star



reduced to the plane of the ecliptic. From *F* ∥ to the tang. *MT*, take $Fd : Fa :: FA : FO$, and from *a* upon *Fa* take $ca = dc$, with which as rad. describe the semicircle *adb*, and the arc *ad* will be the apparent locus of the star reduced to the ecliptic, whilst the earth moves from *A* to *M* in an hyperbola; but when the orbit is a parabola, *ca* the rad. of the circ. must be taken $= Fc = \frac{1}{2}Fa$; which circ. will

will likewise be the locus in this case.——For, by the nature of aberration and central motion (vide prop. 2. pa. 3. of Simp. Essays), d is the apparent place of the star, reduced to the ecliptic, when the earth is at M as by construc.), because the velocity of the earth at M is to the velocity when at A, as FA is to FO, per cor. 2d to prop. 1st on pa. 24. of Simp. Essays aforesaid, and (drawing AS to meet FO so that AS and SO may be equal, and joining the points A, O and a, d) the $\angle$s dFa, AFO being equal, and their including sides proportional (by construc.) the $\triangle$s AFO, dFa, and dca, ASO will be similar, and consequently $Fa : Fc :: FO : FS$; but the ratio of FO to FS is always the same (being in the constant ratio of FB to FC, as is easily demonstrated, the circumf. $AGBH$ being the locus of the point O, per prop. 21. part 3. of Steel's Conic-Sections), and therefore the ratio of Fa and Fc will be always the same likewise, and Fa being constant, Fc and ca will be always the same, and the point d be always found in the circumf. of the same circ. adb, as per construction.——But in the case of a parabolic orbit, AC being infinite, the locus of the point O becomes a right line $\perp$ to FA, and as the triangle FAO may be then circumscribed by a semicircle, $AS = SO$ will be $= FS$, and consequently $Fc = ca$.

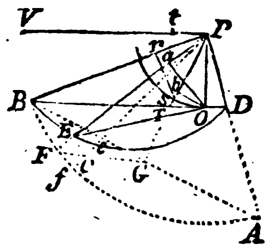
COROLLARY. The proportion $FB : FC :: FO : FS :: Fa : Fc$ also holds when the orbit is an ellipsis; only AC being then greater than FC, SO must be greater than FS, and ca than Fc; wherefore the circ. arc ad will in this case fall partly beyond the axis, and be compleated in one revolution of the earth. But in the case of the hyperbola above, it is evident that the point d, in moving along the arc ad, can never arrive at the position wherein Fd would be $\parallel$ to the asymptote, nor at the point F when the orbit is a parabola.

We have also received very ingenious, elegant geometrical solutions to it from Mess. *Burrow*, *Caughron*, and *Dalby*; and *Peter Puzzlem* has obliged us with a curious analytical solution, but through great want of room, we have been forced, with much regret, to leave it out.

XIII. QUEST. 654 *answered by the Rev. Mr.* Wildbore, (*the Proposer*).

Suppose PO to represent the object, BE the path of the shad. of P from midnight to the time required: draw $PS \parallel$ to the earth's axis, till it cuts the transverse diameter BD. With PO (1) rad. describe the arc OS cutting the axis in S, and

and having joined PD, PB, PE, and OE, let fall $Oa \perp$ to PE, Ob to PS, and Sr to PB. Then 'tis well known that rS is the sine of the co-declination BPS, Ob that of the co-lat. OPS, aO the sine, EO the tang. PE the secant of the co-alt. EPO, and BOE the azim. from the north at the time required. Moreover, the dist. described at E, during the infinitely small instant of time that the velocity can be considered as uniform, is, it is well



known, $=\sqrt{\dot{E}O^2+EO^2\times\angle B\dot{O}E^2}$. But, by Simp. Flux. p. 282, as $B\dot{O}E : E\dot{P}O = \frac{\dot{E}O}{EP^2}$ :: (in the oblique spherical triangle terminated by P the pole, Z the zenith, and $\odot$ the sun, *see fig.* Q. 651) the co-tang. $\angle\odot$: s. $Z\odot$ (the co-alt. $= EPO) = aO = \frac{EO}{EP}$; consequently $B\dot{O}E =$ $\frac{\dot{E}O\,.\,\text{co-t.}\,\odot}{EO\,.\,PE}$: Moreover, as s. $P\odot \times$ cos. $\odot$: s. $Z\odot$ :: $\angle\dot{Z} : \dot{P}$ the fluxion of the time $= \frac{\dot{Z}\times \text{s.}\,Z\odot}{\text{s.}\,P\odot\times\text{cos.}\,\odot}$ $= \frac{B\dot{O}E\,.\,aO}{rS\,.\,\text{cos.}\,\odot} = \frac{\dot{E}O\,.\,\text{co-t.}\,\odot}{rS\,.\,PE^2\,.\,\text{cos.}\,\odot}$. But, by the laws of motion, the distance $\sqrt{\dot{E}O^2+EO^2\times B\dot{O}E^2} = \dot{E}O\times$ $\sqrt{1+\frac{\text{co-t.}\,\odot^2}{PE^2}}$ $\div$ by the time $\frac{\dot{E}O}{rS\,.\,PE^2\,.\,\text{s.}\,\odot}$ is $=$ to the velocity of the uniform motion during that time $= rS.$ $PE\sqrt{PE^2\,.\,\text{s.}\,\odot^2+\text{cos.}\,\odot^2}$, which by the quest. is to be a max. or, because $PE^2 = 1+EO^2$, s. $\odot^2$ + cos. $\odot^2 = 1$, $PE\sqrt{1+EO^2\,.\,\text{s.}\,\odot^2} = PE\sqrt{1+EO^2-EO^2\,.\,\text{cos.}\,\odot^2}$, $PE^2\times\overline{PE^2-EO\;.\,\text{cos.}\,\odot^2}$ is a max. But it is proved by writers on spherics, that the cos. $\angle\odot\times$ s. $Z\odot\times$ s. $P\odot$ $=$ cos. $ZP -$ cos. $Z\odot\times$ cos. $P\odot =$ cos. $\odot\times aO\times rS =$ $Pb - rP\,.\,aP$; but $aO = \frac{EO}{EP}$, and $aP = \frac{1}{EP}(\frac{PO^2}{EP})$, $\because$ cos. $\odot\times EO = \overline{Pb\,.\,EP - RP}\times rS^{-1}$: Consequently

PE^2

$PE^2 \times \overline{PE^2 - Pb.PE - rP|^2 .rS^{-2}} = PE \times \overline{PE + PE.Pb - rP.rS^{-1}} \times PE \times \overline{PE - PE.PB - rP.rS^{-1}}$ is a maximum, or $PE \times \overline{PE.rS + Pb - rP} \times PE \times \overline{PE.rS - Pb + rP}$ a max. take any where, $Pt = \frac{rP}{Pb + rS}$ and $PV = \frac{rP}{Pb - rS}$, and $PE \times \overline{PE - Pt} \times PE \times \overline{PV - PE}$ is a max. wherefore, per Euc. II. 5. and v. 4. $\overline{2PE - Pt} \times PE \times \overline{PV - PE} = \overline{2PE - PV} \times PE \times \overline{PE - Pt}$; whence PE may be readily found by a geometrical construction, or otherwise, $= 89\cdot79712 =$ the co-secant of 38^d 43″, the ☉'s alt. at the time required.*

Ingenious answers to this question have also been received from Mess. *G. Cougbron*, *J. Dalby*, and *W. Wales*.

The

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* As several of our correspondents have complained of the obscurity of the solution above given to this quest. we shall here, at their request, subjoin another solution a little different from it, which may perhaps be better understood by some readers than the above original one.

In order to this, let the following additions, in dotted lines, be made to the figure above, viz. complete the cone PBA, and produce its axe PS to cut BD in T and the diam. AB in the center G of its base; also produce PE to the circumf at F, and draw another line Pef indefinitely near it, on which demit the ⊥ Ec; and, lastly, join G, F.

Then, conceiving the right-angled △ PGB to revolve with an uniform motion about the axe PG till it comes to the position PGF, it is evident that BGF is the angle of time corresponding to the ∠ BOE of azimuth and PEO of altitude; also Ff is as the constant flux. of the time or arc BF, Ee the flux. of the arc BE described by the shadow of P on the horizontal plane BED, ec the flux. of PE, and Ec the flux. of an arc whose rad. is PE. And when the shadow of P moves quickest, then Ee will be greatest in respect of Ff, and therefore we are to find $Ee \div Ff$ a max.

Now put a and b for the sine and cosine of the lat. PTO or BTG, c and d for the sine and cosine of the declin. PBG, and z the sine of the alt. PEO, to the rad. 1. Then, in the spherical △ PZ☉ or PZS in the fig. to the solution of quest. 651, all the sides are expressed, to find the ∠ P of the time, whose cosine therefore, or the cosine of BGF in our figure, will, by spherics, be found $= \overline{ac - z} \div bd$, and therefore the flux. of BGF, or of

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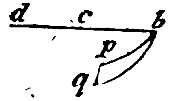
The PRIZE QUESTION *answered by* Peter Puzzlem, (*the Proposer*).



Let bp, bq be circular arcs described from the cent. c in different planes cbp, cbq; and, the $\angle pbq$ being supposed indefinitely small, let pq be an indefinitely small particle of the curve described by the body. Put r, y, and s to denote the sines of the arcs bp, bq, and the angle bpq respectively; also, put u for the angular velocity of the circ. in which the body is always to be found, about bcd, measured at a distance from c = to the rad. cb, and v for the velocity of the body along the said circle. Call the said rad. a, and considering the required force as compounded of two forces, one acting at right angles to the plane of the circ. in which the body is always found, and the other in the direct. of the tang. to the said circ. at the point where the body at any time is; let f and g denote those two forces respectively.

Then $a : u :: y : w \left(= \frac{uy}{a}\right)$, the velocity at q at right angles to bq being denoted by w. Also, $y : s :: r : \frac{\dot r s}{y} =$ sine

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the arc whose cosine is this quantity and radius 1, is $\dot z \div \sqrt{bbdd - \overline{ac-z}|^2}$, which drawn into the rad. BG, or m, gives $F\dot f = m\dot z \div \sqrt{bbdd - \overline{ac-z}\,^2}$. Again, $d : 1$ (rad.) :: m $(BG) : m \div d = PB = PF$, and $z : 1 :: n\ (PO) : n \div z =$ PE; hence $(PF : PE :: F\dot f :)\ E\dot c = nd\dot z \div z\sqrt{bbdd - \overline{ac-z}|^2}$; also $\dot c e = P\dot E = -n\dot z z^{-2}$; ∵ $E\dot e \ (= \sqrt{E\dot c^2 + \dot c e^2}) =$ $\frac{n\dot z}{zz}\sqrt{\frac{bbdd - aacc + 2acz - cczz}{bbdd - \overline{ac-z}|^2}}$; consequently $E\dot e \div F\dot f =$ $\frac{n}{mzz}\sqrt{bbdd - aacc + 2acz - cczz}$ a minimum; whose flux. being equated to 0, we obtain $z = \frac{3a - \sqrt{8dd + aa}}{2c} =$ the sine of 38' 17'', supposing the declin. to be 23° 29'. But if the true present declin. 23° 28' be used, z will come out = the sine of 36° 58'' for the alt. when the shadow of the summit moves fastest.—— And hence the time from midnight is easily found, its cosine being $= \overline{ac - z} \div bd$.

fine of bqp, and $\frac{rs}{y} : w :: a : \frac{awy}{rs}$ the absolute velocity of the body at q along the curve pq; which would be invariable if no force were to act on the body but the tension of the string. Therefore in that case, a, r, and s being considered as invariable whilst w and y vary, $\dot{w}y + w\dot{y}$ would would be $= 0$, and $\dot{w} = -\frac{w\dot{y}}{y} = -\frac{u\dot{y}}{a}$. But the force f continuing to act on the body, $\dot{w}$ will be $= \frac{\dot{u}y}{a} + \frac{u\dot{y}}{a}$ consequently $\frac{\dot{u}y}{a} + \frac{2u\dot{y}}{a}$, the excess of $\frac{\dot{u}y}{a} + \frac{u\dot{y}}{a}$ above $-\frac{u\dot{y}}{a}$, will be the flux. of the velocity of the body at right angles to bq, occasioned by the action of the force f; and therefore, the fluxion of the time being manifestly $= \frac{a\dot{y}}{v\sqrt{aa-yy}}$, $f \times \frac{a\dot{y}}{v\sqrt{aa-yy}}$ will be $= \frac{\dot{u}y}{a} + \frac{2u\dot{y}}{a}$, and $f = \frac{v\sqrt{aa-yy}}{aa} \times \frac{\dot{u}y + 2u\dot{y}}{\dot{y}}$.——Moreover $\sqrt{vv+ww}$, the velocity in the curve pq, would be invariable, if the forces f and g were to cease acting; therefore $v\dot{v} + w\dot{w}$ would in such case be $= 0$, and $\dot{v} = -\frac{w\dot{w}}{v} = \frac{uuy\dot{y}}{aav}$: consequently we have $g \times \frac{a\dot{y}}{v\sqrt{aa-yy}} = \dot{v} - \frac{uuy\dot{y}}{aav} = \frac{aav\dot{v} - uuy\dot{y}}{aav}$, and $g = \frac{\sqrt{aa-yy}}{a^3} \times \frac{aav\dot{v} - uuy\dot{y}}{\dot{y}}$.

Thus is the question solved generally, the required force being compounded of the forces f and g found above; from whence its quantity and direction may be known, and the solution may be easily adapted to particular cases. I shall here only take notice of one particular case, and that is when u is invariable, which is the same as lem. 2. p. 3. of the late Mr. Simpson's Miscell. Tracts. In which case $\dot{u}$ being $= 0$, f appears to be $= \frac{2vu\sqrt{aa-yy}}{aa}$, and its greatest

 value

value (when $y = o$) $= \frac{2vu}{a}$. The gent. I have just now mentioned has considered v as invariable, without taking any notice that it will not be so, unless the body be acted on by a force $g = \mp \frac{uu\dot{y}\sqrt{aa-yy}}{a^3}$. Indeed he has considered the velocity u as very small, and then g will be very small, but not absolutely $= o$, nor yet indefinitely small, for u (though small) being finite, g will be also finite. The solution here given is true whether the velocity u be small or great.——If, u being invariable, g be $= o$, $aav\dot{v}$ will be $= uuy\dot{y}$, and $v = \frac{\sqrt{aaee+uuyy}}{a}$, e being put for the velocity of the body at the commencement of the motion. Therefore it appears that f will in that case be $= \frac{2u\sqrt{aa-yy} \times \sqrt{aaee+uuyy}}{a^3}$; which, when u is very small, is nearly $= \frac{2eu\sqrt{aa-yy}}{a^2}$, the value of f as computed by Mr. *Simpson*.

If u be invariable, and only one force act on the body, that force must be $= \frac{\sqrt{4aavv+uuyy} \times \sqrt{aauu-uuyy}}{a^3}$; and the direction in which it must act, inclined to the plane of the circle in which the body is, in an angle whose sine is to rad. as $2v$ to $\frac{\sqrt{4aavv+uuyy}}{a}$.——Mr. Simpson seems not to have been aware that the force g must necessarily act to keep the velocity v from varying!

Mr *De la Lande*, in his Astronomy (art. 3457) proposing to explain Mr. Simpson's solution, has observed, that, only the force f acting, v will not be invariable when u is so. ——But, without computing the necessary force g, or the exact value of the force f when g is $= o$, he neglects a part of the force f, and entirely neglects the force g, as being what he calls *infiniments petits du troisiême ordre*; whereas they are not generally *infiniments petits* of any order whatever, being assignable quantities which may be considerable, and therefore should not be neglected without first computing their values, and shewing that, in the case in question, they are inconsiderable.

The consequence is obvious, when a number of bodies, kept from flying from the center c by any attractive force, follow one another in the revolving circ. from the point b, each

each having the same velocity (e) when at that point.—— The force f being computed as above, if only that force acts on each, they may be always found in one and the same circle, forming a kind of a ring, but will not revolve uniformly therein, unless each be also acted on by the force g.

The Rev. Mr. *Wildbore* has also favoured us with an ingenious solution to this question, and the prizes of 8 and 12 Diaries for the solution of it are the respective claims of that gentleman and the proposer *Peter Puzzlem*.

The Eclipses calculated for 1773.

This year will present us with two eclipses of the sun and two of the moon, according to Mayer's Tables.

The first will be of the sun, on March the 23d, in the morning, invisible in England, but visible in Asia.

The second will be of the moon, on April the 7th, in the morning, invisible at London, but visible in the pacific ocean.

The third will be of the sun, on September the 16th, in the afternoon, invisible at London, but visible in South America.

The fourth will be of the moon, on September the 30th, in the evening, part visible at London: Beginning 4h. 33m. 25s. Middle 6h. 2m. 44s. End 7h. 32m. 3s. Digits eclipsed 7° 58′ apparent time. Isaac Tarratt.

New Questions.

I. Question 655, *by Mr.* John Shadgett.

Ye ladies, court'ous, kind, and fair,
Who oft mysterious truths declare,
From hence * the name of him you'll find,
Whose heav'n-born muse charm'd all mankind.
Be quick, concise, correct, and true;
Next year I'll do as much for you.

* Given $\left\{\begin{array}{l} y^5 z + x + xy^3 u = 4505639, \\ xu^2 z^2 + yx^2 = 81690, \\ ux^2 + zxy^2 = 18690, \\ x^3 y = 35535 + u^2 z^2; \end{array}\right\}$ to find the values of u, x, y, and z, which denote the places of the letters in the alphabet which compose the poet's name.

II. QUESTION 656, *by Mr.* Wm. Spicer.

Given the legs of a plane triangle equal to 90 and 100, and the rectangle contained by the base and the line bisecting the vertical angle and terminating in the base a maximum, to determine the triangle.

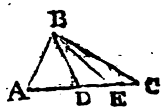
III. QUESTION 657, *by Mr.* Wm. Wilkin.

Required the number of ale gallons contained in a cask composed of two equal conic frustums of the least resistance, lying with its axis parallel to the horizon and the liquor just covering its ends; the bung diameter being 7 and the length 11 feet?

IV. QUESTION 658, *by Mr.* Moss.

To determine all the different ways it is possible to pay 50l. with pistoles at 17s. each, guineas, moidores, and six-and-thirties.

V. QUESTION 659, *by Mr.* Wm. Wilkin.



In the plane triangle ABC, there is given the angle at C, and the parts or segments of the base AD, AE, to construct the triangle so, that, if BD be drawn, the $\angle ABD$ may be a maximum, and BC to EC as m to n.

VI. QUESTION 660, *by Mr.* Steph. Hodges, *at the Right Hon. the Earl* Spencer's, *at* Althorp.

Of all the plane triangles having the same given base and perpendicular, to determine geometrically, that whose vertical angle shall be the greatest.

VII. QUESTION 661, *by Mr.* Moss.

The difference of the sides including a known angle of a plane triangle being given, and also the sum of one of those sides and that opposite the given angle, to construct the triangle.

VIII. QUES-

VIII. Question 662, *by Mr.* Isaac Dalby.

Given the vertical angle, the line bisecting it, and the difference of the segments of the base made thereby, to construct the triangle.

IX. Question 663, *by Mr.* Steph. Ogle.

While a given circular wheel is trundling on an horizontal plane, it is required to determine its point of contact therewith such, that the sum of the altitudes of any four given points in its circumference above the said plane, may be equal to a given quantity M; and to ascertain the limits within which the solution is possible.

X. Question 664, *by the Rev. Mr.* Crakelt.

Through the point of intersection of two given circles it is required to draw a right line in such a manner, that the sum of the respective rectangles under the parts thereof intercepted between the said point and their peripheries, and given lines M and N, may be equal to a given square, R^2.

XI. Question 665, *by the Rev. Mr.* Lawson.

Having given the base and vertical angle of a plane triangle, it is required to find the locus of the extremity of the line that continually bisects the said vertical angle, and is an arithmetical mean between the two sides comprehending the same.

XII. Question 666, *by Mr.* Todd.

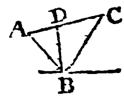
To determine the nature of the curve which will cut at right angles, any number of parabolas having the same vertex and axis.

XIII. Question 667, *by the Rev. Mr.* Wildbore.

To redintegrate (or find the whole fluent of) the expression $\frac{\dot{x}^n}{1-xx}$; where n is any whole positive number whatever.

The PRIZE QUESTION, *by* Peter Puzzlem.

Let ABC be a given triangle, and let the line BD meet AC in D, ſo that BC being greater than AB, AD may be greater than CD: It is required to find with what velocity the ſaid triangle, conſidered as a very thin plate of heavy metal, muſt revolve about BD as an axis, that, whilſt it is ſo revolving, the ſaid line BD ſhall (if poſſible) always keep in an upright poſition, with the angular point B reſting on an horizontal plane?



AN
APPENDIX:

CONTAINING

Additional Solutions to some of the Questions, and the Corrections of such material Errata as have been discovered in the three preceding Volumes.

VOL. I.

PAGE 30, line 5, for ♂ read ♀.——Pa. 71, l. 19, for z read az.

Pa. 74. We shall here supply a solution to Quest. 22 on the principles of Mercator's Sailing. In order to which let $a = BD = 150$; $b =$ the latitude, in minutes, of A the place sailed from; $m = 3437\frac{3}{4}$ the reciprocal of the length of an arc of 1 min. to the rad. 1; $c = m \times : b + \frac{1}{6}b^3 + \frac{1}{24}b^5 + \frac{61}{5040}b^7$ &c. $=$ the merid. parts in the lat. b, or c might be taken from a table of merid. parts; $d = 60$, the dif. between the dif. of lat. and longitude; also put $x =$ the lat. of C, the place arrived at.

Then $m \times : x + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \frac{61}{5040}x^7$ &c. $=$ the merid. parts in the lat. x.

Hence $x - b = AB$, the proper dif. of lat. and $-c + m \times : x + \frac{1}{6}x^3 + \frac{1}{24}x^5$ &c. $=$ the merid. dif. of lat.

Now $AD = \sqrt{\overline{x-b}|^2 - a^2} : DB :: (AB : BC ::)$ merid. dif. of latitude : dif. of long. $= -\dfrac{ac}{\sqrt{\overline{x-b}|^2 - a^2}} + \dfrac{ma}{\sqrt{\overline{x-b}|^2 - a^2}} \times : x + \frac{1}{6}x^3$ &c. But the same dif. of long. is $= AB + 60 = x - b + d$. Then this value being equated to the other, there will result an equation from which the value of x may be found, and thence every thing else.

Pa.

Pa. 102, l. 28, for $5x^3$ read $\overline{5x}|^3$.——Pa. 226, l. 16, for $z =$, read $x =$ ——Pa. 337, as also, some other places, for *Geo. Anderson*, read *Geo. Auderson*.

VOL II.

P. 11, l. 6, for — *diminitive*, read 8128 *perfect*. Also dele all the numbers but 6, 28, 496, and 8128, which are the only perfect numbers.——Pa. 12, l. 9, for *exponent* read *number of terms*. Also l. 19, for 8123 read 8128. And dele all the rest of the line after, as also part of the next line to the number 2196128 inclusive.

In the solution of the former part of the prize quest. given at pa. 17 by the Editor, it must be noted that it is adapted to that case in which the cyl. is of no sensible thickness, or a mathematical line; and therefore after the word *cylinder* in the 9th line from the bottom, add the words *when considered as of no thickness*. Also instead of the last6 lines of the solu. add as here follows: viz. $\frac{c+z}{z}\sqrt{bb+zz} = AF$, *which must be a minimum. This put into fluxions, &c. we get* $z^3 - bbc = 0$; *hence* $z = \sqrt[3]{bbc} = AD$. *And therefore the length of the rod or line AF in this case is* $\frac{c+bq}{q}\sqrt{1+qq}$, *where q is* $= \sqrt[3]{\frac{c}{b}}$.

Pa. 34, l. 14, for $= \frac{18m+4}{19}$ read *or* $\frac{18m+4}{19}$. Also l. 20, for $= \frac{14n+18}{15}$ read *or* $\frac{14n+18}{15}$.

Pa. 65, l. 5 from the bottom, for $\frac{x}{2}$ read $\frac{1}{2}x : x$.

Pa. 148, l. 22, before y^2 write $=$.

Pa. 151, l. 24, 25, and 26, for $\cdot 2^2$, $\cdot 3^3$, and $\cdot 4^4$, read $\cdot 2^{\cdot 2}$, $\cdot 3^{\cdot 3}$, and $\cdot 4^{\cdot 4}$.

QUESTION 262 *answered by Mr.* J. Landen, *F. R. S.*

Of $x^5 - y^5 + axy = 0$, the equation of the curve, take the fluxions; and for x and y put the invariable quantities m and n respectively: repeating the operation till you get an equation consisting only of invariable quantities. By so doing you will get

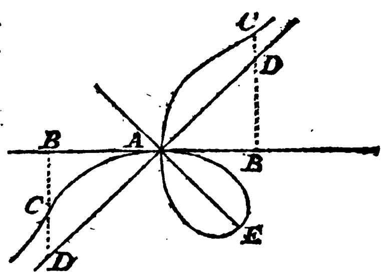
5mx⁴

$$5mx^4 - 5ny^4 + amy + anx = 0,$$
$$10m^2x^3 - 10n^2y^3 + amn = 0,$$
$$m^3x^2 - n^3y^2 = 0,$$
$$m^4x - n^4y = 0,$$
$$m^5 - n^5 = 0:$$

each of which equations, ſhewing the relation of x and y, will (as appears by chap. IX of my *Reſid. Anal.*) expreſs a line which will be an aſymptote to the curve expreſſed by the given equation, when $\frac{m}{n}$ is determined by the final equation $m^5 - n^5 = 0$.

Now by that final equation $\frac{m}{n}$ is $= 1$: therefore each of the equations $5x^4 - 5y^4 + ay + ax = 0$, $10x^3 - 10y^3 + a = 0$, $xx - yy = 0$, $x - y = 0$, is an aſymptotic equation with reſpect to the propoſed curve.

The 4th aſymptotic equation correſponds to a right line, paſſing through the point where x begins, making an angle of $45°$ with the baſe on which x is meaſured. The 3d aſymptotic equation correſponds to two right lines; one of which is the ſame as that expreſſed by the 4th aſymptotic equation; and the other, expreſſed by $x + y = 0$, is (by what is ſaid chap. X of my *Reſid. Anal.*) a diameter of the propoſed curve; which diameter cuts the aſymptote at right angles. The 2d aſymptotic equation correſponds to a line of the 3d order (which is Sir Iſ. Newton's 45th ſpecies) having the ſame aſymptote as the propoſed curve. The 1ſt aſymptotic equation likewiſe appertains to a line of the 3d order (of the ſpecies juſt now mentioned), having the ſame aſymptote as the propoſed curve, and to the right line which I have obſerved is the diameter of that curve. And the form of the propoſed curve, with its rectilineal aſymptote and diameter, is (by chap. VIII, IX, and X of my *Reſid. Anal.*) as in the marginal figure: x being meaſured on AB (a tangent at the node A) to which the ordinate BC (y) is perpendicular; AD the aſymptote; AE the diameter.



☞ Mr. *Farrer*'s Anſwer is very erroneous!

Pa.

Pa. 233. If any person be not satisfied with the solutions to the Prize Q. for 1746, here given, and in the *Miscel. Curios.* therein referred to; he may consult Mr. *Landen's Lucubrations*, where the subject is very fully handled.

P. 248, l. 8, for $\frac{4a}{3}$ read $\frac{4}{3}$. Ibid. l. 13, f. $+xx$, r. $-xx$. Ib. l. 14, f. $+4mx$, r. $-4nx$; also here and in the following parts of the solution, for m write n. Line 15, for $\overline{+4nx}$ and $+xx$, r. $-4nx$ and $-xx$. Line 16, f. $\sqrt{mx+2xx}$, r. $\sqrt{nx}$. Line 18 and 19, f. $8\sqrt{4x+4xx}$, r. $8\sqrt{nx}$. Also in l. 19, for $+4nx$, r. $-4nx$.——It may also be remarked that the solidity required in this question, may be found, without an infinite series, by the rules at p. 335, &c. of my *Mensur.* And using the numbers given in this question, the solidity by those rules will become $= 6{\cdot}48510957$ very accurately.

Pa. 253, l. 19 and 21, for $4hhxx$, read $4bbxx$. And l. 21, f. $bbtt$, r. $3bbtt$. Also the *N.B.* near the bottom is a mistake; for when $x = TL$, then the fluxion of the segment DTH is $n\dot{x} \times bb \times \frac{tx - xx}{tt}$.

But it seems the principle of the solution is not accurately true, but only an approximation, as will appear from the following

Solution of Question 293, *by the Rev. Mr.* Wildbore.

In the original solution to this question, it is said that the greater the vacuity, the greater will be the quantity of liquor, and consequently the sum of both; therefore when the ullage is a max. the whole cask will be also a max. But this method of reasoning is not altogether to be depended on: for, retaining the notation there given, viz. $x = D.m = Gt$, $d = BD$, and $c = Bt$; then the conjugate semiaxes of the surface of the liquor, being the semiordinate passing through t of the circle whose diam. is BG, must be $= \sqrt{cx}$; whence, ($x - c$ being the head, $x + c$ the bung diam. $2\sqrt{dd - xx}$ = the length of the cask, and $p = 3{\cdot}14159$) by Hutton's Mensuration, pa. 278 and 287, we have $\frac{1}{6}p \times \overline{3xx + 2cx + 3aa}\sqrt{dd - xx}$ = the content of the whole cask, and $\frac{1}{6}p \times \frac{3ccx + c^3}{\sqrt{cx}}\sqrt{dd - xx}$ = that of the empty part; consequently the diff. of these two must be the ullage.

But these expressions are so very different, that it is evident at first sight that the one will not be a max. when the other

other is ſo; and conſequently their diff. cannot be a max. when either of them is ſo. But the queſtion expreſsly requires the ullage to be a max. and though this, by reaſon of the ſmallneſs of the vacuity in this particular caſe, be nearly a max. when the caſk is a max. it will not be exactly ſo: and caſes may be put in which they will be very different: and in the preſent, though there be but little difference in the content, the form of the caſk will come out conſiderably different, when the ullage is a max. from what it does when the caſk is ſo. Thus, putting $cx = vv$, when the ullage is a max. then $\overline{\frac{3v^4}{cc} + 2vv + 3cc - 3cv - \frac{c^3}{v}} \times \sqrt{dd - \frac{v^4}{cc}}$ is ſo; the fluxion, reduced, gives $v^9 + \frac{4c^2v^7}{9} - \frac{c^3v^6}{2} - \frac{2d^2c^2 - c^4}{3}v^5 - \frac{c^5v^4}{18} - \frac{2d^2c^4v^3}{9} + \frac{d^2c^5v^2}{6} - \frac{c^7d^2}{18} = 0$; in numbers, $v^9 + 36v^7 - 364\frac{1}{2}v^6 - 161163v^5 - 32804\frac{1}{4}v^4 - 4410450v^3 + 3307837\frac{1}{2}v^2 - 89356612\frac{1}{2} = 0$. Whence $v = 20{\cdot}1424$; $x = 45{\cdot}07958$, head-diam. $= 36{\cdot}07958$, bung-diam. $54{\cdot}07958$, length $= 63{\cdot}01844$, and ullage $= 384{\cdot}4266$ ale gallons.

But if the vacuity be a max. the form of the caſk will be ſtill more different, and the content ſtill leſs. For $\overline{3x + c} \times \sqrt{\frac{dd}{x} - x}$ being a max. its fluxion, reduced, is $3ddx - 9x^3 - cd^2 - cx^2 = 0$; in which caſe therefore x is only $= 29{\cdot}625$.

C. W.

Pa. 270, l. 27, for 350, read 250.——Pa. 274, l. 11, for $1 + z$, r. $v + y$.——Pa. 275, l. 31, f. *retarding*, r. *accelerating*. Line 32, f. *accelerating*, r. *retarding*. And to the note at the bottom add, *Alſo in Turner's Exerciſes, pa.* 77 *No.* 3.

Question 307 *anſwered by Mr.* J. Landen, *F. R. S.*

Let b be the angular velocity of the plane per ſecond, meaſured on an arc of a circle whoſe radius is 1; y the ſine of the inclination of the plane to the horizon, to the ſame radius; z the arc whoſe ſine is y; and x the diſtance of the ball from the axis about which the plane revolves.

Then bx will be the circulatory, or paracentric, velocity of the ball; and bbx will be its centrifugal force.

Moreover, denoting $16\frac{1}{12}$ feet by g, $2gy$ will be the force of gravity urging the ball down the plane. Therefore bbx

$+ 2gy$

$+2gy$ will be the whole force urging the ball from the axis of motion. Suppose its velocity from that axis to be v: then then will $\overline{bbx+2gy}\times\frac{\dot{z}}{b}$ be $=\dot{v}$; and $\frac{\dot{z}}{b}=\frac{\dot{x}}{v}$, or $v=\frac{b\dot{x}}{\dot{z}}$.

Hence, $\dot{v}$ being $=\frac{b\ddot{x}}{\dot{z}}$ when $\dot{z}$ is supposed invariable, we get by substitution $\overline{bbx+2gy}\times\frac{\dot{z}}{b}=\frac{b\ddot{x}}{\dot{z}}$. It appears therefore that $\frac{2g}{bb}y\dot{z}^2$ is $=\ddot{x}-x\dot{z}^2$: whereof the equation of the fluents is $n^z-n^{-z}=\frac{2bbx}{g}+2y$, n being the number whose *hyp. log.* is 1; as appears by what follows.

Multiplying each side of the equation $\frac{2g}{bb}y\dot{z}^2=\ddot{x}-x\dot{z}^2$ by n^z, we have $\frac{2g}{bb}yn^z\dot{z}^2=n^z\ddot{x}-xn^z\dot{z}^2$: whence, by taking the fluents, we find the fluent of $\frac{2g}{bb}yn^z\dot{z}^2=n^z\dot{x}-xn^z\dot{z}$.

Now, denoting the cosine $\sqrt{1-yy}$ by w, $\dot{w}$ is $=-y\dot{z}$, and $\dot{y}=w\dot{z}$; therefore $y\dot{z}$ is $=-\dot{w}-w\dot{z}+\dot{y}$, and it is obvious that, by adding $y\dot{z}$ on each side and multiplying by $\frac{gn^z\dot{z}}{bb}$, we shall get $\frac{2g}{bb}yn^z\dot{z}^2=\frac{g}{bb}\times\overline{yn^z\dot{z}^2+\dot{y}\dot{z}n^z-\dot{w}n^z\dot{z}^2-\dot{w}\dot{z}n^z}$. Whence it appears that the fluent of $\frac{2g}{bb}yn^z\dot{z}^2$ is $=\frac{g}{bb}\times\overline{yn^z\dot{z}-wn^z\dot{z}+\dot{z}}$ $=n^z\dot{x}-xn^z\dot{z}$ by what is said above. Hence, by division, we have $n^{-z}\dot{x}-xn^{-z}\dot{z}=\frac{g}{bb}\times\overline{n^{-2z}\dot{z}+yn^{-z}\dot{z}-wn^{-z}\dot{z}}$ $=\frac{g}{bb}\times\overline{n^{-2z}\dot{z}+yn^{-z}\dot{z}-n^{-z}\dot{y}}$: from which equation, by again taking the fluents, we get $xn^{-z}=\frac{g}{bb}\times\overline{\tfrac{1}{2}-\tfrac{1}{2}n^{-2z}-yn^{-z}}$; and consequently $n^z-n^{-z}=\frac{2bbx}{g}+2y$, as expressed above.

By

By means of this equation of the curve described by the ball, the velocity $v\ (=\frac{b\dot{x}}{\dot{z}})$ is found $=\frac{g}{2b}\times\overline{n^{z}+n^{-z}-2w}$.

To find the pressure against the plane: let p be the perpendicular from the center of motion to the tangent to the curve described by the ball, the ray from that center to the point of contact being x; and let q be the absolute velocity of the ball in the curve at that point. If then the gravity ceased to act on the ball, it would proceed in the direction of the said tangent, and (p and q being in that case invariable) the fluxion of $(\frac{pq}{x})$ its circulatory velocity would be $=$ $\frac{-pq\dot{x}}{xx}$; which expression is $=-b\dot{x}$, $\frac{pq}{x}$ being $=bx$. But the gravity acting, and the ball pressing against the plane, the fluxion of (bx) its circulatory velocity is $=b\dot{x}$. Therefore $2b\dot{x}=2v\dot{z}$ is the fluxion of the circulatory velocity arising from $(2gw-P)$ the excess of the force of gravity at right angles to the ray x above P the pressure against the plane. Consequently $2v\dot{z}$ is $=\overline{2gw-P}\times\frac{\dot{z}}{b}$: whence P $=2gw-2bv=g\times\overline{4w-n^{z}-n^{-z}}$.

When the ball quits the plane P is $=0$, and $4w=n^{z}+n^{-z}$; from which equation z is found to be, at that instant, equal to (·823766) an arc of 47° 11′ 54″. Mr. *Simpson*, finding the fluents by infinite series, makes the angle 47° 9′.

Remark. If, instead of the plane, a *tube* be substituted, continued both ways from the axis of motion, just capable of receiving the ball so that it may move freely therein; and, the instant the tube begins to revolve from a horizontal position, the ball be made to move therein from the said axis, along the ascending part of the tube, with a velocity $=\frac{g}{b}$; the fluxionary equation adapted to this case will be $\frac{2g}{bb}y\dot{z}^{2}=x\dot{z}^{2}-\ddot{x}$: and, by correcting the fluents accordingly, x will from thence be found equal to the simple expression $\frac{gy}{bb}$! Which equation suggests this *very remarkable inference*: The ball being at first put in motion with the

the velocity $\frac{g}{b}$, it will revolve uniformly in a circle (whose diameter is $\frac{g}{bb}$) touching the horizontal line with which the tube at first coincides! and it will continue so to revolve (moving up and down alternately in the different branches of the tube) so long as the motion of the tube is continued, making two complete revolutions whilst the tube makes one revolution! and the uniform velocity ($\frac{g}{b}$) wherewith the ball so revolves in the circle will be to its velocity along the tube every where as 1 to w. J. L.

A solution to Quest. 311 may be seen in Ozanam's Dictionary, and in Euler's Algebra. We shall also here insert the following solutions to it, viz.

I. *By Mr.* J. Landen, *F. R. S.*

To find three such numbers, that the sum and difference of any two of them shall be square numbers. Supposing x, y, and z to be the required numbers; we may assume

1st, $x = \frac{1}{2} \times \overline{f^4g^4 + g^4 + f^4 + 1}$,

$y = \frac{1}{2} \times \overline{f^4g^4 - g^4 - f^4 + 1} + 2ffgg$,

$z = \frac{1}{2} \times \overline{f^4g^4 - g^4 - f^4 + 1} - 2ffgg$;

whence the values of $x+y$, $x-y$, $x+z$, $x-z$, and $y-z$, are manifestly squares. Therefore, to solve the question, it only remains to find $f^4g^4 - g^4 - f^4 + 1$ (the value of $y+z$) a square.

Or 2dly we may assume

$x = ffgg + 1$,

$y = ff + gg$,

$z = 2fg$;

where the values of $x+z$, $x-z$, $y+z$, $y-z$, are manifestly squares. Therefore, after this assumption, we have to find $ffgg + gg + ff + 1$ (the value of $x+y$) a square, and $ffgg - gg - ff + 1$ (the value of $x-y$) a square.

It is obvious, that, according to this assumption, the value of the expression $f^4g^4 - g^4 - f^4 + 1$ ($= x+y \times \overline{x-y}$) must be a square as in the 1st assumption where it is $= y+z$.——To find that expression a square, substitute $f+r$ instead of g; it will then bcome $\overline{f^4-1}|^2 \times$

$1 + \frac{4f^3r + 6ffrr + 4fr^3 + r^4}{f^4 - 1}$. It appears therefore

that

that $1+\frac{4f^3r+6ffrr+4fr^3+r^4}{f^4-1}$ must be a square: supposing it $=\overline{1+\frac{2f^3r}{f^4-1}+\frac{f^6-3ff\times rr}{\overline{f^4-1}|^2}}|^2$, from that equation r will be found $=\frac{4f\times\overline{f^8-1}}{1+6f^4-3f^8}$, and $g\ (=f+r)$ $=\frac{f\times\overline{f^8+6f^4-3}}{1+6f^4-3f^8}$. Therefore f being any number whatever in the 1st assumption, and g as here found, the question will be answered.

It is plain that, $f^4g^4-g^4-f^4+1$ being a square, its factors $ffgg+gg+ff+1$ and $ffgg-gg-ff+1$ will be squares if $\frac{gg\pm1}{ff\pm1}$ be a square. Now, taking g equal to its value found above, it will appear that the value of $\frac{gg\pm1}{ff\pm1}$ is $=\overline{\frac{f^8\pm4f^6-6f^4\pm4f^2+1}{1+6f^4-3f^8}}|^2$. Consequently taking f any number whatever, and g as just now mentioned, the question will be answered by the 2d assumption in much lower terms than by the 1st assumption.——If f be taken $=2$, the numbers given in the Diary 1750 will be obtained.

There is another way to find $f^4g^4-g^4-f^4+1$ $(=\overline{f^4-1}\times\overline{g^4-1})$ a square. Assume it $=\overline{f^4-1}|^2\times\overline{g^2+1}|^2$, or $=\overline{f^4-1}|^2\times\overline{g^2-1}|^2$: whence $g=\frac{ff}{\sqrt{2-f^4}}$, or $=\frac{ff}{\sqrt{f^4-2}}$. Consequently, if, in the 1st assumption, f be so taken that $2-f^4$, or f^4-2, be a square, and $g=\frac{ff}{\sqrt{2-f^4}}$, or $\frac{ff}{\sqrt{f^4-2}}$ respectively, the quest. will be answered.

To find $2-f^4$ a square, I write $1-d$ for f: by which means the expression becomes $1+4d-6d^2+4d^3-d^4$, which being supposed $=\overline{1+2d-5dd}|^2$, we find $d=\frac{12}{13}$, $f\ (=1-d)=\frac{1}{13}$, and $g=\frac{1}{239}$. Therefore, if, in the 1st assumption, f be $=\frac{1}{13}$ and $g=\frac{1}{239}$, or $f=13$ and $g=239$, the quest. will be answered.

To find $f^4 - 2$ a square, I suppose it $= \overline{ff - hh}^2$; whence $ff = \frac{4 + 2h^4}{4hh}$. Therefore $4 + 2h^4$ must be a square.

To find it so, I suppose it $= \overline{2 + bbhh}^2$; and, by this supposition, I find $hh = \frac{4bb}{2 - b^4}$; where, if b be $= 1$, hh will be $= 4$; and consequently $f = \frac{3}{2}$, $g = \frac{9}{7}$.

Having found $2 - f^4$ a square when f is $= \frac{3}{17}$, it is obvious that hh (and consequently $f^4 - 2$) will be found a square by taking $b = \frac{3}{17}$. And other values of f and g may be found in like manner.

II. *By the Rev. Mr.* Wildbore.

Suppose $2abxy$, $aaxx + bbyy$, and $aayy + bbxx$ to be the three numbers required. Then the sums and differences $aaxx \pm 2abxy + bbyy$ and $aayy \pm 2abxy + bbxx$ being necessarily squares, it only remains to make $aaxx + bbyy + aayy + bbxx$ and $aaxx + bbyy - aayy - bbxx$ squares; their product therefore $= a^4x^4 - b^4x^4 - a^4y^4 + b^4y^4$ must be a square: make $x = z - \frac{ay}{b}$; then substituting this value for x in the last expression, it will thence appear that $a^8y^4 - 2a^4b^4y^4 + b^8y^4 - 4a^7by^3z + 4a^3b^5y^3z + 6a^6bbyyzz - 6aab^6yyzz - 4a^5b^3yz^3 + 4ab^7yz^3 + a^4b^4z^4 - b\ z^4$ must be a square. Suppose its side $= \overline{a^4 - b^4} . yy - 2a^3byz + \frac{a^4 - 3b^4}{a^4 - b^4} aabbzz$; this squared and made equal to the other, will give, by reduc. $-4a^9y + 4ab^8y = -3a^8bz + 6a^4b^5z + b^9z$; whence $z = 4a^9 - 4ab^8$, and $y = 3a^8b - 6a^4b^5 - b^9$, and consequently $x = a^9 + 6a^5b^4 + 3ab^8$. These values being substituted in the assumed expressions, they will become respectively

$$6a^{18}b^2 + 24a^{14}b^6 - 92a^{10}b^{10} + 24a^6b^{14} + 6a^2b^{18},$$
$$a^{20} + 21a^{16}b^4 - 6a^{12}b^8 - 6a^8b^{12} + 21a^4b^{16} + b^{20},$$
$$10a^{18}b^2 - 24a^{14}b^6 + 60a^{10}b^{10} - 24a^6b^{14} + 10a^2b^{18}.$$

Which are general theorems for infinite answers, where a and b may be taken any numbers at pleasure; the only limitation being that $6a^4b^4 + b^8$ must be less than $3a^8$. And when $a = 2$, and $b = 1$, the three numbers answering the question are 2288168, 2399057, and 1873432, as put down at page 284.

III. By

III. *By the* Editor.

Three numbers anſwering the conditions may be found by the following eaſy proceſs.

Aſſume $4x$, $4+xx$, and $1+4xx$ for the three numbers; where the ſum and diff. of the firſt and each of the other two being all four ſquares, we have only to make the ſum and diff. of the two latter to be ſquares, viz. $5xx+5$; and $3xx-3$ each a ſquare. Their product $15x^4-15$ will therefore be a ſquare. Which it will evidently be when $x=2$; for then it is $15^2=225$. And then the three aſſumed numbers become 8, 8, and 17, which anſwer the conditions. But as two of the numbers are equal to each other, make $x=z-2$; then $15x^4-15$ is $=225-480z+360z^2-120z^3+15z^4=$ a ſquare $=$ ſuppoſe $\overline{15-az+bzz}|^2=225-30az+\left.\begin{matrix}30b\\+aa\end{matrix}\right\}z^2-2abz^3+b^2z^4$. Here equate the 2d term of the one of theſe to the 2d of the other, and a is found $=16$; then equate the 3d to the 3d, and there reſults $b=\frac{52}{15}$; and finally the laſt two terms of the one equated to the laſt terms of the other, we have $z=\frac{2040}{671}$. Hence $x=z-2=\frac{698}{671}$; which ſubſtituted in the aſſumed expreſſions, they become $4\times\frac{698}{671}$, $4+\frac{698^2}{671^2}$, and $1+\frac{4.698^2}{671^2}$; or, by multiplying each by 671^2, they become 1873432, 2288168, and 2399057, the very ſame numbers as before.

Pa. 289, in Queſt. 316, when the given dividing line is conſidered as making given angles with the two ſides of the triangle about a given vertical angle, it is conſtructed by theor. 8 pa. 199 Simpſon's Geom. And when the area is a given quantity inſtead of a min. it is conſtructed in prob. 5 pa. 214.

QUESTION 318 *anſwered by the Rev. Mr.* Wildbore.

As no notice whatever is taken of the given breadth of the viſto, in the original ſolution, it cannot be right. For, though the $\triangle$s GeE, oHF in this particular example be ſmall in reſpect of the $\square eoFE$, the breadth of the viſto may be ſuppoſed increaſed till they even infinitely exceed

it. These three things (the breadth and two Δs) must not therefore be omitted in the solution, which may be thus.

The area of the visto is evidently equal to the sum of these two Δs and $\square$. And Ge being, by sim. triangles, $=\frac{ED.Ee}{FD}$, and $Ho=\frac{FD.Ee}{ED}$; the visto $=\frac{ED.Ee^2}{2FD}$ $+\frac{FD.Ee^2}{2ED}+FE.Ee$ a minimum, or $\frac{Ee}{2}\times\frac{ED}{FD}+\frac{FD}{ED}$ $+\sqrt{FD^2+ED^2}$ a min. At B erect $BS\perp FD$ meeting FE in S. Then, by sim. Δs, $SB=ED\times\frac{FD-BD}{FD}$ and, by conics, $SB=\frac{Tt^2}{ED}$; hence $ED^2=\frac{FD.Tt^2}{FB}=$ $Tt^2+\frac{BD.Tt^2}{FB}$.

Make now $FB=u$, $BD=2b$, $Tt=c$, and $Ee=d$. Then $FD=2b+u$, $ED=c\sqrt{\frac{2b+u}{u}}$, $\frac{ED}{FD}=\frac{c}{\sqrt{2bu+uu}}$, and $\frac{dc}{2\sqrt{2bu+uu}}+\frac{d\sqrt{2bu+uu}}{2c}+\sqrt{\overline{2b+u}^2+cc.\frac{2b+u}{u}}$ is a min. The flux. made $=0$, &c. there results $d\times\frac{b+u}{2c}$ $\times\frac{2bu+uu-cc}{2b+u}=\frac{bcc-uu.\overline{2b+u}}{\sqrt{2bu+uu+cc}}$. Whence u may be found by an equation of the 8th power, or by the well-known rule of position; and thence the rest. Thus in the present case $u=24{\cdot}863187$. Whereas when EF only is a min. u is $=25{\cdot}26757$, $EF=153{\cdot}702$, and the mean length $=157{\cdot}552$. But the mean length determined as above is $=157{\cdot}545$, and the quantity of land in the visto $\cdot02545$ parts of a square pole less than when EF is a minimum.

C. W.

Pa. 332, l. 16, f. $2c+4d$, r. $\frac{2c+4d}{3}$.

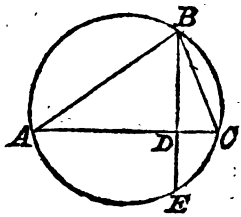
Pa. 335, l. 3, f. HI, r. HF. Line 5, f. $2aabb$, r. $2abbxx$]

Pa. 336, l. 7, in the value of $2v^2$, after $2bac$ write $+ss$ in the denominator.

QUES-

QUESTION 340 *Constructed by the Rev. Mr.* Wildbore.

Having made two rectangles equal to the given ones, so that the given perpendicular may be the shortest side of the greater and the longest side of the less, upon the longest side of the greater as a diam. describe the circ. ABC, any where in which apply $BE =$ the sum of the two sides of the less rectangle, on which take $BD =$ the given perpendicular; through D draw $AC \perp BE$, cutting the circle in A and C; then, AB, BC being drawn, ABC is the required triangle.



For, by Simpson's Geom. III. 25, calling the diam. of the circle D, the rectangle $D \times DB = AB \times BC =$ the given one by construc. and, by III. 21, $AD \times DC = BD \times DE$ $=$ the less given one by constr. and BD being the given perp. ABC must be the triangle required.

COROL. Since $D \times DB = AB \times BC$, and $DE \times BD = AD \times DC$, the two given rectangles are in the ratio of D to DE; $\therefore$ when, instead of the perpendicular, the vertical angle is given; AC being assumed at pleasure, and the circle described to contain thereon the given angle; and DE taken to D in the given ratio of the rectangles, and continued to B; ABC will be a triangle similar to the required one. C. W.

Pa. 343, l. 17 and 18, for ·243 read ·226.——Pa. 387, l. 17, f. p, r. q; and f. q, r. r.——Pa. 390, l. 8, f. $aa + aa$, r. $aa + cc$.

VOL. III.

Pa. 16, l. 13, f. $PP + 1$, r. $P.\overline{P+1}$.——Pa. 34, l. 8, f. $x^5 + x^5$, r. $u^5 + x^5$.——Pa. 45, l. 1, f. is the time, r. is as the time. Line 26, f. ABD, r. ADB.——Pa. 61, l. 2, f. $4ab$, r. $4abs$.——Pa. 67, l. 12, f. $\surd$&c. r. $y\surd$&c.——Pa. 93, l. 13, f. 10°, r. 19°.——Pa. 96, l. ult. f. $4pxx$, r. $4px$.——Pa. 123, l. 3, after *consequently* put $y =$.——Line 4, for *we*, r. *as*.——Pa. 160, l. 19, f. 298, r. 289. Lines 21 and 25, for the decimal point write the sign of multiplication.

As a second *Corol.* to the solution to quest. 511 at pa. 184, it may be remarked that, as td is $= DC$ in the fig. pa. 186, therefore the area or illuminated surface tdd becomes $d \times DC$; that is, the diam. of the cyl. drawn into the alt. of the illuminated surface. Which agrees with the rule proved in corol. 1, p. 148 of my Mensuration, from whence the surface might have been at first easily found.

QUESTION 579 *answered by Mr.* J. Landen, *F. R. S.*

If $\overline{1+x}|^4 \times \overline{1+x^2}|^4 \times \overline{1+x^3}|^4 \ (9) \times \overline{1+x^{10}}|^{16}$ be actually involved, the coefficient of that power of x whose exponent is 15 will be the number sought.

To find the coefficient with facility, observe that the above expression is $= \overline{1+x}|^4 \times \overline{1+x^2}|^4 \times \overline{1+x^4}|^4 \times \overline{1+x^8}|^4 \times \overline{1+x^3}|^4 \times \overline{1+x^6}|^4 \times \overline{1+x^5}|^4 \times \overline{1+x^7}|^4 \times \overline{1+x^9}|^4 \times \overline{1+x^{10}}|^{16} = \overline{1+x+x^2+x^3 \ldots\ldots x^{15}}|^4 \times \overline{1+x^3+x^6+x^9}|^4 \times \overline{1+x^5}|^4 \times \overline{1+x^7}|^4 \times \overline{1+x^9}|^4 \times \overline{1+x^{10}}|^{16}$.

Now $\overline{1+x+x^2+x^3 \ldots\ldots x^{15}}|^4$ being $= \overline{\frac{1-x^{16}}{1-x}}\Big|^4 = \overline{1-4x^{16}+\frac{4.3}{1.2}x^{32}\ \&c.} \times \overline{1+4x+\frac{4.5}{1.2}x^2+\frac{4.5.6}{1.2.3}x^3\ \&c.}$ it appears, that, in the value of $\overline{1+x+x^2+x^3 \ldots\ldots x^{15}}|^4$ the terms wherein the exponents of the powers of x are not greater than 15 are $1+4x+\frac{4.5}{1.2}x^2+\frac{4.5.6}{1.2.3}x^3$ &c. $(16) = \frac{16.17.18}{1.2.3}x^{15}+\frac{15.16.17}{1.2.3}x^{14}+\frac{14.15.16}{1.2.3}x^{13} \ldots\ldots\ldots 4x+1$. It appears also (by an easy involution) that, in the value of $\overline{1+x^3+x^6+x^9}|^4 \times \overline{1+x^5}|^4 \times \overline{1+x^7}|^4 \times \overline{1+x^9}|^4 \times \overline{1+x^{10}}|^{16}$, the terms wherein the exponents of the powers of x are not greater than 15 are $1+4x^3+4x^5+10x^6+4x^7+16x^8+24x^9+38x^{10}+40x^{11}+63x^{12}+128x^{13}+102x^{14}+212x^{15}$. It is obvious therefore, that the coefficient sought is $= 1 \times \frac{16.17.18}{1.2.3} + 4 \times \frac{13.14.15}{1.2.3} + 4 \times \frac{11.12.13}{1.2.3} + 10 \times \frac{10.11.12}{1.2.3}$ &c. $(13) = 816+1820+1144+2200+660+1920+2016+2128+1400+1260+1280+408+212 = 17264$.

Pa.

Pa. 340. Concerning Mr. Wildbore's solution there given to the prize quest. for the year 1770, he 'desires it may 'be observed that in the solu. to that quest. the motion of 'the bucket is supposed to commence when the water therein, and that in the reservoir, are both at rest, with their 'surfaces on the same horizontal level.'

Question 653 *answered by Mr.* J. Landen, *F. R. S.*

Lemma 1. If $\frac{2d}{1 \backsim ee}$ and $\frac{2d}{\sqrt{1 \backsim ee}}$ be the axes of a conic section, the curve will be a circle if e be $= 0$; and, according as e is less, equal to, or greater than 1, the curve will be an ellipsis, a parabola, or a hyperbola.

The distance of the focus f from the vertex a, of any conic section, will be $= \frac{d}{1+e}$: and, y being put to denote the ray fp from the said focus to any point p of the curve, the perpendicular fq, from the same focus upon the tangent pq, will be $= d\sqrt{\frac{y}{2d + \overline{ee-1} \cdot y}}$. Moreover $\sqrt{\frac{2dy + \overline{ee-1} \cdot yy - dd}{2dy + \overline{ee-1} \cdot yy}}$ will be the sine of the angle pfq, radius being 1; and, that sine being always to the sine of the angle afq in the constant ratio of e to 1, the sine of the last-mentioned angle will be $= \frac{1}{e}\sqrt{\frac{2dy + \overline{ee-1} \cdot yy - dd}{2dy + \overline{ee-1} \cdot yy}}$.

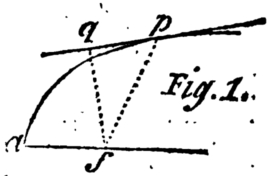


Fig. 1.

Lemma 2. The earth being supposed to describe a conic section whose axes are as expressed in the preceding lemma, let its velocity at the vertex a be denoted by $b \times \overline{1+e}$: then will its velocity at any other point p of its trajectory be $= b\sqrt{\frac{2d + \overline{ee-1} \cdot y}{y}}$, being reciprocally as the perpendicular from the focus upon the tangent.

PROPOSITION.

The velocity of the earth in its trajectory being to the velocity of light as $b\sqrt{\frac{2d+\overline{ee-1}\cdot y}{y}}$ to c; it is proposed to determine the linear aberration of a star from its true place, measuring such aberration on a plane passing through the star parallel to the plane of the said trajectory?

Let s be the star, and p the earth in its trajectory moving from the vertex a; and, qkp being a tangent to the trajectory, let pk be to ph as the velocity of the earth at p to the velocity of light, *i. e.* as $b\sqrt{\frac{2d+\overline{ee-1}\cdot y}{y}}$ to c: then, phs being the line joining the earth and the star, if the angle spt be equal to the angle phk, st parallel to pk will be the linear aberration sought, at the time the earth is at p, the star appearing to be at t instead of its true place s.

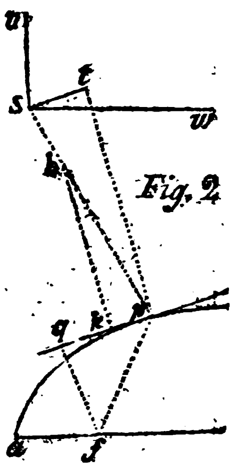


Fig. 2

Hence it follows that ps (the distance of the star from the earth) being denoted by D, and st by z, z will be $=\frac{bD}{c}\sqrt{\frac{2d+\overline{ee-1}\cdot y}{y}}$ $=r\sqrt{\frac{2d+\overline{ee-1}\cdot y}{y}}$, r being put for $\frac{bD}{c}$.

Moreover, if sw be parallel to the axis af, and sv perpendicular to sw; the angle tsv will, it is obvious, be equal to the angle afq made by af and the perpendicular fq on the tangent pq.——Let the sine of the angle tsv (or afq) be denoted by v: then, by lemma 1, v will be $=\frac{1}{e}\sqrt{\frac{2dy+\overline{ee-1}\cdot yy-dd}{2dy+\overline{ee-1}\cdot yy}}$; whence $y=\frac{d}{1-ee}\mp\frac{de\sqrt{1-vv}}{\overline{1-ee}\cdot\sqrt{1-eevv}}$.

But

But from the equation $z = r\sqrt{\frac{2d + \overline{ee-1}.y}{y}}$ (found above) we get $y = \frac{2drr}{zz + \overline{1-ee}.rr}$. From which two values of y it appears that z will be $= r\sqrt{1-eevv} \pm er\sqrt{1-vv}$.

Let AEB be a semicircle whose center is C; and in the diameter ACB (or the continuation thereof) take any point D; moreover let CE, DE be drawn and denoted by r and z respectively: then, the sine of the angle CDE being denoted by v, and the distance CD by m, $z\sqrt{1-vv} \mp \sqrt{rr-vvzz}$ will be $= m$. Hence $z = r\sqrt{1-\frac{mmvv}{rr}} \pm m\sqrt{1-vv}$.

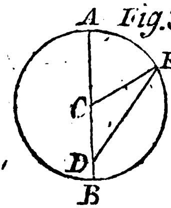


Fig. 3.

Comparing this equation with that which we just now found above, we may observe that they will be exactly alike if m be equal to er.——It follows therefore, that, s being the true place of the star, sw parallel to the axis (af) of the earth's trajectory, sv perpendicular to sw, and st so drawn that the sine of the $\angle tsv$ be equal to v: if sc be taken (upon sv) equal to er ($= \frac{beD}{c}$); and from the center c, with a radius equal to r ($= \frac{bD}{c}$), an arc be described intersecting st in t; the point t so determined will be the apparent place of the star in the plane passing through the star parallel to the earth's trajectory, when the earth is at such a point p thereof that the sine of the angle afq (fig. 2) be $= v$.

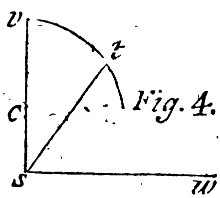


Fig. 4.

If D be invariable, (which can only be when the earth's trajectory is a circle, and the star is situated in a line erected perpendicular to the plane of the orbit from the center thereof) the curve of aberration, on our said plane parallel to the trajectory, will be a circle whose center is c, e being then $= 0$.

If the trajectory be an ellipsis not very eccentric, and the distance of the star from the earth be at any time *very great* in comparison with the distance of the earth from the focus; D may be considered as continuing of *nearly* the same value whilst

whilst the earth describes its whole orbit: our curve of aberration will therefore, in such case, be *nearly* (but not exactly) a circle.

Whether D be considered as invariable or variable, z (or the star's apparent place) will be truly determined by our construction; and by means thereof the star's true place may be readily found from its observed apparent place.

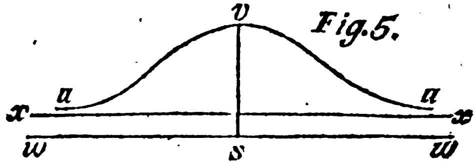


Fig. 5.

Supposing the trajectory to be a parabola, and g the distance of the star from the vertex thereof; let s be the star's true place, and $sv = \frac{2bg}{c}$: then will sw, at right angles to sv, be parallel to an asymptote (xx) to the curve of aberration uvu, which will always have two branches extending ad infinitum from the point v.——The distance of the asymptote from s depends on the situation of the star with respect to the earth's trajectory.

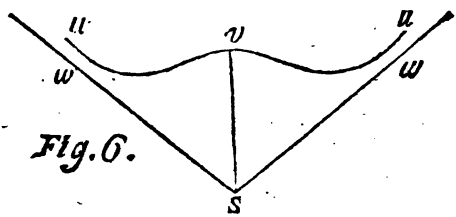


Fig. 6.

If the trajectory be a hyperbola; and, s being the star's true place, sv be $= \frac{bg}{c} \times \overline{1+e}$; the right lines sw, sw, so drawn that the sine of the angle vsw be $= \frac{1}{e}$, will be asymptotes to the curve of aberration uvu.

The two branches of the curve of aberration excurring ad infinitum from the point v (fig. 5 and 6) will be similar when the star is situated any where in a plane at right angles to the plane of the earth's trajectory, the common section of

of the two planes being the principal axis of the trajectory. ——If the star be not so situated, the two branches will take different forms; but their asymptotes will make equal angles with the line sv on different sides thereof, and form one continued right line (as in fig. 5) or intersect each other at s (as in fig. 6).

These conclusions respecting the curve of aberration are so obvious, upon considering the value of st and its position in fig. 4, that it is unnecessary to be more explicit on that head. But it may be worth while to add a remark or two concerning *the angle of aberration*, i. e. the angle under which the line of aberration above found would appear to an observer on the earth; which, from what is done above, may be readily computed when the situation of the earth with respect to the star is known.

It is observable, that, in a parabola, as the earth recedes from the vertex, the angle of aberration will after some time (if not immediately) continually decrease, and at length may become less than any assignable angle: and it will decrease in like manner when the trajectory is a hyperbola, if c be greater than $b\sqrt{ee-1}$. But if c be $= b\sqrt{ee-1}$, the angle of aberration may, as the earth so recedes, increase nearly to 90°; and if c be less than $b\sqrt{ee-1}$, that angle may increase still more.

The Rev. Mr. *Wildbore* sent also a corrected solution on the supposition that the parallax is sensible, &c.

INDEX

Of the Names of the Persons who have Proposed and Answered the Questions.

(N. B. *The numbers affixed to the names shew which quest. each answered or proposed.*)

Boydall,

Edens,

Hingeſton,

Om-

Wilkin,

FINIS.